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INFLUENCE OF DISSIPATIVE ENTRY TEMPERATURE ON LAMINAR HEAT TRANSFER IN THERMALLY DEVELOPING REGION OF A CIRCULAR PIPE WITH VISCOUS DISSIPATION

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ABSTRACT

Steady laminar forced convection heat transfer in the thermal entrance region of a circular pipe including viscous dissipation has been studied assuming the flow to be hydrodynamically fully developed and thermally developing. The circular pipe is subjected to constant wall temperature. Two entry temperatures have been considered. 1) A temperature that varies with the radial coordinate obtained in an adiabatic pipe due to viscous dissipation while the flow is hydrodynamically developing. This temperature is termed as the dissipative entry temperature. 2) A uniform temperature equal to the bulk mean value of the dissipative entry temperature. It has been shown that the difference in the Nusselt numbers and heat transfer obtained with these two entry temperatures is insignificant. Thus, the simplicity of classical assumption of uniform entry temperature can be retained when the entry temperature has been chosen as the bulk mean of the dissipative entry temperature.

INTRODUCTION

Laminar forced convection heat transfer through circular pipes has been the subject of several investigations owing to its present day applications in fuel cells, catalytic reactors and solar receivers or absorbers. Inclusion of the effect of viscous dissipation is warranted when studies deal with fluid flow for high Prandtl number. After the pioneering work done by Graetz [1, 2] and Nusselt [3], excellent accounts of the developments till 1978 are available in Shah and London [4]. Similar account is also available in Kakac, Shah and Aung [5].

Laminar forced convection through circular pipe including viscous dissipation effects subjected to prescribed wall temperature can be found in [6]. Similar studies when the pipe is subjected to constant heat flux are available in [7-12]. Studies reported in [13] deal with both types of boundary conditions and convective boundary conditions has been addressed in [14, 15]. Laminar forced convection including axial conduction has been examined in [16-20]. Jambal et al. [21] examined the laminar forced convection through circular pipes for power law fluids and the solutions for Newtonian fluids are obtained as a special case.

In order to assess the commonly employed assumption of the fluid entering a pipe with fully developed velocity profile and uniform entry temperature, when viscous dissipation is included, Barletta and Magyari [22, 23] introduced the concept of adiabatic preparatory zone. In the adiabatic preparatory zone, the flow develops hydrodynamically and gets heated only due to viscous dissipation, since the duct is kept adiabatic. The hydrodynamically fully developed flow with the temperature generated in the adiabatic preparatory zone now enters the duct where forced convection in the thermally developing zone occurs. This entry temperature, a function of the radial coordinate is termed the dissipative entry temperature in the present article. Barletta and Magyari [22] gave an analytical expression for this temperature. Further, Barletta and Magyari [22] examined the influence of this dissipative entry temperature on local Nusselt numbers only. They compared the results obtained with those obtained with the conventional uniform temperature that existed at the entry to the hypothetical adiabatic preparatory zone, i.e. a temperature no different had the flow developed with no dissipation. This is misleading the difference in the Nusselt numbers is not only due to the entry temperature but also essentially due to differing Brinkman numbers.

The present article is an attempt to evaluate the effect of the dissipative entry temperature not only on local Nusselt numbers but also on temperature profiles, local wall heat transfer and energy gain (or loss) by the fluid up to a desired axial distance in the thermally developing region of circular pipes kept at constant temperature. It is proposed that two entry temperatures be considered, a) the dissipative entry temperature and b) the bulk mean of the dissipative entry temperature. To the best knowledge of the authors, the wall heat transfer and the energy gain (or loss) by the fluid in the developing region of the circular pipes including viscous dissipation have not been presented in the literature. Numerical solutions to the governing equations have been obtained employing the Successive Accelerated Replacement (SAR) scheme which ha been described in Satyamurty [24], and extensively used in [25, 26]. Recently this scheme has been employed for the present class of problems by Satyamurty and Bhargavi [27] and Ramjee and Satyamurty [28].

