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VALIDITY OF THE TAYLOR HYPOTHESIS FOR LINEAR KINETIC WAVES IN THE WEAKLY COLLISIONAL SOLAR WIND

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ABSTRACT

The interpretation of single-point spacecraft measurements of solar wind turbulence is complicated by the fact that the measurements are made in a frame of reference in relative motion with respect to the turbulent plasma. The Taylor hypothesis—that temporal fluctuations measured by a stationary probe in a rapidly flowing fluid are dominated by the advection of spatial structures in the fluid rest frame—is often assumed to simplify the analysis. But measurements of turbulence in upcoming missions, such as *Solar Probe Plus*, threaten to violate the Taylor hypothesis, either due to slow flow of the plasma with respect to the spacecraft or to the dispersive nature of the plasma fluctuations at small scales. Assuming that the frequency of the turbulent fluctuations is characterized by the frequency of the linear waves supported by the plasma, we evaluate the validity of the Taylor hypothesis for the linear kinetic wave modes in the weakly collisional solar wind. The analysis predicts that a dissipation range of solar wind turbulence supported by whistler waves is likely to violate the Taylor hypothesis, while one supported by kinetic Alfvén waves is not.

Subject headings: turbulence — solar wind

1. INTRODUCTION

Developing a thorough understanding of turbulence in space and astrophysical plasmas is a grand challenge that has the potential to impact a wide range of research frontiers in plasma physics, space physics, and astrophysics. The effort to unravel the complex plasma physical processes that govern the evolution and impact of plasma turbulence is greatly aided by the ability to make *in situ* spacecraft measurements of turbulence in the weakly collisional solar wind plasma. But the interpretation of spacecraft measurements of the turbulent plasma and electromagnetic field fluctuations is complicated by the unavoidable fact that the measurements are made in a frame of reference (the spacecraft frame) that is in relative motion with respect to the frame of reference of the solar wind plasma (the plasma frame).

For each spatial Fourier mode with wavevector $\mathbf{k}$, the transformation from the frequency ω in the plasma frame to the frequency ω_{sc} in the spacecraft frame yields the relation $\omega_{sc} = \omega + \mathbf{k} \cdot \mathbf{v}_{sw}$, as shown in §2.1. The first term on the right-hand side is the plasma-frame frequency, and the second term is the advection term accounting for the frequency arising from the sweeping of a spatial fluctuation with wavevector $\mathbf{k}$ past the spacecraft at velocity $\mathbf{v}_{sw}$. Taking advantage of the typically super-Alfvénic velocity of the solar wind, $v_{sw} \gg v_A$, observers have historically adopted the Taylor hypothesis (Taylor 1938), assuming that $|\omega| \ll |\mathbf{k} \cdot \mathbf{v}_{sw}|$, so that the spacecraft-frame frequency of fluctuations is interpreted to be related directly to the wavenumber of the spatial fluctuations in the plasma frame, $\omega_{sc} \simeq \mathbf{k} \cdot \mathbf{v}_{sw}$ (Matthaeus & Goldstein 1982; Perri & Balogh 2010).

The validity of using the Taylor hypothesis to transform from frequency to wavenumber is threatened as upcoming spacecraft missions push turbulence measurements into new regimes where the Taylor hypothesis may be violated. Specifically, the Taylor hypothesis may fail in two distinct regimes: (i) *Slow Flow Regime*: when the upcoming *Solar Probe Plus* mission samples up to and within the Alfvén critical point, the solar wind flow velocity will drop to and fall below the Alfvén velocity, $v_{sw} \lesssim v_A$; and (ii) *Dispersive Regime*: the high cadence of turbulence measurements on the upcoming *Magnetospheric Multiscale (MMS)*, *Solar Orbiter*, and *Solar Probe Plus* missions effectively samples spatial fluctuations of ever smaller scale, length scales where the linear plasma response becomes dispersive, leading to a more rapid than linear increase of plasma-frame fluctuation frequency with increasing spatial wavenumber (decreasing length scale). In either of these cases, the temporal variation of the turbulent fluctuations in the plasma frame may become non-negligible compared to the temporal variation due to the sweeping of spatial structure past the spacecraft, $|\omega| \gtrsim |\mathbf{k} \cdot \mathbf{v}_{sw}|$, thereby violating the Taylor hypothesis. The aim of this paper is to explore the limits of validity of the Taylor hypothesis in the study of turbulence in the solar wind.

To evaluate the validity of the Taylor hypothesis for solar wind turbulence, it is necessary to estimate the plasma-frame frequency of the turbulent fluctuations. A fundamental premise of this study is that *the frequency of the turbulent fluctuations is well characterized by the frequency of the linear waves supported by the solar wind plasma*. This concept is contained within a more general hypothesis for the modeling of plasma turbulence, the *quasilinear premise* (Klein et al. 2012; Howes et al.

2014). The quasilinear premise[1] states simply that *some* characteristics of turbulent fluctuations in a magnetized plasma may be usefully modeled by a superposition of randomly-phased, linear wave modes. The nonlinear interactions inherent to the turbulent dynamics may be considered to transfer energy among these linear wave modes—therefore, the model is quasilinear. Here, we simply adopt the premise that linear wave modes adequately characterize the frequency response of the turbulent plasma, and we defer a detailed discussion of the quasilinear premise and supporting evidence to a subsequent work (Howes et al. 2014). Note that the weakly collisional nature of the solar wind plasma requires that kinetic plasma theory, rather than commonly used fluid descriptions such as magnetohydrodynamics (MHD), must be used to describe the relevant linear wave modes in the solar wind (Klein et al. 2012). Therefore, in this paper, we evaluate the validity of the Taylor hypothesis for the linear kinetic wave modes in the weakly collisional solar wind plasma.

The low-frequency wave modes of interest in the study of the inertial range of solar wind turbulence are the kinetic counterparts of the fast, Alfvén, and slow MHD wave modes (Klein et al. 2012). The slow wave is generally considered to be strongly damped by collisionless mechanisms for the finite ion temperatures typical of the solar wind (Barnes 1966), so previous investigations have generally focused on the fast and Alfvén wave modes only. Note, however, that recent analyses of correlations between different components of the turbulent fluctuations in the inertial range suggests that the compressible fluctuations arise from kinetic slow waves (Smith & Zhou 2007; Howes et al. 2012; Klein et al. 2012), so further investigation of the role of slow waves in solar wind turbulence is warranted. But since the frequency of the slow wave scales in the same way as the frequency of the Alfvén wave, the conditions for the validity of the Taylor hypothesis for the slow wave will be similar to that for the Alfvén wave. Therefore, we focus here on the remaining two modes, the Alfvén waves and the kinetic fast waves in the weakly collisional solar wind plasma at length scales corresponding to both the inertial range and the dissipation range of solar wind turbulence.

As the turbulent cascade enters the dissipation range, around the scale of the characteristic ion kinetic length scales, these wave modes transition to a variety of different dispersive wave modes, with properties dependent on the region of wavevector space inhabited by each mode. Figure 1 depicts these modes, and the regions of wavevector space in which they exist, for the Alfvén and fast wave branches. Since the ion cyclotron waves of the Alfvén branch are strongly damped by cyclotron resonance with the ions and ion Bernstein waves of the fast wave branch are dominantly electrostatic in nature,[2] the electromagnetic fluctuations observed in the solar wind dissipation range are believed to be one, or a mixture, of the two remaining linear kinetic wave modes: whistler waves and kinetic Alfvén waves (Howes 2009).

