

Baryons and String Creation from the Fivebrane Worldvolume Action

Curtis G. Callan, Jr.¹, Alberto Güijosa² and
Konstantin G. Savvidy³

Joseph Henry Laboratories
Princeton University
Princeton, New Jersey 08544

Abstract

We construct BPS-exact solutions of the worldvolume Born-Infeld plus WZW action of a D5-brane in the background of N D3-branes. The non-trivial background metric and RR five-form field strength play a crucial role in the solution. When a D5-brane is dragged across a stack of N D3-branes a bundle of N fundamental strings joining the two types of branes is created, as in the Hanany-Witten effect. Our solutions give a detailed description of this bundle in terms of a D5-brane wrapped on a sphere. We discuss extensions of these solutions which have an interpretation in terms of gauge theory multi-quark states via the *AdS/CFT* correspondence.

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¹callan@viper.princeton.edu

²aguijosa@princeton.edu

³ksavvidi@princeton.edu

1 Introduction

In this paper we will construct solutions of the Born-Infeld action for a D5-brane in the background of a stack of N D3-branes. By building on some recent work of Imamura [1], we can find BPS-saturated solutions which presumably correspond to exact solutions of string theory. Using the general approach of [2, 3], these solutions use D5-branes wrapped in various ways to describe branes and strings attached to each other. The primary object we construct this way is a D5-brane joined to N D3-branes by a bundle of fundamental strings. Our solutions give a detailed description of the creation of these strings as the fivebrane is dragged across the threebranes.

As has been pointed out by various people [7, 8], in the context of the anti-de Sitter/conformal field theory (*AdS/CFT*) correspondence there are general reasons to expect N fundamental strings to join together on a D5-brane wrapped on a five-sphere in the throat region of the threebrane geometry (i.e. in the *AdS* geometry). This is the string theory counterpart to the gauge theory $SU(N)$ baryon vertex, representing a bound state of N external quarks. The baryon vertex has been studied in [9], where the strings and the fivebrane are described in terms of separate Nambu-Goto actions. That approach ignores the worldvolume gauge field on the fivebrane. Its inclusion leads to the Born-Infeld action, which allows a unified description of the fivebrane and the strings. When restricted to the *AdS* background, our solutions provide an explicit string theory representation of the baryon vertex. The Born-Infeld action for the worldbrane dynamics of the fivebrane in a threebrane background is an accessible and instructive way to go after the energetics of this problem.

2 The Setup

We set up the equations for the Born-Infeld D5-brane in the background geometry of a stack of N D3-branes. The metric in a standard coordinate system is

$$ds^2 = H(r)^{-1/2}(-dt^2 + dx_{\parallel}^2) + H(r)^{1/2}(dr^2 + r^2 d\Omega_5^2), \quad H(r) = a + R^4/r^4 .$$

We have chosen to express $H(r)$ in terms of an auxiliary constant a , in order to treat the asymptotically flat D3-brane ($a = 1$) and the $AdS_5 \times \mathbf{S}^5$ ($a = 0$) geometries in parallel. The worldvolume action is the Born-Infeld action calculated using the induced metric (including the worldvolume gauge field)

$$g_{\alpha\beta}^{ind} = g_{MN} \partial_{\alpha} X^M \partial_{\beta} X^N + \mathcal{F}_{\alpha\beta},$$

plus the WZW term induced by the five-form field strength; the latter is basically a source term for the worldvolume gauge field. The explicit action we will use is

$$S = -T_5 \int d^6 \xi \sqrt{-det(g^{ind})} + T_5 \int d^6 \xi A_{\alpha} \partial_{\beta} X^{M_1} \wedge \dots \partial_{\gamma} X^{M_5} G_{M_1 \dots M_5} ,$$

where T_5 is the D5-brane tension and the second term is the explicit WZW coupling of the worldvolume gauge field A to the background five-form field strength G . We use the target space time and \mathbf{S}^5 spherical coordinates as worldvolume coordinates for the fivebrane, $\xi_\alpha = (t, \theta_\alpha)$.

We pick a five-sphere surrounding a point on the threebrane stack and look for static solutions of the form $r(\theta)$ and $A_0(\theta)$ (with all other fields set to zero), where θ is the polar angle in spherical coordinates. Non-static solutions are of interest too, but we will not deal with them in this paper. On substituting explicit forms for the threebrane metric and the five-form field strength, the action (for static configurations) simplifies to

$$S = T_5 \Omega_4 \int dt d\theta \sin^4 \theta \{ -r^4 H(r) \sqrt{r^2 + (r')^2 - F_{0\theta}^2} + 4A_0 R^4 \}, \quad (1)$$

where $\Omega_4 = 8\pi^2/3$ denotes the volume of the unit four-sphere.

The gauge field equation of motion following from this action reads

$$(\sin \theta)^{-4} \partial_\theta \left[-\sin^4 \theta \frac{(ar^4 + R^4)E}{\sqrt{r^2 + r'^2 - E^2}} \right] = 4R^4,$$

where we have set $E = F_{0\theta}$ and the right-hand side is the source term coming from the WZW action. It is helpful to repackage this in terms of the displacement D (the variation of the action with respect to E):

$$D = \frac{\sin^4 \theta (ar^4 + R^4)E}{\sqrt{r^2 + r'^2 - E^2}} \Rightarrow \partial_\theta D(\theta) = -4R^4 \sin^4 \theta. \quad (2)$$

Obviously, we can integrate the equation for D to find it as an explicit function of θ . The result is

$$D(\theta) = R^4 \left[\frac{3}{2}(\nu\pi - \theta) + \frac{3}{2} \sin \theta \cos \theta + \sin^3 \theta \cos \theta \right], \quad (3)$$

where the integration constant has been written in terms of a parameter $0 \leq \nu \leq 1$, whose meaning will be elucidated below. Notice that the sign of the WZW term in (1) reflects the choice of a particular fivebrane orientation. Choosing the opposite orientation therefore reverses the sign of the source term in (2), and consequently the sign of D .