NOMENCLATURE

a	[m]	radius of the circular pipe
A_c	[m^2]	cross sectional area of the circular pipe
Br	[-]	Brinkman number, Eq. (14)
C_p	[J/kgK]	specific heat of the fluid
h_x	[W/m^{2}K]	local wall heat transfer coefficient
k	[W/mK]	thermally conductivity of the fluid
$\dot{m}$	[kg/s]	mass flow rate of the fluid, Eq. (27)
M	[-]	position of the node along axial direction used in the numerical scheme
MD	[-]	number of divisions along axial direction used in the numerical scheme
N	[-]	position of the node along the radial direction used in the numerical scheme
ND	[-]	number of divisions along radial direction used in the numerical scheme
Nu_x	[-]	local Nusselt number
Pe	[-]	Peclet number, Eq. (11)
Q_{xw}	[W]	dimensional heat transferred from (or to) the wall up to a distance x, Eq. (25)
Q_{xf}	[W]	heat gain (or loss) by the fluid up to a distance x, Eq. (29)
$\bar{Q}_{xw}$	[-]	non-dimensional heat transferred from (or to) the wall, Eqs. (26) and (28)
$\bar{Q}_{xf}$	[-]	non-dimensional heat gain (or loss) by the fluid, Eqs. (30) and (31), W
r	[-]	radial coordinate
R	[-]	non-dimensional radial coordinate
Re	[-]	Reynolds number of the fluid
T	[K]	temperature of the fluid
T_{ent}	[K]	fluid entry temperature
$T_{de}(r)$	[K]	dissipative entry temperature of the fluid, Eq. (7)
T_b	[K]	bulk mean temperature of the fluid, Eq. (22)
T_w	[K]	wall temperature
$T_{0,a}$	[K]	wall temperature at $r = a$ at the end of the adiabatic preparatory zone
$\bar{T}_{de}$	[K]	bulk mean temperature of the dissipative entry temperature, Eq. (8)
u	[m/s]	fully developed velocity of the fluid, Eq. (2)
u_{avg}	[m/s]	average velocity of the fluid
U	[-]	non-dimensional fully developed velocity of the fluid
x	[-]	axial coordinate
X	[-]	non-dimensional axial coordinate
X^*	[-]	non-dimensional normalized axial distance, X/Pe
Special characters		
α	[m^2/s]	thermal diffusivity of the fluid
ε	[-]	error tolerance limit
θ	[-]	non-dimensional temperature of the fluid, Eq. (10)
θ_{ent}	[-]	non-dimensional fluid entry temperature
$\theta_{de}(R)$	[-]	non-dimensional dissipative entry temperature of the fluid, Eq. (19)
θ^*_{de}	[-]	non-dimensional bulk mean temperature of the dissipative entry temperature of the fluid, Eq. (20)
θ^*	[-]	non-dimensional bulk mean temperature of the fluid, Eq. (23)
μ	[Ns/m^2]	dynamic viscosity of the fluid
ν	[m^2/s]	kinematic viscosity of the fluid
ρ	[kg/m^3]	density of the fluid
ϕ	[-]	variable used to describe the numerical scheme
$\tilde{\phi}$	[-]	error in the governing equation due to guessed profiles for the variable
ω	[-]	acceleration factor in the numerical scheme
Subscripts		
1	[-]	when the fluid enters with dissipative entry temperature, $T_{de}(r)$
2	[-]	when the fluid enters with a temperature equal to the bulk mean temperature of the dissipative entry temperature, $\bar{T}_{de}$

MATHEMATICAL FORMULATION

The physical model that of a circular pipe of radius, a, along with the coordinate system in dimensional and non-dimensional form are shown in Figure 1 (a) and (b) respectively. r is the radial coordinate and x is the axial coordinate. The circular pipe is subjected to a constant wall temperature of T_w. The fluid enters the pipe with a fully developed velocity of $u(r)$ and one of the entry temperatures: a) dissipative entry temperature as obtained in [22], $T_{de}(r)$ or, b) uniform temperature, $\bar{T}_{de}$, equal to the bulk mean temperature of the dissipative entry temperature, $T_{de}(r)$. These are indicated by T_{ent} in Figure 1.

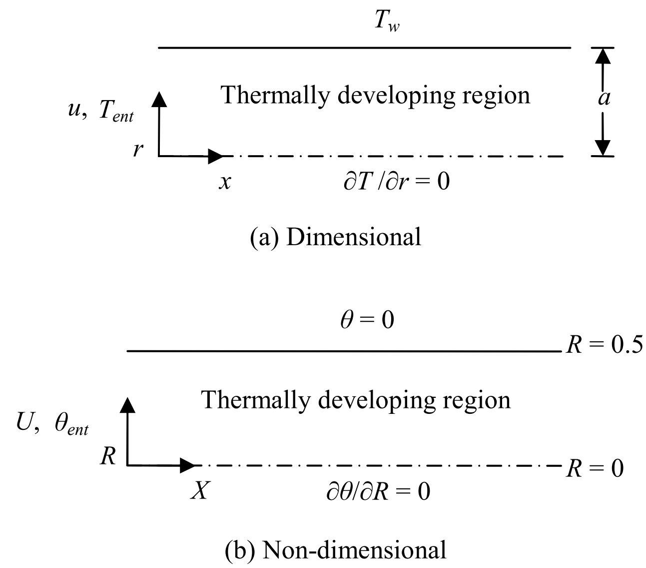


Figure 1 Physical model and coordinate system

The flow is assumed to be steady, incompressible and laminar and the fluid properties are constant. Further, the flow is hydrodynamically fully developed and enters the pipe with one of the entry temperatures described above. Let T_1 (x, r) and T_2 (x, r) be the dimensional temperatures at any (x, r) corresponding to the two entry temperatures, $T_{de}(r)$ and $\bar{T}_{de}$ described above. Neglecting axial conduction and including viscous dissipation, the conservation of thermal energy equation in the thermally developing region, is given by,

$$\rho C_P u \frac{\partial T_{1,2}}{\partial x} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_{1,2}}{\partial r}\right) + \mu\left(\frac{\partial u}{\partial r}\right)^2 \quad (1)$$