[1] Note that the quasilinear premise is not the same as *quasilinear theory* in plasma physics, the rigorous application of perturbation theory to explore the long-time evolution of weakly nonlinear systems.

[2] Although, see Sahraoui et al. (2012) for a more in-depth exploration of the magnetic signature of ion Bernstein wave.

Here we derive simple analytical expressions for the validity of the Taylor hypothesis for the study of plasma turbulence in the solar wind, focusing on the turbulence at scales smaller than the ion length scales, consisting of whistler waves or kinetic Alfvén waves. In §2, we derive a useful form of the Taylor hypothesis and derive the general condition for its validity. In §3, we derive simplified analytical expressions for the conditions necessary for the Taylor hypothesis to be valid for MHD Alfvén waves, kinetic Alfvén waves, ion cyclotron waves, kinetic fast MHD waves, and whistler waves. These expressions use simple approximate dispersion relations for the Alfvén and kinetic Alfvén waves and for the whistler waves that are validated against numerical solutions of the linear Vlasov-Maxwell dispersion relation in §4. We discuss the consequences of these findings in §5 for the study of both the inertial range and the dissipation range of solar wind turbulence. Finally, we conclude in §6 with the primary prediction of this paper that a dissipation range of turbulence supported by whistler waves will violate the Taylor hypothesis, while a dissipation range consisting of kinetic Alfvén waves typically will not. The qualitative and quantitative effects of the violation of the Taylor hypothesis on the magnetic energy spectrum in the solar wind that will be measured by upcoming missions, such as *Solar Probe Plus*, are discussed in a companion paper, Klein et al. (2014a).

2. THE TAYLOR HYPOTHESIS

In this section, we derive the relationship between the plasma-frame and spacecraft-frame frequency, express the validity condition for the Taylor hypothesis in a convenient normalization, and discuss the conditions under which the Taylor hypothesis may be violated.

2.1. *Transforming from Plasma-Frame to Spacecraft-Frame Frequency*

The magnetic field in the solar wind plasma frame as a function of position and time, $\mathbf{B}(\mathbf{r}, t)$, may be expressed in general as a sum of Fourier components in wavevector $\mathbf{k}$ and frequency ω by

$$\mathbf{B}(\mathbf{r}, t) = \sum_{\mathbf{k}} \sum_{\omega} \hat{\mathbf{B}}(\mathbf{k}, \omega) e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}. \quad (1)$$

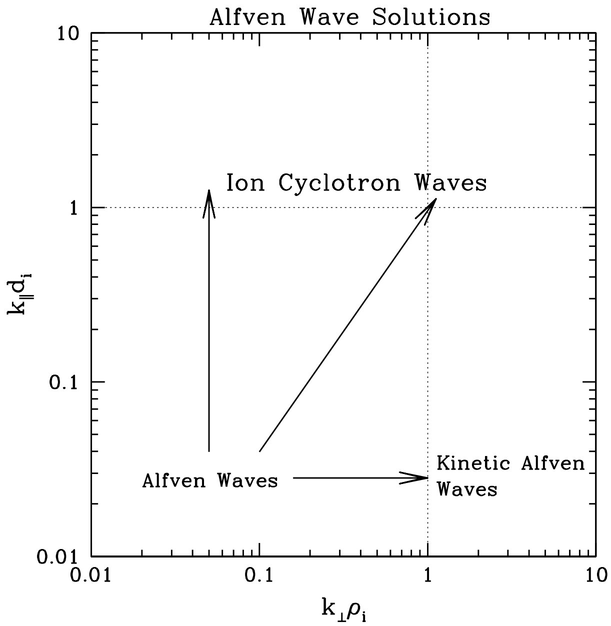
Note that, in general, each wavevector $\mathbf{k}$ can have separate contributions at different frequencies, ω.

We sample this solar wind magnetic field at a spacecraft moving at velocity $-\mathbf{v}_{sw}$, equivalent to spacecraft measurements where the solar wind is streaming past a stationary spacecraft at velocity $\mathbf{v}_{sw}$. The position of the spacecraft as a function of time is given by $\mathbf{r} = -\mathbf{v}_{sw} t$, so the time series of magnetic field measurements in the spacecraft frame is given by $\mathbf{B}(t) = \mathbf{B}(\mathbf{r}, t)|_{\mathbf{r}=-\mathbf{v}_{sw}t}$, yielding

$$\mathbf{B}(t) = \sum_{\mathbf{k}} \sum_{\omega} \hat{\mathbf{B}}(\mathbf{k}, \omega) e^{-i[\mathbf{k}\cdot\mathbf{v}_{sw} + \omega]t}. \quad (2)$$

Finally, we Fourier transform the spacecraft-frame magnetic field time series to obtain the signal in terms of the spacecraft-frame frequency ω_{sc} given by $\mathbf{B}(\omega_{sc}) = (1/2\pi) \int dt\ \mathbf{B}(t) e^{i\omega_{sc} t}$, or

$$\mathbf{B}(\omega_{sc}) = \sum_{\mathbf{k}} \sum_{\omega} \hat{\mathbf{B}}(\mathbf{k}, \omega) \delta[\omega_{sc} - \mathbf{k}\cdot\mathbf{v}_{sw} - \omega]. \quad (3)$$



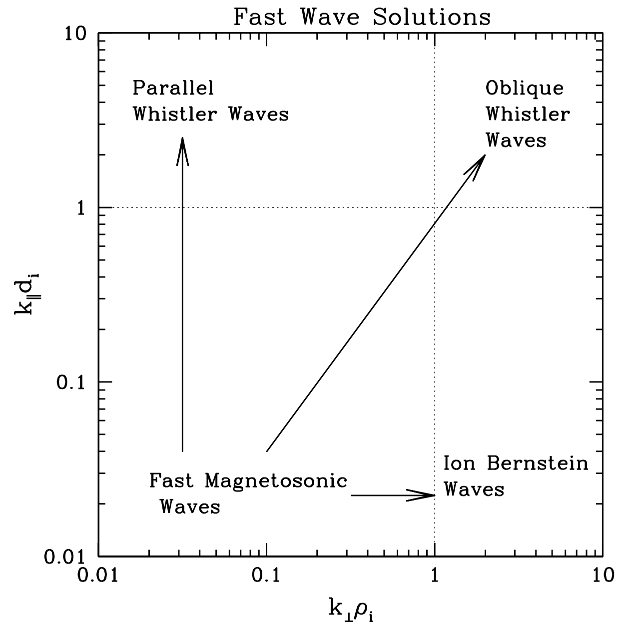


FIG. 1.— (a) Diagram of the region of $(k_\perp, k_\parallel)$ wavevector space inhabited by the linear wave modes of the Alfvén wave branch for a collisionless kinetic plasma governed by the Vlasov-Maxwell equations. (b) Same for the fast wave branch.