Since D , unlike E , is completely unaffected by the form of the function $r(\theta)$, it makes sense to express the action in terms of D and regard the result as a functional for $r(\theta)$. It is best to do this by a Legendre transformation, rewriting the original Lagrangian as

$$U = T_5 \Omega_4 \int d\theta \{ D \cdot E + \sin^4 \theta (ar^4 + R^4) \sqrt{r^2 + (r')^2 - E^2} \}.$$

Integrating the DE term by parts using $E = -\partial_\theta A_0$, one reproduces (with a sign switch) the original Lagrangian. Using (2) we can eliminate E in favor of D to get the desired functional of $r(\theta)$ alone:

$$U = T_5 \Omega_4 \int d\theta \sqrt{r^2 + (r')^2} \sqrt{D^2 + (ar^4 + R^4)^2 \sin^8 \theta}. \quad (4)$$

This functional is reasonably simple, but complicated by the fact that there is explicit dependence on θ . Hence there is no simple energy-conservation first integral that we can use to solve the equations (or at least analyse possible solutions). For future reference, we record the Euler-Lagrange equations that follow from (4):

$$\begin{aligned} \frac{d}{d\theta} \left(\frac{r'}{\sqrt{r^2 + r'^2}} \sqrt{D^2 + (ar^4 + R^4)^2 \sin^8 \theta} \right) &= \frac{r}{\sqrt{r^2 + r'^2}} \sqrt{D^2 + (ar^4 + R^4)^2 \sin^8 \theta} \\ &+ \frac{\sqrt{r^2 + r'^2}}{r} \frac{4ar^4(ar^4 + R^4) \sin^8 \theta}{\sqrt{D^2 + (ar^4 + R^4)^2 \sin^8 \theta}}. \end{aligned} \quad (5)$$

Supersymmetry considerations will allow us to go rather far in analysing the solutions of this formidable-looking equation.

When we discuss solutions in more detail, we will see that it will not be possible to wrap the fivebrane smoothly around a sphere. Even if $r(\theta) \sim r_0$ for most θ , we will find that for $\theta \rightarrow \pi$ (or 0), r shoots off to infinity in a way that simulates a bundle of fundamental strings in the manner described in [2, 3]. Using (4) we can already verify that the energy of such a spike is consistent with its interpretation as a bundle of strings. Suppose that the spike sticks out at $\theta = \pi$; then D will take on some finite value $D(\pi)$ at $\theta = \pi$. As we go into the spike, r' will dominate r and the D term will dominate $\sin^8 \theta$. It is clear then that the spike has a ‘tension’ (i.e. an energy per unit radial coordinate distance) $T_5 \Omega_4 |D(\pi)|$. Using the known facts that $D(\pi) = 3\pi(\nu - 1)R^4/2$ and $T_5 \Omega_4 R^4 = 2NT_f/3\pi$, it follows that the ‘tension’ of the spike is that of n fundamental strings, nT_f , where $n = (1 - \nu)N$. This gives meaning to the parameter ν .

3 Supersymmetry Issues

We are interested in placing a D5-brane in a D3-brane background and finding a structure that looks like fundamental strings attached to the D3-branes. Insight into what is possible is often obtained by looking for brane orientations such that the various brane supersymmetry conditions are mutually compatible for some number of supersymmetries. In type IIB supergravity in ten-dimensional flat space, there are 32 supersymmetries generated by two 16-component constant Majorana-Weyl spinors η_L, η_R of

like parity ($\Gamma_{11}\eta_{L,R} = \eta_{L,R}$). In the presence of branes of various kinds, the number of supersymmetries is reduced by the imposition of further conditions. Explicitly,

$$\begin{aligned}
\text{F - string} & : \quad \Gamma^{09}\eta_L = -\eta_L, & \Gamma^{09}\eta_R = +\eta_R, \\
\text{D3 - brane} & : \quad \Gamma^{0123}\eta_L = +\eta_R, & \Gamma^{0123}\eta_R = -\eta_L, \\
\text{D5 - brane} & : \quad \Gamma^{045678}\eta_L = +\eta_R, & \Gamma^{045678}\eta_R = +\eta_L,
\end{aligned}
\tag{6}$$

where the particular gamma matrix products are determined by the embedding of the relevant branes into ten-dimensional space. For instance, the D3-brane condition refers to a brane that spans the 123 coordinate directions. Conditions can be multiplied by an overall sign by changing brane orientation.⁴ The existence of a BPS state containing more than one type of brane depends on the existence of simultaneous solutions of more than one of the above equations. The relevant point for our discussion is that the conditions precisely as written above, corresponding to mutually perpendicular D3-branes, D5-branes and F-strings, are compatible with eight supersymmetries ($\mathcal{N} = 2$ in usual parlance). The supersymmetry argument suggests that mutually orthogonal branes spanning a total of eight dimensions joined by a fundamental string running along the one remaining dimension (perpendicular to both branes) should in fact be a stable BPS state. An interesting aspect of our Born-Infeld worldvolume approach is that we will explicitly see how the fundamental strings are created and destroyed as the D-branes are moved past each other in the ninth direction (the Hanany-Witten effect [10, 11, 12]).

The above analysis has been carried out in flat space. To make contact with the *AdS/CFT* correspondence, one would want to consider N superposed D3-branes with N large, in which case the background geometry is not flat and the supersymmetry analysis given above is at least incomplete. Imamura [1] has analysed the supersymmetry conditions associated with a D5-brane stretched over some surface in the ‘throat’ of the D3-brane (where the geometry is $AdS_5 \times S^5$ and there is a flux of the RR five-form field strength through the S^5). There are several new features here: first, the unbroken supersymmetries of type IIB supergravity in this particular background are 32 position-dependent spinors (as opposed to constant spinors in flat space); second, because of the RR five-form field strength, there is a worldbrane gauge field induced on the worldvolume of the D5-brane; third, the condition that a particular element of the D5-brane worldvolume preserve some supersymmetry involves the local orientation of the brane, the value of the induced worldvolume gauge field and the local value of the supergravity supersymmetry spinors. Since the D5-brane is embedded in some nontrivial way in the geometry, the supersymmetry condition is in principle different

⁴Notice, however, that to maintain a supersymmetric configuration one must simultaneously reverse the orientation of two of the three objects.

at each point on the worldvolume and it is far from obvious that it can be satisfied everywhere.