In Eq. (1), ρ, C_p, k, and μ are the density, specific heat, thermal conductivity and dynamic viscosity of the fluid respectively.

In Eq. (1), u (r), the fully developed Hagen-Poiseuille velocity, is given by,

$$u = 2u_{avg}\left[1 - (r/a)^2\right] \quad (2)$$

In Eq. (2), u_{avg} is average velocity of the fluid. Eq. (1) is subjected to the following boundary conditions,

$$T_{1,2} = T_w \text{ at } r = a \text{ for } x > 0 \quad (3)$$

$$\left(\partial T_{1,2}/\partial r\right) = 0 \text{ at } r = 0 \text{ for } x > 0 \quad (4)$$

The entry temperature, T_{ent}, is one of,

$$T_{ent} = T_{de}(r), \text{ at } x = 0 \text{ for } 0 \le r \le a\text{, OR} \quad (5)$$

$$T_{ent} = \overline{T}_{de}, \text{ at } x = 0 \text{ for } 0 \le r \le a \quad (6)$$

In Eq. (6), $T_{de}(r)$ adapted from Barletta and Magyari [22], is given by,

$$T_{de}(r) = T_{(0,a)} - \frac{2\mu u_m^2}{k}\left[\frac{r^4}{a^4} - 2\frac{r^2}{a^2} + 1\right] \quad (7)$$

In Eq. (7), $T_{(0,\,a)}$, is the temperature at $r = a$ and $x = 0$, i.e., at the beginning of the thermally developing region. In principle, $T_{(0,\,a)}$, can be perceived to be dependent on the length of the adiabatic duct needed for the flow to become fully developed. However, $T_{(0,\,a)}$ has not been related to the development length in [22]. The scope of the present article, as a first step, is to examine the influence of the entry temperature not only on the local Nusselt number, but also on temperature profiles, wall heat transfer and the energy gain or loss by the fluid.

In Eq. (6), $\overline{T}_{de}$, the bulk mean temperature of $T_{de}(r)$, is obtained from,

$$\overline{T}_{de} = \int_0^a T_{de} ur.dr \Big/ \int_0^a ur.dr \quad (8)$$

Using Eqs. (2) and (7), in Eq. (8),

$$\overline{T}_{de} = T_{0,a} - \frac{\mu u_{avg}^2}{k} \quad (9)$$

Eq. (1) is rendered non-dimensional with the following non-dimensional variables,

$$\left.\begin{aligned} &X^* = x/(2aPe);\ R = (r/2a);\ U = (u/u_{avg}) \\ &\theta_{1,2} = (T_{1,2} - T_w)/(T_{0,\,a} - T_w) \end{aligned}\right\} \quad (10)$$

In Eq. (10), Pe, the Peclect number, product of the Reynolds number based upon hydraulic diameter (= $2a$) and the Prandtl number, is given by,

$$Pe = RePr = u_{avg}2a/\alpha,\ Re = u_{avg}2a/\nu \text{ and } Pr = \nu/\alpha \quad (11)$$

In Eq. (11), ν and α are the kinematic viscosity and thermal diffusivity of the fluid. Introducing the non-dimensional variables defined in Eq. (10), Eq. (1), takes the following form.

$$U\frac{\partial \theta_{1,2}}{\partial X^*} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \theta_{1,2}}{\partial R}\right) + Br\left(\frac{\partial U}{\partial R}\right)^2 \quad (12)$$

It may be noted that θ_1 and θ_2 are the non-dimensional temperatures obtained with the entry temperatures $T_{de}(r)$ and $\overline{T}_{de}$ respectively. In Eq. (12), U, the non-dimensional fully developed velocity, is given by,

$$U(R) = 2\left[1 - 4R^2\right] \quad (13)$$

In Eq. (12), Br is the Brinkman number defined by,

$$Br = \mu u_{avg}^2 \Big/ \left[k\left(T_{0,\,a} - T_w\right)\right] \quad (14)$$

With reference to $T_{(0,\,a)}$, $Br < 0$ represents fluid getting heated in the pipe and $Br > 0$ represents fluid getting cooled in the pipe.