Thus we find that the spacecraft-frame frequency is given by the argument of the delta function,

$$\omega_{sc} = \omega + \mathbf{k} \cdot \mathbf{v}_{sw}. \tag{4}$$

Note that this condition must be applied separately to each Fourier mode $\mathbf{k}$. *A serious limitation of single-point spacecraft measurements is that it is not, in general, possible to separate these two contributions to the measured spacecraft-frame frequency, ω_{sc}.*

2.2. *Applying the Quasilinear Premise*

For all possible linear wave modes m with frequency $\omega = \omega_m(\mathbf{k})$, we may apply the premise that the frequency of the turbulent fluctuations is well characterized by the frequency of the linear waves by summing over all possible linear wave modes m and multiplying by the constraint $\delta[\omega - \omega_m(\mathbf{k})]$ to obtain

$$\mathbf{B}(\omega_{sc}) = \sum_m \sum_{\mathbf{k}} \hat{\mathbf{B}}_m(\mathbf{k}) \delta[\omega_{sc} - \mathbf{k} \cdot \mathbf{v}_{sw} - \omega_m(\mathbf{k})]. \tag{5}$$

To simplify the notation, we have defined $\hat{\mathbf{B}}_m(\mathbf{k}) \equiv \hat{\mathbf{B}}[\mathbf{k}, \omega_m(\mathbf{k})]$ since, according to our premise, the plasma-frame frequency is completely determined by the wave mode m and wavevector $\mathbf{k}$. With the implicit understanding that the plasma-frame frequency ω is constrained to be given by the linear dispersion relation for mode m, $\omega = \omega_m(\mathbf{k})$, the relation between the spacecraft frame frequency and the plasma-frame frequency is the same as that given in equation (4).

2.3. *Validity Condition for the Taylor Hypothesis*

We can better understand the relative contributions to ω_{sc} of the two terms on the right-hand side of equation (4) if we normalize the equation by a characteristic frequency for waves in a magnetized plasma, kv_A, yielding

$$\frac{\omega_{sc}}{kv_A} = \frac{\omega}{kv_A} + \frac{v_{sw}}{v_A} \cos\theta, \tag{6}$$

where we have specified a wavevector $\mathbf{k}$ at an angle θ with respect to the solar wind velocity, such that $\mathbf{k} \cdot \mathbf{v}_{sw} = kv_{sw} \cos\theta$.

Single-point spacecraft measurements are often analyzed by taking advantage of the fact that the solar wind flows past the spacecraft at generally super-Alfvénic velocities, $v_{sw} \gg v_A$, where a typical velocity ratio in the solar wind is $v_{sw}/v_A \simeq 10$ (Tu & Marsch 1995; Bruno & Carbone 2005). In contrast, the first term on the right-hand side of equation (6) has a typical magnitude $\omega/(kv_A) \lesssim 1$, so this first term is often negligible. For a system in which temporal fluctuations are measured by a stationary probe in a rapidly flowing fluid, the Taylor hypothesis makes the approximation that the frequency measured by the probe is dominated by the advection of spatial fluctuations past the probe, $|\omega| \ll |\mathbf{k} \cdot \mathbf{v}_{sw}|$ (Taylor 1938). In the case of the solar wind, this means taking $\omega_{sc} \simeq \mathbf{k} \cdot \mathbf{v}_{sw}$. Therefore, the Taylor hypothesis is valid in the limit

$$\frac{v_{sw}}{v_A} \cos\theta \gg \frac{\omega}{kv_A}. \tag{7}$$

It is important to point out that this condition must be evaluated for each wavevector $\mathbf{k}$ and wave mode m that makes up the turbulent distribution of power in three-dimensional wavevector space.

2.4. *Violation of the Taylor Hypothesis*

Violation occurs when $|\omega| \gtrsim |\mathbf{k} \cdot \mathbf{v}_{sw}|$, which can occur in the *Slow Flow Regime* or the *Dispersive Regime*. In the Slow Flow Regime, nondispersive waves may violate the Taylor hypothesis when the wave velocity is of order or greater than the flow velocity. In the Dispersive Regime, dispersive effects causing the wave frequency to increase more rapidly than linearly with the wavevector[3] may lead to high-frequency turbulent fluctuations that violate the Taylor hypothesis.

Since turbulence theories typically predict that turbulent power fills a region of three-dimensional wavevector space, it is generally possible that turbulent power exists in wavevectors $\mathbf{k}$ that are perpendicular to the solar wind velocity, $\theta \to \pi/2$. In this case, the factor $\cos\theta \to 0$, and therefore the validity condition for the Taylor hypothesis equation (7) is not satisfied (unless the plasma-frame frequency $\omega = 0$, *i.e.* a convected structure that does not evolve in time in the plasma frame, such as a pressure-balanced structure or an entropy mode fluctuation). In fact, for all wavevectors in the volume of wavevector space with angles such that $\cos\theta \ll 1$, the Taylor hypothesis will be violated. But, this volume of wavevectors that violate equation (7) is often vanishingly small.

For example, consider the case of turbulence that fills wavevector space isotropically, such as is predicted for hydrodynamic turbulence or the fast wave component of MHD turbulence (Cho & Lazarian 2003). In this case, the turbulent amplitudes vary with spherical radius but do not depend on the azimuthal or polar angles in spherical coordinates. For the purpose of this illustrative example, let us assume that $\omega/(kv_A) \sim 1$, so the validity condition equation (7) will be significantly violated when $\cos\theta \leq v_A/v_{sw}$. For a typical ratio of $v_{sw}/v_A = 10$, this significant violation will occur only for angles $\theta > \theta_c = \cos^{-1}(v_A/v_{sw}) = 84°$. The ratio of the volume of wavevector space which significantly violates the Taylor hypothesis to the total volume of wavevector space is given by $\cos\theta_c = v_A/v_{sw} = 0.1$, so only 10% of the turbulent power violates the Taylor hypothesis in this example.

The case for magnetized plasma turbulence is significantly more complicated because the turbulent power is predicted theoretically (Goldreich & Sridhar 1995; Boldyrev 2006; Howes et al. 2008a; Howes 2008; Schekochihin et al. 2009), shown numerically (Shebalin et al. 1983; Cho & Vishniac 2000; Maron & Goldreich 2001; Cho & Lazarian 2004; Saito et al. 2008; Gary et al. 2012; TenBarge & Howes 2012), and measured observationally (Sahraoui et al. 2010; Narita et al. 2011; Roberts et al. 2013) to fill wavevector space anisotropically, with most power concentrated in wavevectors nearly perpendicular to the local mean magnetic field. Note that the angle of the wavevector with respect to the local mean magnetic field is *not* the same as the angle θ of the wavevector with respect to the solar wind velocity. However, in the near-sun enivronment that will be probed by *Solar Orbiter* and *Solar Probe Plus*, the Parker spiral magnetic field is nearly radial, so in this region the long-time-averaged magnetic field is indeed more often aligned with the solar wind plasma flow. In this case, wavevectors nearly perpendicular to the averaged magnetic field will also have $\theta \to \pi/2$. Note, however, that at the outer scale of the turbulent inertial range (based on spacecraft measurements at heliocentric distances of 0.3 AU and greater), the magnetic field fluctuations have $|\delta\mathbf{B}| \sim |\mathbf{B}_0|$, so although the average magnetic field may be aligned with the solar wind flow, instantaneously the *local* magnetic field direction may have a wide range of angles about the Parker spiral value. In addition, the anisotropic distribution of turbulent wave power also leads to lower plasma-frame frequencies for Alfvénic fluctuations: since $k_\parallel \ll k_\perp$ and $\omega = k_\parallel v_A$, this means that $\omega/(kv_A) \ll 1$, leading to a much narrower volume of wavevector space in which the Taylor hypothesis may be significantly violated. Therefore, one needs a more sophisticated approach to evaluate the validity of the Taylor hypothesis for anisotropic plasma turbulence. In our companion work, Klein et al. (2014a), we employ the synthetic spacecraft data method (Klein et al. 2012) both to investigate the conditions for the validity of the Taylor hypothesis and to predict the quantitative effect on the measured magnetic energy spectrum in spacecraft-frame frequency when the Taylor hypothesis is violated.