However, Imamura [1] was able to show that these conditions boil down, at least in the AdS_5 ($a = 0$) background, to a first-order equation for the embedding of the D5-brane into the space transverse to the D3-brane stack. In our language, his BPS condition can be written

$$\frac{r'}{r} = \frac{R^4 \sin^5 \theta + D(\theta) \cos \theta}{R^4 \sin^4 \theta \cos \theta - D(\theta) \sin \theta} , \quad (7)$$

where $r = r(\theta)$ is the D5-brane embedding in the transverse space and $D(\theta)$ is the ‘displacement’ field describing how the worldvolume gauge field varies from point to point. It is easy to show that any function $r(\theta)$ that satisfies this condition automatically satisfies the Euler-Lagrange equation (5) with $a = 0$; in that sense it is a first integral of the usual second-order equations. Note that, as mentioned above, the structure of the action is such that there is no trivial energy first integral. The BPS condition (7) can be integrated analytically to obtain a two-parameter family of curves that describe BPS embeddings of a D5-brane into the $AdS_5 \times \mathbf{S}^5$ geometry. These solutions will be discussed in the next section.

We are also interested in exploring the analogous solutions in the full asymptotically flat D3-brane background ($a = 1$). In this background the interpretation and energetics of solutions should be quite straightforward. What is less obvious is how to find BPS solutions. To follow Imamura’s approach, one would first find the supersymmetry spinors in the D3-brane background, use them to construct local supersymmetry conditions for an embedded D5-brane and from this find the condition on the embedding for there to be a global worldvolume supersymmetry. This is no doubt perfectly feasible but we have not had the patience to try it. Instead, we have simply guessed a generalization of the $AdS_5 \times \mathbf{S}^5$ BPS condition that automatically provides a solution of the Euler-Lagrange equations in the full D3-brane background. The generalized BPS condition is obtained by making the (very plausible) replacement $R^4 \rightarrow R^4 + r^4$ in (7),

$$\frac{r'}{r} = \frac{(R^4 + r^4) \sin^5 \theta + D(\theta) \cos \theta}{(R^4 + r^4) \sin^4 \theta \cos \theta - D(\theta) \sin \theta} . \quad (8)$$

It is easy to verify, using only (2), that this equation implies the full Euler-Lagrange equation (5) with $a = 1$, so it is certainly a first integral. Given its derivation, it is almost certainly the BPS condition as well. It is rather surprising that things work so smoothly, and we take this as another evidence of the special nature of the D3-brane background. The first-order equation (8) must be integrated numerically (as far as we know) and yields a two-parameter family of solutions whose structure is quite non-trivial. Exploration of these and the AdS solutions will be the subject of the rest of the paper.

Before closing this section we note that in either background one can obtain an ‘alternative’ BPS condition by reversing the signs in front of D in equations (7) or (8). The resulting condition would guarantee the preservation of a different set of supersymmetries, albeit just as many. In order to still have a first integral of the Euler-Lagrange equation (5), D must satisfy (2) with the opposite sign for the source term. Such oppositely oriented fivebrane configurations will actually have the same embedding $r(\theta)$.

4 Solution Overview and Interpretation

4.1 AdS background: Born-Infeld Baryons

We start with a discussion of the solutions of the AdS BPS equation (7) for the supersymmetric embedding of a fivebrane in the $AdS_5 \times \mathbf{S}^5$ geometry, with topology $\mathbf{S}^4 \times \mathbf{R}$. Fortunately, the BPS equation has the following simple analytic solution:⁵

$$r(\theta) = \frac{A}{\sin \theta} \left[\frac{\eta(\theta)}{\pi(1-\nu)} \right]^{1/3}, \quad \eta(\theta) = \theta - \pi\nu - \sin \theta \cos \theta, \quad (9)$$

where the scale factor A is arbitrary, and ν is the integration constant in (3). The freedom of changing A is a direct consequence of the scale invariance of the AdS background: if $r(\theta)$ is a solution of (7), then so is $\lambda r(\theta)$ for any λ . Note that $\eta > 0$ (so that the solution makes sense) only for $\theta_{min} < \theta < \pi$, where θ_{min} is defined by

$$\pi\nu = \theta_{min} - \sin \theta_{min} \cos \theta_{min}. \quad (10)$$

This critical angle increases monotonically from zero to π as ν increases from zero to one. Furthermore, when $\theta_{min} > 0$, $r(\theta_{min}) = 0$, a fact whose consequences will be explored below.

The fact that (if $\nu \neq 1$) the solution diverges as $r \sim A/(\pi - \theta)$ when $\theta \rightarrow \pi$ means that a polar plot of $r(\theta)$ has, asymptotically, the shape of a ‘tube’ of radius A . (This way of describing the surface is a bit misleading as to the intrinsic geometry, but helps in visualization.) This tube is to be interpreted as a bundle of fundamental strings running off to infinity in the space transverse to the D3-branes. As explained in section 2, the asymptotic ‘tension’ of the tube equals that of $(1 - \nu)N$ fundamental strings. For the classical solutions ν is a continuous parameter, but at the quantum level ν should obey the quantization rule $\nu = n/N$.