Boundary conditions given by Eqs. (3) to (6), take the non-dimensional form as,

$$\theta_{1,2} = 0 \text{ at } R = 0.5 \text{ for } X^* > 0 \quad (15)$$

$$\left(\partial \theta_{1,2}/\partial R\right) = 0 \text{ at } R = 0 \text{ for } X^* > 0 \quad (16)$$

a) When the fluid enters with T_{de},

$$\theta_1 = \theta_{de}(r) = \frac{T_{de}(r) - T_w}{T_{0,a} - T_w}, \text{ at } X^* = 0 \text{ for } 0 \le R \le 0.5 \quad (17)$$

b) When the fluid enters with $\overline{T}_{de}$,

$$\theta_2 = \theta_{de}^* = \frac{\overline{T}_{de} - T_w}{T_{0,a} - T_w} =, \text{ at } X^* = 0 \text{ for } 0 \le R \le 0.5 \quad (18)$$

Using Eq. (7) for T_{de}, into Eq. (17) for $\theta_{de}(r)$, it is obtained as,

$$\theta_{de}(r) = 1 - 2Br\left(16R^4 - 8R^2 + 1\right) \quad (19)$$

Introducing the non-dimensional variables defined in Eq. (10), into Eq. (9), the non-dimensional bulk mean temperature, θ_{de}^*, of Eq. (18) has been obtained from,

$$\theta_{de}^* = \int_0^{0.5} \theta_{de} UR.dR \Big/ \int_0^{0.5} UR.dR = (1 - Br) \quad (20)$$

Nusselt Number

The local heat transfer coefficients, $h_{1,2x}$, corresponding to the two entry temperatures $T_{de}(r)$ and $\overline{T}_{de}$ are defined by,

$$k\left(\partial T_{1,2}/\partial r\right)\Big|_a = h_{1,2x}\left(T_w - T_{b1,2}\right) \quad (21)$$

In Eq. (21), $T_{b1,2}$ are the bulk mean temperatures of the fluid at any x corresponding to the two entry temperatures $T_{de}(r)$ and $\overline{T}_{de}$. $T_{1,2\underline{b}}$, are evaluated from,

$$T_{1,2b} = \int_0^a uT_{1,2}r.dr \Big/ \int_0^a ur.dr \quad (22)$$

The corresponding non-dimensional bulk mean temperatures, $\theta_{1,2}^*$, are obtained from,

$$\theta_{1,2}^* = \frac{\left(T_{b1,2} - T_w\right)}{\left(\overline{T}_{de} - T_w\right)} = \left(1/\theta_{de}^*\right)\left(\int_0^{0.5} U\theta_{1,2}R.dR \Big/ \int_0^{0.5} UR.dR\right) \quad (23)$$

In Eq. (23), θ_{de}^* is given by Eq. (20)

On non-dimensionalizing Eq. (21), the local Nusselt numbers become,

$$Nu_{1x,2x} = \left(h_{1x,2x}2a/k\right) = -\left(1/\theta_{de}^*\right)\left(1/\theta_{1,2}^*\right)\left(\partial \theta_{1,2}/\partial R\right)\Big|_{R=0.5} \quad (24)$$

Instead of the more common expression like $Nu_x = (h_x 2a/k) = -(1/\theta^*)\,(\partial\theta/\partial R)|_{R\,=\,0.5}$, appearance of two terms θ_{de}^* and $\theta_{1,2}^*$ in Eq. (23) is a consequence of non-dimensionalizing $T_{1,2}$ with $\left(T_{0,a} - T_w\right)$ and $T_{b1,2}$ with $\left(\overline{T}_{ae} - T_w\right)$.

Heat Transfer

Wall heat transfer from (or, to) the pipe wall to (or, from) the fluid, $Q_{1,2xw}$, for the two entry temperatures, up to a desired axial distance, x, can be obtained from,

$$Q_{1,2xw} = 2\pi a\left(\int_0^x k\left(\partial T_{1,2}/\partial r\right)\Big|_a dx\right) \tag{25}$$

The non-dimensional wall heat transfer, $\bar{Q}_{1xw}$, when the fluid enters with $T_{de}(r)$, has been defined and obtained from,

$$\bar{Q}_{1xw} = \frac{Q_{1xw}}{\dot{m}C_p(T_w-\bar{T}_{de})} = \frac{4}{\theta^*_{de}}\int_0^{X^*}\left(\partial\theta_1/\partial R\right)\Big|_{R=0.5}dX^* \tag{26}$$