[3] For resonances in the dispersive regime, the Taylor hypothesis becomes increasingly well satisfied at smaller scales since the wave frequency $\omega \to$ constant as $k \to \infty$, so the plasma-frame term in equation (4) remains constant while the advection term increases linearly with k. This is shown quantitatively for ion cyclotron waves in §3.1.3.

3. VALIDITY OF THE TAYLOR HYPOTHESIS FOR KINETIC WAVE MODES

In this section, we will use the properties of the linear wave modes in a weakly collisional plasma to evaluate the validity of the Taylor hypothesis. The goal is to derive simple expressions for the observational identification of conditions in which a particular wave mode may violate the Taylor hypothesis. We focus on the kinetic counterparts of the Alfvén and fast waves, at length scales both above and below the characteristic ion length scales: (1) the thermal ion Larmor radius, $\rho_i = v_{ti}/\Omega_i$, where the ion thermal velocity is defined by $v_{ti}^2 = 2T_i/m_i$ (absorbing Boltzmann's constant to measure temperature in energy units) and $\Omega_i = q_i B_0/(m_i c)$ is the ion cyclotron frequency; and (2) the ion inertial length, $d_i = v_A/\Omega_i$, where the Alfvén velocity in a magnetic field with equilibrium magnitude B_0 is given by $v_A^2 = B_0^2/(4\pi n_i m_i)$. The relation between the ion Larmor radius and ion inertial length is given by $\rho_i = d_i\sqrt{\beta_i}$, where the ion plasma beta is $\beta_i = 8\pi n_i T_i/B_0^2$. Note that, throughout this paper, $k_\parallel$ and $k_\perp$ are defined as the parallel and perpendicular components of the wavevector with respect to the direction of the *local* mean magnetic field $\mathbf{B}_0$.

We emphasize here that we use analytical approximations for the linear wave mode frequencies that are either rigorous limits of kinetic theory (Howes et al. 2006; Schekochihin et al. 2009) or empirical expressions based on the numerical results from the linear Vlasov-Maxwell dispersion relation (Quataert 1998; Howes et al. 2006), as verified in §4. The use of analytical expressions from fluid theories, such as Hall MHD (Hirose et al. 2004; Ito et al. 2004) or two-fluid theory (Stringer 1963), can lead to a mixed up identification of wave modes from the fluid theory with those arising from the more broadly applicable kinetic theory. As an example, as shown in Figure 3 of Howes (2009), for the finite ion temperature conditions relevant to the study of the solar wind, Hall MHD (as well as two-fluid theory (Stringer 1963)) connects the slow wave to the ion cyclotron resonance as $k_\parallel d_i \to 1$, whereas the Vlasov-Maxwell results show that this resonance is actually associated with the Alfvén wave. Such occasional mixing up of the identification of wave modes between fluid theory and kinetic theory has been noted in the literature (Krauss-Varban et al. 1994; Yoon & Fang 2008; Howes 2009), but this fact is not necessarily widely known. The bottom line is that, to ensure the correct identification of wave modes using fluid theory, one should always confirm the results using kinetic theory for the appropriate plasma parameters. All simple analytical expressions used in this study originate from the results of kinetic theory, and we caution the reader that conflicting information about wave mode properties from fluid theories should be carefully checked against kinetic results.

3.1. *Alfvén Modes*

At parallel length scales $k_\parallel d_i \ll 1$, the Alfvén wave frequency is well estimated by

$$\omega = k_\parallel v_A \sqrt{1 + \frac{(k_\perp \rho_i)^2}{\beta_i + 2/(1+T_e/T_i)}}, \tag{8}$$

(Howes et al. 2006). We numerically verify this dispersion relation in §4. Note that in the regime $k_\parallel d_i \gtrsim 1$, the Alfvén mode becomes the heavily damped ion cyclotron wave with frequency $\omega \simeq \Omega_i$ (see Figure 1); this limit is discussed separately in §3.1.3. Substituting this frequency into the validity condition given by equation (7), we obtain

$$\frac{v_{sw}}{v_A}\cos\theta \gg \frac{k_\parallel}{k}\sqrt{1 + \frac{(k_\perp \rho_i)^2}{\beta_i + 2/(1+T_e/T_i)}}. \tag{9}$$

We determine below simplified versions of this validity condition for limits of $k_\perp \rho_i$.

3.1.1. *Alfvén Wave Limit, $k_\perp \rho_i \ll 1$*

In the limit $k_\perp \rho_i \ll 1$, the Alfvén mode is the collisionless counterpart of the usual MHD Alfvén wave[4] with frequency $\omega = k_\parallel v_A$. Therefore, the validity condition for the Alfvén waves of the solar wind inertial range simplifies to

$$\frac{v_{sw}}{v_A}\cos\theta \gg \frac{k_\parallel}{k}. \tag{10}$$

In addition to the fact that it is always true that $k_\parallel/k \leq 1$, the turbulence in magnetized plasmas is widely observed to cascade anisotropically to smaller scales perpendicular than parallel to the magnetic field, leading to the condition $k_\perp \gg k_\parallel$ (Shebalin et al. 1983; Cho & Vishniac 2000; Maron & Goldreich 2001; Cho & Lazarian 2004; Sahraoui et al. 2010; Narita et al. 2011; TenBarge & Howes 2012; Roberts et al. 2013). In this case, $k_\parallel/k \simeq k_\parallel/k_\perp \ll 1$. Since the super-Alfvénic conditions of the solar wind typically satisfy the relation $v_{sw}/v_A \gg 1$, we predict that the Alfvénic fluctuations in the inertial range of solar wind turbulence do not typically violate the Taylor hypothesis.

[4] Because the Alfvén wave is incompressible, kinetic effects do not modify the linear wave properties in the limit $k_\parallel d_i \ll 1$; in fact, it has been proven that the reduced MHD description of Alfvén waves is a rigorous limit of the kinetic theory for anisotropic fluctuations with $k_\parallel \ll k_\perp$ (Schekochihin et al. 2009).