In Fig. 1 we have plotted (9) for some representative values of ν . Consider first the special case $\nu = 0$, which yields a tube with the maximal asymptotic tension NT_f

⁵We thank Ø. Tafjord for help in finding this solution.

and corresponds to the classic ‘baryon’ vertex. In this case the solution starts at a finite radius $r(0) = (2/3\pi)^{1/3}A$, with $r'(0) = 0$, and then $r(\theta)$ increases monotonically with θ . The initial radius $r(0)$ represents another way of setting the overall scale of this scale-invariant solution. The fact that the fivebrane surface stays away from the horizon at $r = 0$ suggests that it is well-decoupled from degrees of freedom living on the threebranes.

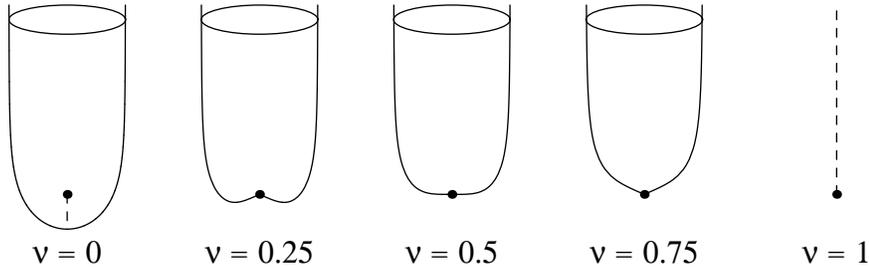


Figure 1: Polar plots of $r(\theta)$ for AdS ‘tube’ solutions corresponding to $(1 - \nu)N$ strings (with $\theta = \pi$ at the top of the plots). A cross-section of each ‘tube’ is an S^4 .

As far as the BPS equation is concerned, it seems to make sense to consider the $\nu > 0$ solutions as well. They have instructive features, although we will eventually conclude that they are on a less sound footing than their $\nu = 0$ cousins. For large r , the solution asymptotes to the familiar tube with a tension corresponding to $(1 - \nu)N < N$ strings: it corresponds to a general multi-quark state of a $U(N)$ gauge theory. As mentioned above, for $\nu > 0$, the surface intersects $r = 0$ at an angle $\theta_{min} > 0$ defined by (10), leading to the cusp-like configurations displayed in Fig. 1. Note that, because the $r \rightarrow 0$ cusp has a finite opening angle, the fivebrane does not capture all of the five-form flux: this is closely related to the fact that the asymptotic tension is $(1 - \nu)N$ and not N .

As $\nu \rightarrow 0$, the opening angle $\theta_{min} \rightarrow 0$. The approach to the $\nu = 0$ solution, which does not contact $r = 0$, is achieved, as shown in Fig. 1, via a ‘tensionless string’ connecting the minimum radius of the $\nu = 0$ solution to $r = 0$ (indicated as a dotted line in the figure). At the other extreme, $\nu \rightarrow 1$, one has $\theta_{min} \rightarrow \pi$, and the solution collapses to a similar ‘phantom string’, this time running from $r = 0$ to infinity.

One can compare the total energy of these configurations to that of $(1 - \nu)N$ fundamental strings (this was done in [1] for the case of $\nu = 0$). Using the solution (9) in expression (4), the energy of the fivebrane up to an angular cutoff θ_{max} can be put in the form

$$U(\theta_{max}) = NT_f \frac{A}{\pi} \int_{\theta_{min}}^{\theta_{max}} d\theta \left[\frac{\eta(\theta)}{\pi(1 - \nu)} \right]^{1/3} \left\{ \frac{\eta(\theta)}{\sin^2 \theta} - \frac{4}{3} \sin \theta \cos \theta + \frac{4 \sin^4 \theta}{9\eta(\theta)} \right\}. \quad (11)$$

The fundamental string energy, on the other hand, for strings extending from the origin to a radial cutoff $r_{max} = r(\theta_{max})$, is simply $E_{str}(\theta_{max}) = (1 - \nu)NT_f r(\theta_{max})$. It is easy to check numerically that $E_{str}(\theta_{max}) - U(\theta_{max}) \rightarrow 0$ as $\theta_{max} \rightarrow \pi$ ($r_{max} \rightarrow \infty$). The Born-Infeld fivebrane ‘tubes’ can be therefore regarded as threshold bound states of $(1 - \nu)N$ fundamental strings. We emphasize that this holds for any value of the scale parameter A : as $\theta_{max} \rightarrow \pi$, the energy $U(\theta_{max})$ becomes independent of A . The parameter A is therefore a modulus in the space of equal-energy solutions.

A complication for the interpretation of these solutions is that, in general (specifically, when $\nu \neq 0, 1/2, 1$), the total five-form flux captured by the fivebrane differs from the number of fundamental strings, $(1 - \nu)N$, indicated by the asymptotic tension or total energy. The fundamental string charge is the source of the displacement field D , and we can rearrange (2) to show that a fivebrane that runs from $\theta = \theta_{min}$ to $\theta = \pi$ intercepts a total five-form flux

$$Q_{flux} = -\frac{2N}{3\pi R^4} [D(\pi) - D(\theta_{min})] = (1 - \nu)N + \frac{2N}{3\pi} \sin^3 \theta_{min} \cos \theta_{min} .$$

From the value of the tension, we would have expected a total charge $Q_{tot} = (1 - \nu)N$ on the D5-brane. The difference,⁶

$$Q_{missing} = -\frac{2N}{3\pi R^4} \sin^3 \theta_{min} \cos \theta_{min} , \quad (12)$$

is nonzero for $\nu \neq 0, 1/2, 1$ and presumably must be accounted for by a point charge at $r = 0$. Since $r = 0$ is where the fivebrane makes contact with the threebranes, this reminds us that, in order to be fully consistent, we should take into account the possibility of exciting the threebrane worldvolume $U(N)$ gauge fields when we attach fundamental strings to the D3-branes (as in [2, 3]). The case of N strings ($\nu = 0$) is special since they can be in a $U(N)$ singlet which will decouple from the D3-brane worldvolume gauge theory. When $\nu \neq 0$, we are talking about a collection of strings that cannot be $U(N)$ neutral and must excite the D3 gauge fields, which will in turn react back on the metric. Since we have not allowed for this possibility in our construction, the detailed features of our solutions with $Q_{missing} \neq 0$ have to be taken with a certain grain of salt. The case $\nu = 1/2$ is peculiar: it corresponds to $N/2$ strings and so cannot form a $U(N)$ singlet, yet has $Q_{missing} = 0$. We are not sure that it really has the same status as the true singlet $\nu = 0$ solution.