In Eq. (26), $\dot{m}$, the mass flow rate of the fluid, is given by,

$$\dot{m} = \rho A_c u_{avg} = \rho\left(\pi a^2\right)u_{avg} \tag{27}$$

Similarly, the non-dimensional wall heat transfer, $\bar{Q}_{2xw}$, when the fluid enters with $\bar{T}_{de}$, has been defined and obtained from,

$$\bar{Q}_{2xw} = \frac{Q_{2xw}}{\dot{m}C_p(T_w-\bar{T}_{de})} = \frac{4}{\theta^*_{ae}}\int_0^{X^*}\left(\partial\theta_2/\partial R\right)\Big|_{R=0.5}dX^* \tag{28}$$

Energy Gained (or Lost) by the Fluid

Energy gained (or lost) by the fluid, $Q_{1,2xf}$, corresponding to the two entry temperatures, up to any desired x can be obtained from,

$$Q_{1,2xf} = \dot{m}C_P\left(T_{b1,2}\,|_x - T_{b1,2}\big|_{x=0}\right) \tag{29}$$

The non-dimensional energy gained by the fluid, $\bar{Q}_{1xf}$, when the fluid enters with $T_{de}(r)$, has been defined and obtained from,

$$\bar{Q}_{1xf} = \frac{Q_{1xf}}{\dot{m}C_P\left(T_w-\bar{T}_{de}\right)} = \frac{\dot{m}C_P\left(T_{1b}\,|_x - \bar{T}_{de}\right)}{\dot{m}C_P\left(T_w-\bar{T}_{de}\right)} = 1-\theta_1^* \tag{30}$$

The non-dimensional energy gained by the fluid, $\bar{Q}_{2xf}$, when the fluid enters with $\bar{T}_{de}$, has been defined and obtained from,

$$\bar{Q}_{2xf} = \frac{Q_{2xf}}{\dot{m}C_P\left(T_w-\bar{T}_{de}\right)} = \frac{\dot{m}C_P\left(T_{2b}\,|_x - \bar{T}_{de}\right)}{\dot{m}C_P\left(T_w-\bar{T}_{de}\right)} = 1-\theta_2^* \tag{31}$$

In Eqs. (29) and (30), θ_1^* and θ_2^* are given in Eq. (23).

NUMERICAL SCHEME

Numerical solutions to Eq. (12) have been obtained using Successive Accelerated Replacement (SAR) scheme [25-28] as described below in brief. The scheme is basically the successive over relaxation (Gauss-Seidel) method, see, Antia, p.677 [29], though the terminology of SAR has been used by Dellinger [30].

The basic philosophy of the SAR scheme [25-28] is to guess an initial profile for each variable such that the boundary conditions are satisfied. Let the partial differential equation governing a variable, ϕ (X, R), expressed in finite difference form be given by $\tilde{\phi}_{M,N} = 0$, where M, and N represent the nodal points when the non-dimensional axial distance and radius of the annulus are divided into a finite number of intervals MD and ND respectively. The guessed profile for the variable ϕ at any mesh point generally will not satisfy the equation. Let the error in the equation at (M, N) and k^{th} iteration be $\tilde{\phi}^k_{M,N}$. Then the $(k+1)^{th}$ approximation to the variable ϕ is obtained from,

$$\phi^{k+1}_{M,N} = \phi^k_{M,N} - \omega\left(\frac{\tilde{\phi}^k_{M,N}}{\partial\tilde{\phi}^k_{M,N}/\partial\phi_{M,N}}\right) \tag{32}$$

In Eq. (32), ω is an acceleration factor which varies between 0 and 2. $\omega < 1$ represents under relaxation and $\omega > 1$ represents over relaxation. The procedure of correcting the variable ϕ at each mesh point in the entire region of interest is repeated until a set of convergence criteria is satisfied. For example the change in the variable at any mesh point between k^{th} and $(k+1)^{th}$ approximation satisfies,

$$\left|1-\left(\phi^k_{M,N}/\phi^{k+1}_{M,N}\right)\right| < \varepsilon \tag{33}$$

In Eq. (33), ε is a prescribed small positive number.

To correct the guessed profile, each dependent variable has to be associated with one equation. It is natural to associate the variable with the equation, which contains the highest order derivative in that variable. Eq. (12) shall be associated to obtain the numerical solutions for θ_1 and θ_2.