3.1.2. *Kinetic Alfvén Wave Limit, $k_\perp \rho_i \gg 1$*

In the asymptotic range of kinetic Alfvén waves, given by $k_\perp \rho_i \gg 1$, the validity condition becomes

$$\frac{v_{sw}}{v_A}\cos\theta \gg \frac{k_\parallel}{k}\frac{k_\perp \rho_i}{\sqrt{\beta_i + 2/(1+T_e/T_i)}}. \tag{11}$$

The dispersive nature of kinetic Alfvén waves gives rise to the factor of $k_\perp \rho_i$ in this expression and may lead to a violation of the Taylor hypothesis at sufficiently high perpendicular wavenumber. Because kinetic Alfvén waves have $k_\parallel \ll k_\perp$, we may replace $k \simeq k_\perp$ in the denominator. The term $2/(1+T_e/T_i)$ simplifies to the value 0 for $T_i/T_e \ll 1$, to the value 1 for $T_i/T_e = 1$, and to the value 2 for $T_i/T_e \gg 1$, so we take the $T_i/T_e = 1$ result as the case most representative of solar wind conditions. For limits of the ion plasma beta β_i, we find the following approximate results for the validity condition

$$\frac{v_{sw}}{v_A}\cos\theta \gg \left\{ \begin{array}{ll} k_\parallel \rho_i, & \beta_i \lesssim 1 \\ k_\parallel d_i, & \beta_i \gtrsim 1 \end{array} \right. . \tag{12}$$

Note that at $\beta_i \leq 1$, $\rho_i \leq d_i$, and so a conservative constraint at any value of β_i may be written as $v_{sw}/v_A \cos\theta \gg k_\parallel d_i$. Therefore, the condition for kinetic Alfvén waves to satisfy the Taylor hypothesis simplifies to

$$\frac{v_{sw}}{v_A}\cos\theta \gg k_\parallel d_i. \tag{13}$$

Since the kinetic Alfvén wave exists only[5] in the wavevector regime $k_\parallel d_i \ll 1$, we predict that the kinetic Alfvén wave fluctuations in the dissipation range of solar wind turbulence typically do not violate the Taylor hypothesis.

3.1.3. *Ion Cyclotron Wave Limit, $k_\parallel d_i \gtrsim 1$*

In the limit $k_\parallel d_i \gtrsim 1$, the Alfvén mode transitions to the ion cyclotron wave (see Figure 2) and generally suffers very strong ion cyclotron damping. Although the expectation is that the ion cyclotron waves are so strongly damped that they will rarely be observed in the solar wind, we nevertheless estimate the validity of the Taylor hypothesis for these waves. Based on solutions of the Vlasov-Maxwell dispersion relation, we have constructed a simple, rough estimate of the maximum frequency of ion cyclotron waves

$$\omega \lesssim \frac{k_\parallel d_i + (k_\parallel d_i)^2}{1 + (k_\parallel d_i)^2}\Omega_i, \tag{14}$$

valid for $k_\perp \rho_i \lesssim 1$. In the Alfvén wave limit, $k_\parallel d_i \ll 1$, this function simplifies to $\omega = k_\parallel v_A$, as expected. In the ion cyclotron wave limit $k_\parallel d_i \gtrsim 1$, this function asymptotes to the ion cyclotron frequency, $\omega \simeq \Omega_i$. Although strong collisionless damping via the ion cyclotron resonance often leads to Vlasov-Maxwell solutions of the wave frequency that are somewhat below Ω_i (by a factor of order unity), equation (14) provides an upper bound to the frequency and is therefore suitable for exploring the validity of the Taylor hypothesis.

[5] Note, however, that at $\beta_i > 1$ and $k_\perp \rho_i \gg 1$, the kinetic Alfvén wave becomes insensitive to the ion cyclotron resonance and can persist at values $k_\parallel d_i > 1$ (Boldyrev et al. 2013).

Substituting equation (14) into equation (7), we find that the condition for ion cyclotron waves to satisfy the Taylor hypothesis becomes

$$\frac{v_{sw}}{v_A}\cos\theta \gg \frac{k_\parallel}{k}\frac{1+k_\parallel d_i}{1+(k_\parallel d_i)^2}. \quad (15)$$

Since $k_\parallel/k \leq 1$ and the remaining factor gives a value of order unity or less for any value of $k_\parallel d_i$, we predict that the ion cyclotron wave fluctuations in solar wind turbulence typically do not violate the Taylor hypothesis.

3.2. *Fast Modes*

In this section, we evaluate the validity condition of the Taylor hypothesis for the kinetic counterpart of the fast MHD wave (Klein et al. 2012) and for the whistler wave. Because ion Bernstein waves, the solution of the fast wave branch in the regime $k_\parallel d_i \ll 1$ and $k_\perp \rho_i \gtrsim 1$ as shown in Figure 1, are essentially electrostatic in nature (Stix 1992), it has been suggested that they are unlikely to be responsible for the magnetic field fluctuations observed in the dissipation range of solar wind turbulence (Howes 2009). It has been pointed out, however, that at $k\rho_i \sim 1$, the ion Bernstein wave magnetic signature is small but nonzero (Sahraoui et al. 2012). The possibility that ion Bernstein waves contribute significantly to solar wind turbulence merits further consideration, but we leave that to future work and do not consider ion Bernstein waves here.

3.2.1. *Fast Wave Limit, $kd_i \ll 1$*

In the large scale limit $kd_i \ll 1$, the fast wave mode is the kinetic counterpart of the usual MHD fast wave (Klein et al. 2012). An upper limit on the frequency of this wave can be expressed as

$$\omega \leq k\sqrt{v_A^2 + c_s^2} \simeq kv_A\sqrt{1+\beta_i(1+T_e/T_i)}, \quad (16)$$

so the validity condition becomes

$$\frac{v_{sw}}{v_A}\cos\theta \gg \sqrt{1+\beta_i(1+T_e/T_i)}. \quad (17)$$

Observed values of β_i in the near-Earth solar wind (Howes 2011) generally satisfy $\beta_i < 5$, so although this condition as not quite as well satisfied as for the Alfvén modes, we predict that fast waves do not typically violate the Taylor hypothesis in a significant way. Note that, since the kinetic fast wave at $kd_i \ll 1$ is nondispersive, any violation of the Taylor hypothesis is necessarily due to plasma conditions in the Slow Flow Regime.

3.2.2. *Whistler Wave Limit, $k_\parallel d_i \gtrsim 1$*

Whistler waves are the manifestation of the kinetic fast mode in the limit $k_\parallel d_i \gtrsim 1$, and their frequency is well estimated by

$$\omega = kv_A\sqrt{1+(k_\parallel d_i)^2}. \quad (18)$$

We verify numerically in §4 that this simplified dispersion relation provides a good estimate of the whistler wave frequency. Any violation of the Taylor hypothesis is likely to happen in the regime $k_\parallel d_i \gg 1$, leading to a validity condition

$$\frac{v_{sw}}{v_A}\cos\theta \gg k_\parallel d_i. \quad (19)$$

Here we see that if a whistler wave has a sufficiently high value of $k_\parallel d_i$, it will violate the Taylor hypothesis. This violation occurs in the Dispersive Regime. Therefore, given measured values for the Alfvén velocity v_A and solar wind velocity v_{sw}, it is possible to determine the value of the parallel wavevector at which the whistler wave will violate the Taylor hypothesis.