In light of the *AdS/CFT* correspondence [4, 5, 6], the above results are expected to have a gauge theory interpretation. As discussed by several people [7, 8], a baryon (a bound state of N external quarks) in the $SU(N)$ $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory corresponds, in the dual *AdS* description, to N fundamental strings

⁶ It should be clear that Q_{flux} , Q_{tot} , and $Q_{missing}$ all change sign if we reverse the fivebrane orientation.

which join together on a D5-brane wrapped on an \mathbf{S}^5 at some radius. The Born-Infeld $\nu = 0$ fivebrane configuration described above provides a detailed representation of such a baryon. In particular, the absence of binding energy is as expected for a BPS threshold bound state in the $\mathcal{N} = 4$ theory. Our other solutions with $\nu = n/N$ ($0 < n < N$) are also BPS and correspond to threshold bound states of $N - n$ quarks. The existence of color non-neutral states with finite (renormalized) energy is perfectly reasonable in a non-confining theory. To start learning something interesting about these states, we would have to go beyond mere energetics and ask some dynamical questions. To be absolutely clear, we emphasize that in every case discussed here the quarks in the gauge theory are all at the same spatial location.

The solutions that we have discussed so far are naturally restricted to the range of angles $\theta_{min} \leq \theta \leq \pi$ where $\eta(\theta) > 0$. We will call them ‘upper tubes’. A simple modification of (9) is valid for the complementary angular range $0 \leq \theta \leq \theta_{min}$ where $\eta(\theta) < 0$:

$$\tilde{r}(\theta) = \frac{\tilde{A}}{\sin \theta} \left[\frac{-\eta(\theta)}{\pi \nu} \right]^{1/3}. \quad (13)$$

This expression is singular at $\theta = 0$ (where $\tilde{r}(\theta) \sim \tilde{A}/\theta$) and meets the origin at $\theta = \theta_{min}$. It represents a downward-pointing tube of ‘radius’ \tilde{A} whose shape and tension are the same as those of an upward-pointing tube with parameter $1 - \nu$. In other words, $\tilde{r}(\theta; \nu) = r(\pi - \theta; 1 - \nu)$. This ‘lower tube’ solution intercepts a total flux $Q_{flux} = -\nu N + 2N \sin^3 \theta_{min} \cos \theta_{min} / 3\pi$. From the tension of this configuration, we would have expected a total charge $Q_{tot} = -\nu N$, so there is a charge $Q_{missing}$ localized at the origin which is again given by (12). If the ‘upper tube’ solution corresponds to some number of quarks, the ‘lower tube’ solution corresponds to some number of antiquarks.

Finally we want to speculate about constructing new solutions by combining the $\nu \neq 0$ solutions we have been discussing. Specifically, we are interested in obtaining configurations for which the peculiar charge at the origin cancels. Inspection of (12) shows that this can be achieved by merging two tubes whose opening angles are complementary. Using equation (10) this means that if one of the tubes has parameter ν , the other one must have parameter $1 - \nu$. Taking into account the possibility of using ‘upper’ or ‘lower’ solutions, one is thus led to two types of configurations, illustrated in Fig. 2.

The combination of two upper tubes with parameters ν and $1 - \nu$ yields a baryon-like configuration corresponding to a total of N quarks. This system differs from the $\nu = 0$ baryon of Fig. 1 in that the ‘strings’ have been separated into two distinct coaxial tubes. It is interesting to note that this combined structure can be obtained as a single solution of (7), with a unique value of ν , by formally continuing $r(\theta)$ in (9) beyond $\theta = \theta_{min}$ (where $r = 0$) to negative values of r . The continued solution, depicted in

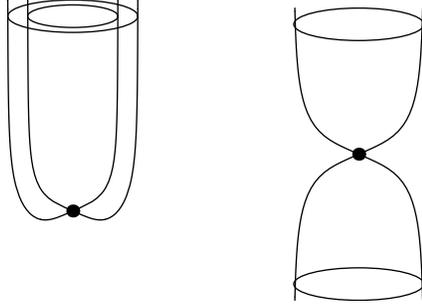


Figure 2: Upper/upper and upper/lower tube combinations. These configurations have vanishing charge at the origin (see text).

Fig. 2, can be interpreted as a single surface which intersects itself at the origin. In this interpretation the parameter ν is an additional modulus of the baryon, controlling how many strings (out of the total of N) ‘lie’ in each tube.

If instead one puts together an upper ν and a lower $1-\nu$ solution, the result represents $1-\nu$ strings which extend from $r = \infty, \theta = 0$ to $r = \infty, \theta = \pi$, and run through the origin. In the gauge theory language this describes a state with $1-\nu$ quarks and the same number of antiquarks. (This is still BPS, because the quarks and antiquarks have opposite $SU(4)$ quantum numbers.) The total charge of the state vanishes.

Judging from the cancellation of the charge at the origin, these combined solutions would appear to have the same status as the baryon. On the other hand, it is unclear to what extent these superposed tubes can be regarded as a single object, given that they are ‘connected’ only at the horizon $r = 0$. The issue is whether fluctuations can propagate from the lower tube to the upper tube in finite ‘gauge theory’ time. Since they would have to pass through a horizon at $r = 0$ to do so, this would appear to be impossible, at least at the classical level. This issue merits further study.