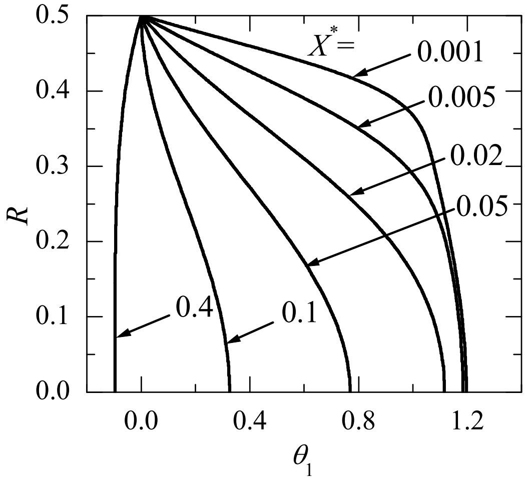
RESULTS AND DISCUSSION

Numerical solutions to Eq. (12), with the boundary conditions given by Eqs. (15) to (17) along with Eq. (19), when the fluid enters with the dissipative entry temperature, $T_{de}(r)$ {referred as Case 1}, have been obtained employing the SAR scheme [25-28] as described above, for Br = -0.1 and 0.1. Similarly, numerical solutions have also been obtained for Eq. (12), with the boundary conditions given by Eqs. (15), (16) and (18) along with Eq. (20) for the case when the fluid enters with a uniform temperature equal to the bulk mean temperature $\bar{T}_{de}$, of the dissipative entry temperature, {referred as Case 2}, for Br = -0.1 and 0.1. Several numerical trials have been conducted to determine suitable values for, the number of grids in X^* and R directions, ω and ε. $\omega < 1$, which represents under-relaxation has been chosen, and a value of 0.8 has been found to be satisfactory. $\varepsilon = 10^{-5}$, MD = 1000 and ND = 80 have been found to be satisfactory, determined by comparing the present values with the values of the Nusselt number available in Shah and London [4], p. 103, for the case of uniform entry temperature neglecting viscous dissipation.

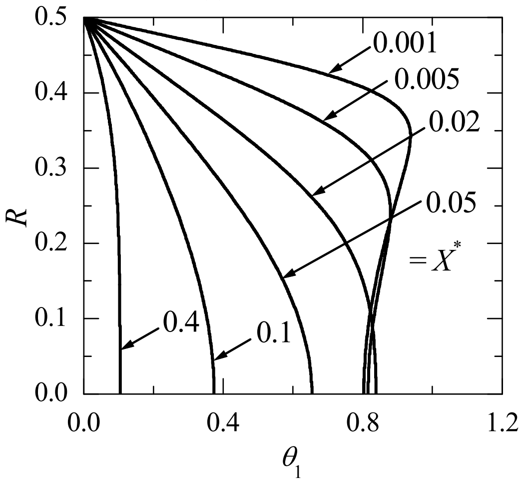
Temperature profiles

Variation of non-dimensional temperature, θ_1 {i.e. for Case 1}, with non-dimensional radius, R, at different non-dimensional axial distance, X^*, for Br = -0.1 and 0.1 are shown in Figure 2 (a) and (b). Similarly, the θ_2 {i.e. for Case 2}, profiles for Br = -0.1 and 0.1 are shown in Figure 3 (a) and (b).

It can be observed by comparing Figure 2 (a) and (b) with Figure 3 (a) and (b) respectively, that the difference between θ_1 and θ_2 is insignificant when $X^* > 0.02$ for all R. From Figure 2 (a) and Figure 3 (a) it can be observed, $\theta_1 > \theta_2$ for lower X^* when Br = -0.1. Similarly, when Br = 0.1, it can be noticed that,

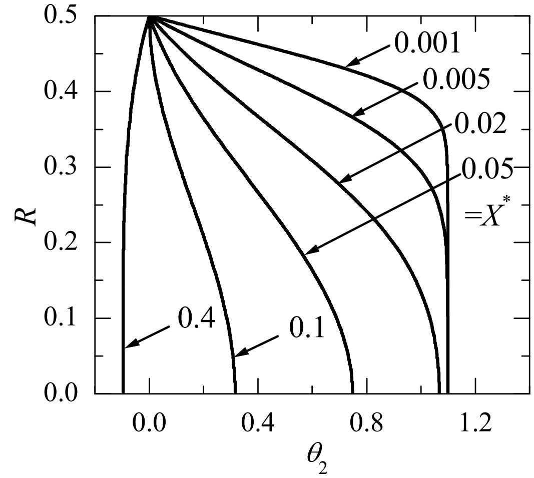


(a) Br = -0.1

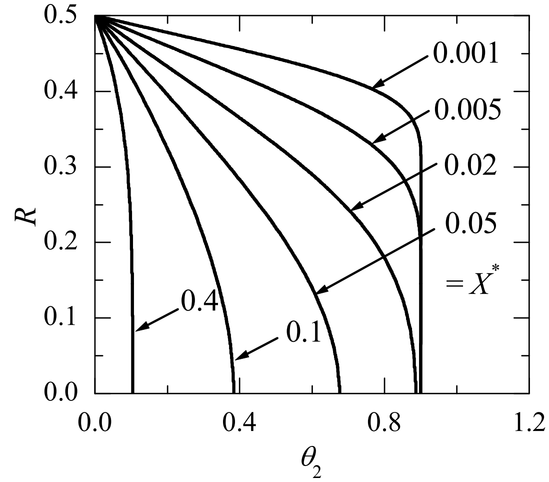


(b) Br = 0.1

Figure 2 Variation of θ_1 with R for different X^*.