Single-point spacecraft measurements, however, do not allow the determination of the parallel and perpendicular components of the wavevector with respect to the *local mean magnetic field*. Therefore, we choose to express the condition in terms of an observable quantity, the effective projection of the wavevector $k_{\rm eff} \simeq k\cos\theta$ along the direction of the solar wind flow.[6] Since whistlers have wavevectors that satisfy $k_\parallel \gtrsim k_\perp$, we first make the simplification $k_\parallel \simeq k$. Then, we approximate the magnitude of the wavevector by its projection along the solar wind flow, $k_{\rm eff} \simeq k\cos\theta$. The resulting condition for the validity of the Taylor hypothesis, in terms of the measured component of the wavevector along the flow direction $k_{\rm eff}$, is

$$\frac{v_{sw}}{v_A}\cos^2\theta \gg k_{\rm eff} d_i, \quad (20)$$

where $k_{\rm eff} \simeq \omega_{sc}/v_{sw}$ is the observable component of the wavevector sampled along the direction of the solar wind flow.

Taking $\cos\theta \sim 1$, we can express the spacecraft-frame frequency at which we expect the Taylor hypothesis to be violated by

$$\omega_{sc} \gtrsim \frac{v_{sw}^2}{v_A d_i}. \quad (21)$$

In the notation used in Klein et al. (2014a), $\overline{V} \equiv v_{sw}/v_A$ and $\omega_*/\Omega_i \equiv (\omega_{sc}/\Omega_i)/\overline{V}$, this condition becomes

$$\frac{\omega_*}{\Omega_i} \gtrsim \overline{V}. \quad (22)$$

3.3. *Slow Modes*

The kinetic counterpart of the MHD slow mode is strongly damped in collisionless plasmas with $T_i \sim T_e$ (Barnes 1966; Klein et al. 2012), so previous analyses have generally dismissed the possibility of kinetic slow waves in the solar wind. Recently, however, the first study to employ Vlasov-Maxwell kinetic theory to investigate the properties of the compressible fluctuations in the inertial range of the solar wind has found strong observational evidence that the compressible fluctuations are almost entirely in the kinetic slow mode (Howes et al. 2012; Klein et al. 2012).

The normalized phase velocity of the kinetic slow wave $\omega/(kv_A)$ is almost always less than the kinetic fast wave—see Figure 1 of Klein et al. (2012)—and has properties generally similar to the Alfvén wave. Since the frequency of the slow mode scales in the same way as the Alfvén mode, we do not separately address the slow mode but use the results for the Alfvén mode as a guide. Therefore, based on our prediction that the Alfvén and fast wave modes do not violate the Taylor hypothesis, we predict that the kinetic slow mode will also not violate the Taylor hypothesis.

[6] Strictly, this is true only when the Taylor hypothesis *is* valid.

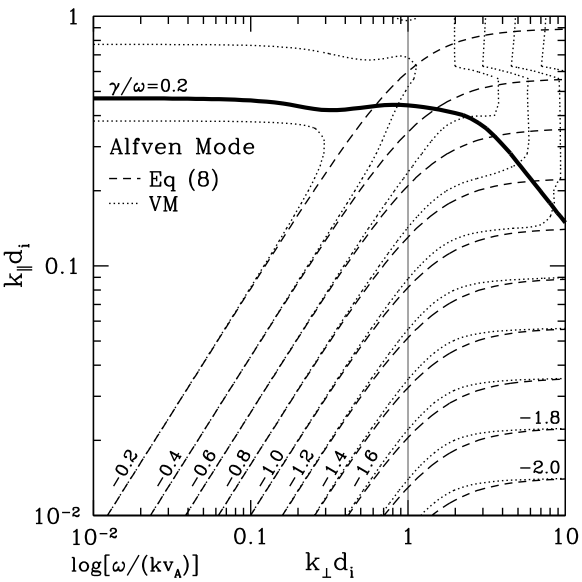


FIG. 2.— Contour plot of $\log[\omega/(kv_A)]$ for the Alfvén mode over the $(k_\perp d_i, k_\parallel d_i)$ plane for a $\beta_i = 1$ and $T_i/T_e = 1$ plasma determined by the linear Vlasov-Maxwell dispersion relation (dotted) and the estimated frequency given by equation (8) (dashed). Collisionless damping is strong at $k_\parallel d_i$ values above the $\gamma/\omega = 0.2$ contour (thick line).

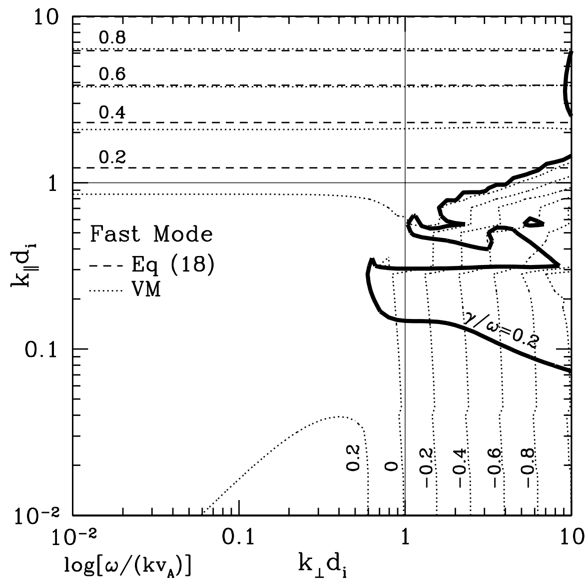


FIG. 3.— Contour plot of $\log[\omega/(kv_A)]$ for the fast mode over the $(k_\perp d_i, k_\parallel d_i)$ plane for a $\beta_i = 1$ and $T_i/T_e = 1$ plasma determined by the linear Vlasov-Maxwell dispersion relation (dotted) and the estimated frequency given by equation (18) (dashed). Collisionless damping is strong at $k_\parallel d_i$ values above the $\gamma/\omega = 0.2$ contour (thick line).

4. NUMERICAL VERIFICATION OF LINEAR WAVE FREQUENCY ESTIMATES

In this section, we verify that the simple expressions for the linear frequencies of the Alfvén, kinetic Alfvén, and whistler waves provide good estimates of the frequencies given by the linear Vlasov-Maxwell dispersion relation (Quataert 1998; Howes et al. 2006). A fully-ionized proton and electron plasma is assumed with isotropic Maxwellian velocity distributions, a realistic mass ratio $m_i/m_e = 1836$, and non-relativistic conditions $v_{ti}/c = 10^{-4}$. For the comparison presented here, we choose plasma parameters that are typical of near-Earth solar wind conditions, $\beta_i = 1$ and $T_i/T_e = 1$. The complex linear Vlasov-Maxwell eigenfrequencies for the Alfvén mode and fast mode are solved on a logarithmically spaced grid in wavevector space spanning $10^{-2} \leq k_\parallel d_i \leq 10$ and $10^{-2} \leq k_\perp d_i \leq 10$. Note that for a $\beta_i = 1$ plasma, the thermal ion Larmor radius and ion inertial length are the same, $\rho_i = d_i$.