4.2 D3-brane background: Hanany-Witten effect

So far, we have looked at the static solutions of D5-branes in the $AdS_5 \times \mathbf{S}^5$ geometry of the ‘throat’ region of the exterior geometry of a large number of D3-branes. As we now know, this limit gives us a supergravity description of $\mathcal{N} = 4$ SYM theory. We can also shed light on some old string theory questions by studying the same types of configurations in the full asymptotically flat geometry of multiple D3-branes.

To examine the character of the solutions in the asymptotically flat D3-brane back-

ground, it is convenient to parametrize the solution by $z = z(\rho)$, where $\rho = r \sin \theta$, and $z = -r \cos \theta$. In these variables, adapted to flat space, the BPS condition (8) reads

$$z'(\rho) = \frac{-D(\arctan(-\rho/z))}{\rho^4 \left(1 + \frac{R^4}{(\rho^2+z^2)^2}\right)}. \quad (14)$$

Solutions to this equation describe D5-brane configurations which asymptote to a flat plane as $\rho \rightarrow \infty$ (equivalently, $\theta \rightarrow \pi/2$). The leading asymptotic behavior following from (14) is

$$z(\rho) = z_{max} + \frac{D(\pi/2)}{3\rho^3} + \mathcal{O}\left(\frac{R^4}{\rho^4}\right), \quad (15)$$

where z_{max} denotes the transverse position of the flat brane. We will be interested in studying how the solution changes as we vary z_{max} .

Figures 3 and 4 show the configurations obtained by integrating (14) numerically for $\nu = 0$ and $\nu = 1/2$ and for a few representative values of z_{max} . The stack of N D3-branes is at the origin, and extends along directions perpendicular to the figure. For any value of z_{max} , the D5-brane captures the same fraction of the total five-form flux, which (in conjunction with a possible point charge at the origin, as discussed in the previous subsection) effectively endows the D5-brane with a total of $(1/2 - \nu)N$ units of charge. Note the shift of $N/2$ units of charge, compared with the analogous situation in *AdS* space: this happens because the asymptotic region of the brane is now at $\theta = \pi/2$ rather than $\theta = \pi$. This will have interesting consequences.

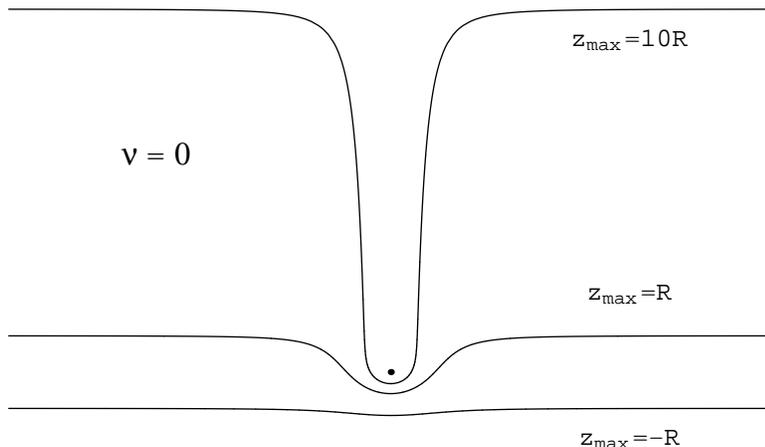


Figure 3: Solutions describing the creation of N fundamental strings as a D5-brane is dragged upward, across a stack of D3-branes. The number of strings connecting the two types of branes changes from 0 to N .

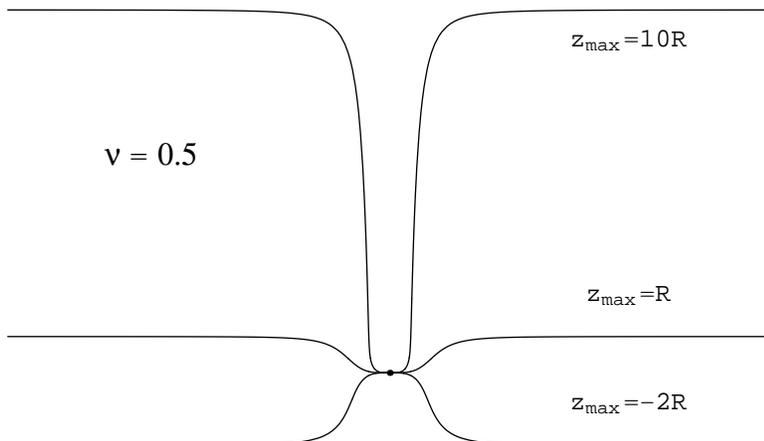


Figure 4: Solutions describing the creation of N fundamental strings as a D5-brane is dragged across a stack of D3-branes. The number of strings connecting the two types of branes changes from $-N/2$ to $+N/2$ (the signs indicate whether the strings originate or terminate on the fivebrane).

Consider first the situation for $\nu = 0$, described graphically in Fig. 3. As $z_{max} \rightarrow -\infty$, the charge density vanishes, and the D5-brane of course becomes flat. As one approaches the stack of threebranes from below ($z_{max} \rightarrow 0_-$), the charge becomes more and more localized near the center of the fivebrane, and the configuration becomes slightly deformed, bending away from the origin. As z_{max} increases, the D5-brane remains ‘hung-up’ on the D3-brane stack at $r = 0$ and a tube of topology $\mathbf{S}^4 \times \mathbf{R}$ gets drawn out. The total charge of the tube itself approaches N as it gets longer and longer and it becomes indistinguishable from a bundle of N fundamental strings. Curiously, when the bundle eventually connects to the flat D5-brane, a region of negative five-form flux is encountered and the total charge intercepted by the fivebrane drops to $N/2$ (for any z_{max}). Altogether, then, this family of solutions provides a very concrete picture of the creation of fundamental strings as a fivebrane is dragged over a stack of threebranes, the Hanany-Witten effect [10, 11, 12].