(a) Br = -0.1



(b) Br = 0.1

Figure 3 Variation of θ_2 with R for different X^*.

$\theta_1 < \theta_2$ from Figure 3 (b) and Figure 3 (b) for lower X^*. Very near the entry, when the fluid enters with $T_{de}(r)$, for $Br < 0$, the fluid gets less heated since the difference between the wall temperature and $T_{0,\,a}$ that exists near the entry, close to the pipe wall, is smaller compared to when the fluid enters with a uniform temperature of $\overline{T}_{de}$. This makes $\theta_1 > \theta_2$ when $Br < 0$ near the entry. A similar argument establishes that $\theta_1 < \theta_2$ near the entry when $Br > 0$. The effect of the difference in the entry temperatures diminishes at larger axial distance, particularly because of heat addition by viscous dissipation.

It is to be borne in mind, that, when, $T_w > T_{0,,a}$, $Br < 0$. $Br < 0$ represents the fluid getting heated for both the cases of entry temperature since, $T_w > T_{0,,a} > \overline{T}_{de}$. $Br > 0$ may not represent cooling for the two cases of entry temperature since $\overline{T}_{de}$ may be lower than T_w, making, $\overline{T}_{de} < T_w < T_{0,\,a}$. Hence, θ_1 and θ_2 profiles may differ at larger X^* also for large positive values of Br.

Bulk mean temperature

Numerical solutions obtained for θ_1 and θ_2 have been used in Eq. (23), to obtain θ_1^* and θ_2^*, the non-dimensional bulk mean temperatures, when the fluid enters with $T_{de}(r)$ and $\overline{T}_{de}$.

Variation of the non-dimensional bulk mean temperatures, $\theta_{1,2}^*$ for the two cases considered, with non-dimensional axial distance, X^*, is shown in Figure 4 (a) for Br = -0.1 and in (b) for Br = 0.1.

θ_1^* and θ_2^* for all values of X^* differ by less than 2 %. The discernable difference between θ_1 and θ_2 that has been noted for small X^* is not seen between θ_1^* and θ_2^*; θ_1^* and θ_2^* being averaged quantities that depend only on X^* and do not depend on R. However a qualitative difference is that, $\theta_1^* > \theta_2^*$ for $Br < 0$ and $\theta_1^* < \theta_2^*$ for $Br > 0$. These inequalities follow from the argument given when θ_1 and θ_2 profiles shown in Figure 2 and Figure 3 have been discussed.

Nusselt number

From the numerical solutions obtained for θ_1, θ_1^*, θ_2 and θ_2^*, along with θ_{de}^*, the Nusselt numbers Nu_{1x} and Nu_{2x} have been calculated using Eq. (24). Nu_{1x} and Nu_{2x} respectively correspond to the two cases of entry temperature T_{de} (r) and $\overline{T}_{de}$.