In Figure 2, we plot contours of the value of $\log[\omega/(kv_A)]$ for the Alfvén mode determined using the linear Vlasov-Maxwell dispersion relation (dotted) and the estimated frequency given by equation (8) (dashed). For a complex linear eigenfrequency $\omega - i\gamma$, the linear collisionless damping of the wave mode becomes strong at a value of $\gamma/\omega \gtrsim 0.2$. Also plotted on Figure 2 is the contour where $\gamma/\omega = 0.2$ (thick solid); for values of $k_\parallel d_i$ above this line, the collisionless damping of the ion cyclotron wave by the cyclotron resonance is sufficiently strong that the waves are not expected to be observed in solar wind turbulence. For this strongly damped region of wavevector space, strong wave-particle interactions lead to deviations in the linear Alfvén mode frequency from the expression in equation (8). In the weakly damped region below the $\gamma/\omega = 0.2$, we see that the agreement between the linear Vlasov-Maxwell real frequency and the estimate by equation (8) is excellent for both the Alfvén wave and kinetic Alfvén wave regimes.

In Figure 3, we plot contours of the value of $\log[\omega/(kv_A)]$ for the fast mode determined using the linear Vlasov-Maxwell dispersion relation (dotted) and the estimated frequency given by equation (18) (dashed). Again, linear collisionless damping is strong for modes to the right (higher values of $k_\perp d_i$) of the $\gamma/\omega = 0.2$ contour (thick solid). In the whistler wave regime at $k_\parallel d_i \gtrsim 1$, agreement between the linear Vlasov-Maxwell real frequency and the estimate by equation (18) is excellent. Note that the wave frequency in the ion Bernstein wave regime ($k_\perp d_i \gtrsim 1$ and $k_\parallel d_i \ll 1$, see Figure 1) is not well estimated by equation (18).

5. DISCUSSION

Here we discuss the implications of the results in §3 for the study of the dissipation range of solar wind turbulence. The primary result of this work is that the whistler wave is the only linear kinetic wave mode that is likely to violate significantly the Taylor hypothesis. This finding relies on the premise that the frequency of the turbulent fluctuations is well characterized by the frequency of the linear waves supported by the solar wind plasma. This premise is contained within a more general hypothesis for the modeling of plasma turbulence, the quasilinear premise; a detailed discussion of the quasilinear premise and a review of supporting evidence is presented in Howes et al. (2014). Adopting this premise, a key question naturally arises about the nature of the dissipation range of solar wind turbulence: do the turbulent fluctuations correspond to a broadband spectrum of kinetic Alfvén waves, a broadband spectrum of whistler waves, or some combination of both types of waves? Since we predict that whistler waves will violate the Taylor hypothesis while kinetic Alfvén waves will not, the answer to this question has important implications for measurements of solar wind turbulence by the upcoming *Magnetospheric Multiscale (MMS)*, *Solar Orbiter*, and *Solar Probe Plus* missions.

The body of observational evidence in support of kinetic Alfvén waves as the dominant contributor to the dissipation range of solar wind turbulence has recently been reviewed in detail by Podesta (2013). Important lines of evidence in support of kinetic Alfvén waves being the dominant contributor to turbulent fluctuations in the dissipation range include enhanced electron density fluctuations around the ion Larmor radius scale (Harmon 1989; Hollweg 1999; Chandran et al. 2009; Chen et al. 2013), measurements of the wave frequency as a function of wavenumber (Bale et al. 2005; Howes et al. 2008a; Sahraoui et al. 2010; Salem et al. 2012), magnetic helicity measurements around the ion Larmor radius scale (Howes & Quataert 2010; He et al. 2011; Podesta & Gary 2011b; Klein et al. 2014b), the magnetic field variance anisotropy at ion kinetic length scales (Belcher & Davis 1971; Harmon 1989; Leamon et al. 1998; Hollweg 1999; Smith et al. 2006; Hamilton et al. 2008; Gary & Smith 2009; TenBarge et al. 2012; Podesta & TenBarge 2012), and the lack of compressible fast-wave fluctuations in the inertial range of solar wind turbulence (Howes et al. 2012; Klein et al. 2012).

In addition to the observational evidence above, there is also evidence from numerical simulations that the solar wind dissipation range consists of a cascade of kinetic Alfvén waves, including gyrokinetic (Howes et al. 2008b; Howes et al. 2011; TenBarge & Howes 2013; TenBarge et al. 2013) and electron reduced MHD simulations (Boldyrev & Perez 2012), both of which reproduce quantitatively the observed magnetic energy spectrum in the dissipation range of the solar wind.

Although, based on the numerous lines of reasoning above, the turbulent cascade of energy from large to small scales appears to be dominated by kinetic Alfvén wave fluctuations, there is strong evidence that kinetic instabilities may generate whistler waves at scales $kd_i \sim 1$ (Kasper et al. 2002; Hellinger et al. 2006; Bale et al. 2009). Measurements of magnetic helicity sorted as a function of the angle of the magnetic field with respect to the solar wind flow show a subdominant contribution of opposite magnetic helicity at small angles. These parallel modes are interpreted to be either whistler or ion cyclotron waves with nearly parallel wavevectors (He et al. 2011; Podesta & Gary 2011b,a; Klein et al. 2014b). Significant energy input into the turbulence at scales $kd_i \sim 1$, however, appears unlikely since structure in the turbulent energy spectra at these scales is not generally observed (see Podesta (2009) and Wicks et al. (2010) for exceptions, where a small bump can be seen in the energetically subdominant $k_\parallel$ spectrum). In addition, modeling of the magnetic helicity as a function of angle using the synthetic spacecraft data method shows that these parallel modes contribute only around 5% of the turbulent power, while the dominant 95% contribution to the turbulent power is due to a spectrum of kinetic Alfvén waves with nearly perpendicular wavevectors (Klein et al. 2014b).

In conclusion, the lack of strong evidence for whistler waves in the dissipation range is good news for planned single-spacecraft missions, such as *Solar Orbiter* and *Solar Probe Plus*: we predict that measurements of a kinetic Alfvén wave dominated dissipation range will not violate the Taylor hypothesis, dramatically simplifying data analysis and interpretation of turbulence measurements for these upcoming missions.

In addition to the magnetic field measurements of turbulence traditionally collected by spacecraft missions in the solar wind, many new spacecraft missions are instrumented to make measurements of the fluctuating turbulent electric fields as well. Relevant to this study of the effect of measurements made in a frame of reference moving with respect to the plasma being measured, it is important to note that one must carefully handle the Lorentz transform of the electric fields, as detailed in Appendix A. The upshot is that, although the magnetic fields may be safely transformed between spacecraft and plasma frames without any complications, the Lorentz transform relating the spacecraft-frame electric field $\mathbf{E}_{sc}$ and the plasma-frame electric field $\mathbf{E}$ for typical solar wind conditions is

$$\mathbf{E}_{sc} = \mathbf{E} + \mathbf{v}_{sw}/c \times \mathbf{B}. \tag{23}$$

The impact of this transformation is made clear in recent work exploring the electric and magnetic field spectra using *Cluster* measurements (Chen et al. 2011), finding that previous electric field spectra reported in the literature (Bale et al. 2005) were dominated by the magnetic field spectrum through the second term in equation (23).