For $\nu > 0$ the story is modified in exactly the same way as in the previous subsection. For either sign of z_{max} , the fivebrane now reaches the origin, $r = 0$, at an angle $\theta = \theta_{min}$ given in terms of ν by equation (10). As $z_{max} \rightarrow -\infty$ the solution describes now a fivebrane connected by νN strings to the stack of threebranes. For definiteness, assume the choice of sign for D (i.e., the orientation of the fivebrane) is such that the strings emanate from the D5-brane and terminate on the D3-branes. Upon moving past $z_{max} = 0$, N strings directed towards the fivebrane are created, and as $z_{max} \rightarrow \infty$

$(1 - \nu)N$ strings directed away from the threebranes extend between the two types of branes. The case $\nu = 1/2$ is portrayed in Fig. 4.

It is instructive to compare the solution presented above to the description of fundamental strings attached to a fivebrane as a Coulomb solution of the fivebrane Born-Infeld theory [2, 3]. In the latter case the parent brane is embedded in flat space and the worldvolume gauge field is simply that of a point charge. For n units of charge, the spike configuration that protrudes from the brane at the location of the charge is of the form

$$z(\rho) = z_{max} - \frac{nc_5}{\rho^3}, \quad (16)$$

where $c_5 = 2\pi^2 g_s (\alpha')^2$ is the quantum of charge. Writing this in terms of the threebrane throat radius $R = (4\pi N g_s (\alpha')^2)^{1/4}$, one can readily verify that the asymptotic form (15) agrees with the solution (16) for $n = (1/2 - \nu)N$ strings. This is precisely as one would expect from the above discussion, for the entire fivebrane captures precisely a fraction $(1/2 - \nu)$ of the total five-form flux. By the same token, it is clear that the present solution is of a more complex nature than that of [2, 3]. The configuration discussed here corresponds roughly to a solution which is locally of the type (16), but where the charge n varies as one changes position on the fivebrane.

One of the more confusing features of the Hanany-Witten effect is its energetics: does the created string exert a force and, if so, how is that consistent with the BPS property? We can shed some light on this by computing the energy of our configurations. In terms of the $z(\rho)$ parametrization, and using the fact that $T_5 \Omega_4 R^4 = 2NT_f/3\pi$, equation (4) becomes

$$U = NT_f \frac{2}{3\pi} \int d\rho \sqrt{1 + (\partial_\rho z)^2} \sqrt{D^2 + \rho^8 \left[1 + \frac{R^4}{(\rho^2 + z^2)^2}\right]^2}. \quad (17)$$

We can use the BPS condition (14) to express the energy integrand for our solutions solely as a function of z and ρ ,

$$U = NT_f \frac{2}{3\pi} \int d\rho \left\{ \frac{D^2}{\rho^4 \left[1 + \frac{R^4}{(\rho^2 + z^2)^2}\right]} + \rho^4 \left[1 + \frac{R^4}{(\rho^2 + z^2)^2}\right] \right\}. \quad (18)$$

Now, the energy of the infinite D5-brane is evidently divergent, so (18) must be regularized. If we do so by placing a cutoff ρ_c on ρ , the leading and subleading contributions to (18) are clearly quintic and linear in ρ_c , respectively. The offending terms, however, are independent of z_{max} , so we choose to simply drop them (thereby removing an infinite constant from U). Altogether, then, we subtract $\rho^4 + R^4$ from the integrand of (18) to obtain the expression

$$\hat{U}(z_{max}, \rho_c) = NT_f \frac{2}{3\pi} \int_0^{\rho_c} d\rho \left\{ \frac{D^2}{\rho^4 \left[1 + \frac{R^4}{(\rho^2 + z^2)^2}\right]} - \frac{2\rho^2 z^2 + z^4}{(\rho^2 + z^2)^2} \right\}, \quad (19)$$

which is finite as the cutoff is removed. Using the numerical solutions, one finds that in fact $\hat{U} \rightarrow (1/2 - \nu)NT_f z_{max}$ for all z_{max} as $\rho_c \rightarrow \infty$.

Since the difference between U and \hat{U} is a constant, it readily follows that there is a net constant force on the fivebrane, independent of z_{max} :

$$\frac{\partial}{\partial z_{max}} U(z_{max}, \rho_c) \rightarrow \left(\frac{1}{2} - \nu\right) NT_f \quad \text{as } \rho_c \rightarrow \infty. \quad (20)$$

The total force equals the tension of $(1/2 - \nu)N$ fundamental strings as a consequence of the fact that the full configuration carries a total of $(1/2 - \nu)N$ units of charge. It might seem surprising at first that there is a force on the fivebrane even for $\nu = 0$ and $z_{max} < 0$ (i.e., before the D5-brane crosses the D3-brane stack and a tube of strings is created), but one must keep in mind that even then there is a charge on the brane, and consequently a position-dependent energy. After the fivebrane is moved past the threebranes, to $z_{max} > 0$, a bundle of fundamental strings is created, and this bundle pulls down on the fivebrane with a force which tends to NT_f as $z_{max} \rightarrow \infty$. The net force on the brane is still $(1/2 - \nu)NT_f$, however, because the outer portion of the brane now carries negative charge, as a consequence of which there is an additional, upward force on the brane. Our approach thus makes it absolutely clear that, contrary to the naive expectation, there is no discontinuity in the force as the branes cross. One is able to find static solutions despite the presence of a constant force because the D5-brane is infinitely massive, and therefore will not move. If desired, this constant force can be cancelled by placing $(1 - 2\nu)N$ additional D3-branes at $z = -\infty$.