Finally, we discuss an important point that has lead to continuing confusion in the literature regarding the relation between the ion cyclotron frequency and spacecraft-frame measurements. The ion cyclotron frequency Ω_i is a characteristic frequency of the plasma *in the plasma frame*—it therefore enters the spacecraft frame frequency through the first term on the right hand side of equation (4). Therefore, purely temporal variation at the ion cyclotron frequency in the plasma frame does not Doppler shift when measured by a probe moving with respect to the plasma. Only spatial variation Doppler shifts to yield a temporal fluctuation when measured by a moving probe. Under typical solar wind conditions, the spacecraft-frame frequency ω_{sc} of the break in the magnetic energy frequency spectrum at the onset of the dissipation range is often roughly coincident with the value of the ion cyclotron frequency Ω_i in the plasma frame. This fact has lead numerous researchers to attribute the steepening of the spectrum to ion cyclotron damping (Coleman 1968; Denskat et al. 1983; Goldstein et al. 1994; Leamon et al. 1998; Gary 1999; Hamilton et al. 2008). But, unless the plasma-frame frequency term competes with, or dominates, the Doppler shift term, $|\omega| \gtrsim |\mathbf{k} \cdot \mathbf{v}_{sw}|$ (thereby significantly violating the Taylor hypothesis), then any measurements of spacecraft-frame frequency at the ion cyclotron frequency, $\omega_{sc} \sim \Omega_i$, are caused by the condition $\mathbf{k} \cdot \mathbf{v}_{sw} \sim \Omega_i$, and are unrelated to ion cyclotron frequency dynamics in the solar wind plasma frame, $\omega \sim \Omega_i$. We have shown in §3.1.3 that the Taylor hypothesis is not generally violated for ion cyclotron waves. Therefore, it is physically incorrect to interpret solar wind turbulent fluctuations that have a spacecraft-frame frequency $\omega_{sc} \sim \Omega_i$ as being related to ion cyclotron frequency dynamics in the plasma frame.

6. CONCLUSION

The Taylor hypothesis is commonly used in the analysis of single-point spacecraft measurements of solar wind turbulence to relate the frequency of turbulent fluctuations measured in the spacecraft frame directly to the spatial wavenumber of turbulent fluctuations in the plasma frame. But, as upcoming missions, such as *Solar Orbiter* and *Solar Probe Plus*, explore new observational regimes, the Taylor hypothesis may fail. Two new regimes threaten to violate the Taylor hypothesis: (i) Slow Flow Regime: when the solar wind flow velocity falls below the Alfvén wave velocity, and (ii) Dispersive Regime: when dispersive effects at small scales cause the wave frequency to increase more rapidly than linearly with the wavevector.

To evaluate the validity of the Taylor hypothesis in these new regimes, we adopt the premise that the frequency of the turbulent fluctuations is well characterized by the frequency of the linear waves supported by the solar wind plasma. Therefore, we examine the validity of the Taylor hypothesis for the linear kinetic wave modes in the weakly collisional solar wind plasma. In particular, we focus on two leading candidate wave modes: (i) the Alfvén wave and its small-scale dispersive extension as the kinetic Alfvén wave, and (ii) the kinetic counterpart of the MHD fast wave and its small-scale dispersive extension as the whistler wave.

We present useful analytical expressions for the kinetic wave modes of interest and numerically verify that these expressions are accurate by direct comparison to solutions for the wave frequencies from the linear Vlasov-Maxwell dispersion relation. We use those analytical expressions to derive simple conditions for the validity of the Taylor hypothesis. Our principle finding is that *the whistler wave is the only wave mode that is likely to violate significantly the Taylor hypothesis* when upcoming missions measure the turbulence in the solar wind. We predict that *the Taylor hypothesis is not likely to be violated significantly by any of the other plasma waves* that may be relevant to turbulence in the solar wind: any limit of the Alfvén mode including kinetic Alfvén waves and ion cyclotron waves, the kinetic fast wave, or the kinetic slow wave.

We emphasize the importance of making the proper Lorentz transformation of electric field measurements to relate the measurements of the spacecraft-frame electric field to the plasma-frame electric field, and we present the appropriate formula for typical solar wind conditions. Finally, we demonstrate that it is physically incorrect to interpret solar wind turbulent fluctuations that have a spacecraft-frame frequency $\omega_{sc} \sim \Omega_i$ as being related to ion cyclotron frequency dynamics in the plasma frame.

Significant evidence exists in support of the kinetic-Alfvén-wave-like nature of the turbulent fluctuations in the dissipation range. This is good news for upcoming single spacecraft missions, such as *Solar Orbiter* and *Solar Probe Plus*, because it means that it will still be possible to apply the Taylor hypothesis to *in situ* measurements of the turbulence to relate the spacecraft-frame frequency of the fluctuations to the spatial wavenumber of fluctuations in the plasma frame, dramatically simplifying the data analysis and interpretation for these upcoming missions.

In a companion paper, Klein et al. (2014a), we employ the synthetic spacecraft data method to explore the conditions leading to the violation of the Taylor hypothesis and to predict the quantitative effect on the measured magnetic energy spectrum in spacecraft-frame frequency when the Taylor hypothesis is violated.

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APPENDIX

LORENTZ TRANSFORM OF ELECTROMAGNETIC FIELD MEASUREMENTS IN THE SOLAR WIND

The general formula for the Lorentz transformation of electric and magnetic fields from the unprimed frame K (at rest) to the primed frame K' moving with velocity $\mathbf{v}$ with respect to frame K are

$$\mathbf{E}' = \gamma(\mathbf{E} + \mathbf{v}/c \times \mathbf{B}) - \frac{\gamma^2}{\gamma+1}\mathbf{v}/c(\mathbf{v}/c \cdot \mathbf{E}), \tag{A1}$$

$$\mathbf{B}' = \gamma(\mathbf{B} - \mathbf{v}/c \times \mathbf{E}) - \frac{\gamma^2}{\gamma+1}\mathbf{v}/c(\mathbf{v}/c \cdot \mathbf{B}), \tag{A2}$$

where $\gamma = 1/\sqrt{1+v^2/c^2}$ (Jackson 1998).

For a velocity of the solar wind (the velocity of transformation between frames) $v \sim 500$ km/s and an Alfvén speed $v_A = 50$ km/s, we have $v/c = 1.6 \times 10^{-3}$ and $v_A/c = 1.6 \times 10^{-4}$, so $\gamma \simeq 1$. For Alfvén waves, the characteristic wave electric fields are

$$\mathbf{E} \sim \frac{v_A}{c}\delta\mathbf{B} \times \hat{\mathbf{z}}, \tag{A3}$$

so $\mathcal{O}(E/\delta B) \sim v_A/c \ll 1$. Taking $v/c \sim \epsilon \ll 1$, the order of the terms on the right-hand side of equation (A1) with respect to δB is ϵ, ϵ, and ϵ^2. So, the last term may be dropped, and we are left with the approximate relation

$$\mathbf{E}' \simeq \mathbf{E} + \mathbf{v}/c \times \mathbf{B}. \tag{A4}$$

The order of the terms on the right-hand side of equation (A2) with respect to δB is 1, ϵ^2, and ϵ^2. Therefore, equation (A2) reduces to

$$\mathbf{B}' = \mathbf{B}, \tag{A5}$$

and we do not need to concern ourselves with the Lorentz transformation of the magnetic field from the spacecraft to the plasma frame.

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