An alternative way to reach the same conclusions is to compute the force by cutting off (17) at ρ_c and differentiating with respect to z_{max} under the integral (regarding $z = z(\rho; z_{max})$). After an integration by parts and an application of the Euler-Lagrange equation, one is left only with a boundary term, which yields the analytic expression

$$\frac{\partial U}{\partial z_{max}}(z_{max}, \rho_c) = NT_f \frac{2}{3\pi} \left\{ \frac{\partial_\rho z}{\sqrt{1 + (\partial_\rho z)^2}} \sqrt{D^2 + \rho^8 \left[1 + \frac{R^4}{(\rho^2 + z^2)^2}\right]^2} \frac{\partial z}{\partial z_{max}} \right\} \Big|_{\rho_c}. \quad (21)$$

Using (21) and (15) one can again conclude that the force on the fivebrane approaches $(1/2 - \nu)NT_f$ as $\rho_c \rightarrow \infty$ at a fixed z_{max} . Furthermore, using the numerical solution in conjunction with (21), one can compute the force on the $\rho < \rho_c$ portion of the brane, for any value of ρ_c . For any fixed ρ_c , it is easy to see that the force tends to $(1 - \nu)NT_f$ (νNT_f) as $z_{max} \rightarrow \infty$ ($z_{max} \rightarrow -\infty$). Taking the limit this way picks out the stress on the string tube part of the configuration and yields the expected tension of $(1 - \nu)N$ (νN) fundamental strings. Nonetheless, the total asymptotic stress on the D5-brane is smaller by $N/2$ fundamental string units, due to the extra five-form flux intercepted by the flat part of the D5-brane.

Notice that for $\nu = 1/2$ the net force on the D5-brane vanishes. This is because in that case the total charge on the brane is zero. As a result, the ρ^{-3} term in (15) has a vanishing coefficient. This configuration has $z(\rho = 0) = z'(\rho = 0) = 0$, $\theta_{min} = \pi/2$, and $D(\theta_{min}) = 0$. It is only for this and the $\nu = 0$ and $\nu = 1$ cases that the point charge at the origin vanishes. This solution (depicted in Fig. 4) describes a bundle of $N/2$ strings which flip their orientation as the fivebrane to which they are attached is moved above or below the threebrane stack. The number of attached strings still changes by N , from $-N/2$ to $+N/2$, as the D5-brane is pulled through the stack. This configuration is thus a realization of the ‘half-string’ ground-state of the system described in [12]. For reasons explained in that paper, and confirmed by our energy analysis, this is the only solution which is in a state of neutral equilibrium.

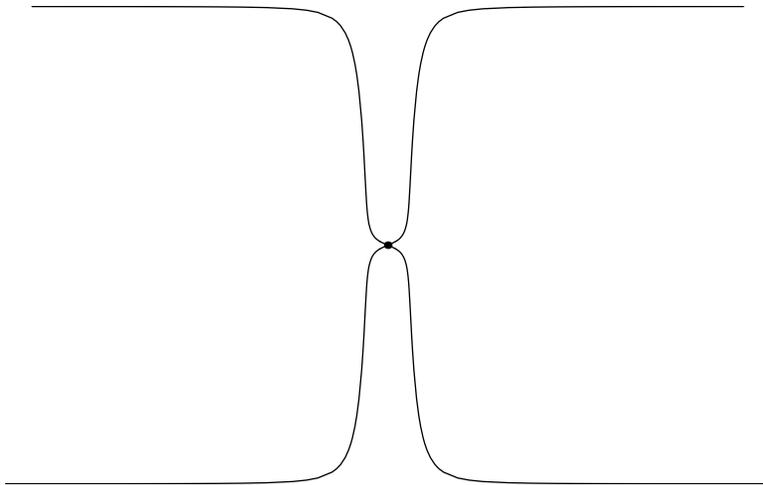


Figure 5: Solution describing a system of two parallel D5-branes connected by $(1-\nu)N$ fundamental strings which run through the D3-branes at $r = 0$. A ‘point W-boson charge’ lies at the origin.

Just as in the previous subsection, one could imagine combining solutions to obtain configurations in which the charge at the origin vanishes. We will focus attention here on the possibility of superposing two solutions with parameters $\nu, z_{max} > 0$ and $1 - \nu, -z_{max}$, respectively. The complete structure obtained this way is illustrated in Fig. 5, and corresponds to a configuration in which two infinite parallel D5-branes with the same orientation, located at $\pm z_{max}$, are connected by $(1-\nu)N$ fundamental strings running through the N D3-branes at the origin. Something interesting has happened here: we have constructed an excitation of a system of two parallel fivebranes, something which should more properly be described by the non-abelian $SU(2)$ Born-Infeld action. We have achieved this effect (perhaps illegitimately!) by gluing together two

$U(1)$ solutions at the singularity provided by the D3-branes. The result is reminiscent of the configurations examined in [13], where the SYM Prasad-Sommerfield monopole solution is used to describe a system of two parallel D3-branes connected by a string. Following that interpretation, the point charges of the component solutions at the origin should be understood not to cancel, but to combine instead into a ‘point W-boson charge’ which interpolates, at no cost in energy, between the two $U(1)$ s of the overall broken $U(2)$ symmetry.

The threebrane background geometry evidently plays a role in facilitating the construction described above. Nevertheless, since the ‘strings’ in the solution are interpreted as merely passing through the D3-branes, it is natural to conjecture that there exist neighboring static fivebrane configurations in which the strings miss the origin. Deforming the system in this manner it would be possible to move the connecting strings arbitrarily far away from the threebranes, thereby producing the analogous flat space configuration. It would be very interesting to pursue this issue further.

5 Conclusions

It is surprising how many subtle aspects of the dynamics of branes and strings can be illuminated by the Born-Infeld worldvolume gauge theory approach. The principal focus of this paper has been the phenomenon of string creation when fivebranes are dragged across threebranes, but there are other issues which we did not discuss here, and which might repay study. It would be straightforward to look at the structurally very similar non-BPS configurations of D5-branes in non-extremal D3-brane backgrounds. Via the *AdS/CFT* correspondence, this would tell us about multi-quark states in confining gauge theories. It would also be very instructive to study the case of multi-center BPS D3-brane configurations in order to understand the effects of Higgs breaking of the underlying $U(N)$ gauge symmetry. We hope to pursue these and other subjects in future work.

Acknowledgements

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