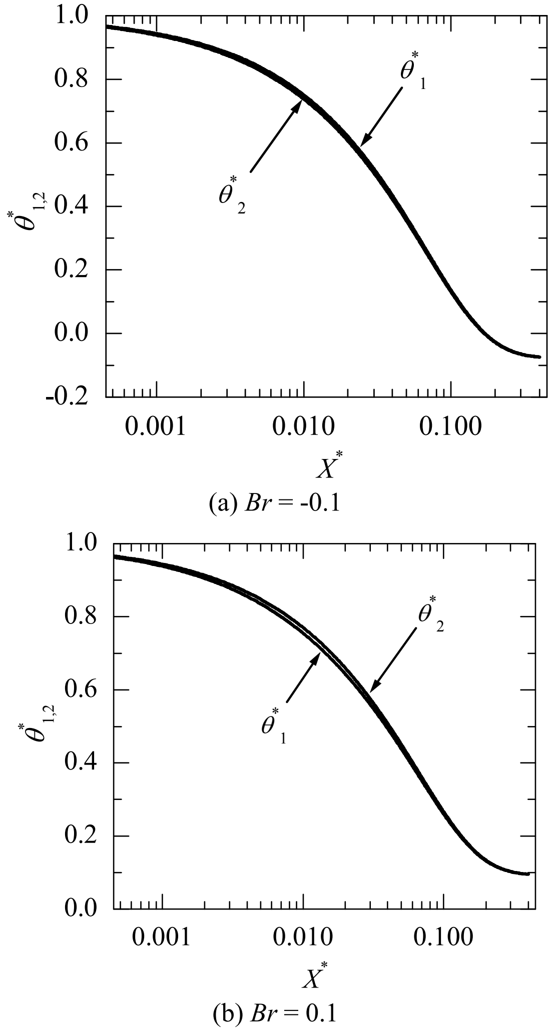


Figure 4 Variation of $\theta^*_{1,2}$ with X^* for Cases 1 and 2

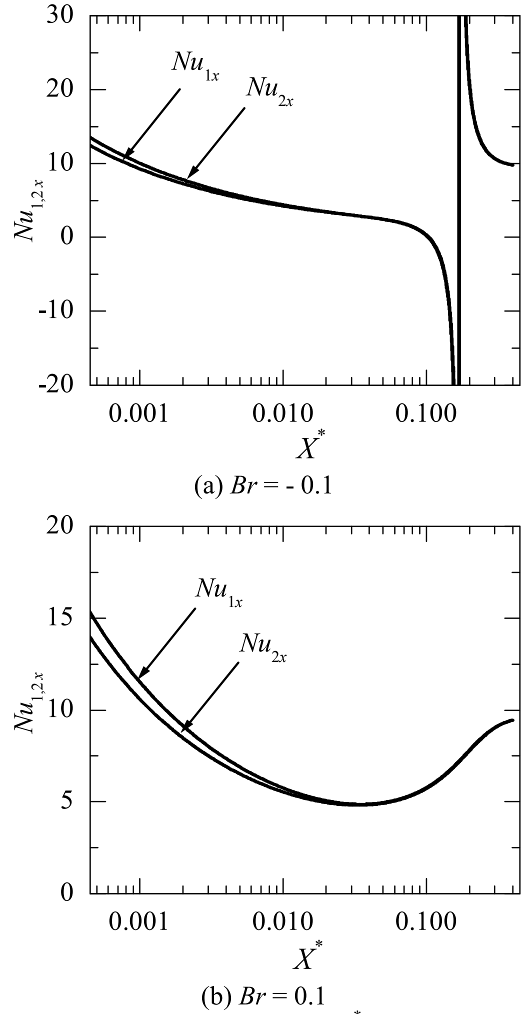


Figure 5 Variation of $Nu_{1,2x}$ with X^* for Cases 1 and 2

Variation of $Nu_{1,2x}$ with X^* is shown in Figure 5 for (a) Br = -0.1 and (b) Br = 0.1. It can be observed from Figure 5, that, Nu_{1x} and Nu_{2x} do not differ much except for lower X^*. Both Nu_{1x} and Nu_{2x} display an unbounded swing for $Br < 0$ at some $X^* = X^*_{sw}$ indicating that T_{1b} and $T_{2b} \rightarrow T_w$. It can be seen from Figure 5 (a) that X^*_{sw} is practically the same for the two entry temperatures. The trend of Nu_{1x} and Nu_{2x} variation with X^* shown in Figure 5 (b) for $Br > 0$, is along the expected lines as reported in Barletta and Magyari [22] for pipes, or Aydin and Avci [19] for channels. Both Nu_{1x} and Nu_{2x} for $Br < 0$ and $Br > 0$ tend to the limiting value of 9.6 for large X^* which is also the value available in [22].

Heat Transfer

From the numerical solutions, heat transferred from (or to) the wall, $\bar{Q}_{1xw}$ and $\bar{Q}_{2xw}$ have been calculated using Eqs. (26) and (28) when the fluid enters with $T_{de}(r)$ or $\bar{T}_{de}$.

Variation of $\bar{Q}_{1,2xw}$ with X^* is shown in Figure 6 for (a) Br = -0.1 and (b) Br = 0.1.

As noted with reference to the bulk mean temperatures θ^*_1 and θ^*_2 for the two cases of entry temperature, there is no significant difference between $\bar{Q}_{1xw}$ and $\bar{Q}_{2xw}$. It can be noticed that the small difference, between $\bar{Q}_{1xw}$ and $\bar{Q}_{2xw}$ is such, that, $\bar{Q}_{1xw} < \bar{Q}_{2xw}$ when $Br < 0$ and $\bar{Q}_{1xw} > \bar{Q}_{2xw}$ when $Br > 0$ which is opposite to the trend in the bulk mean temperatures, $\theta^*_1 > \theta^*_2$ for $Br < 0$ and $\theta^*_1 < \theta^*_2$ for $Br > 0$. This is to be expected when it is realized that the energy gained or lost by the fluid $\bar{Q}_{1,2xf}$ are given by $1-\theta^*_{1,2}$.

A clear difference in the trends of variation of $\bar{Q}_{1,2xw}$ for $Br < 0$ as shown in Figure 6 (a) and in Figure 6 (b) for $Br > 0$ can be noted. $\bar{Q}_{1,2xw}$ monotonically increase with X^* when $Br > 0$, representing the case of fluid getting cooled. This calls for a continuous removal of heat from the wall, even after the fluid reached a limiting temperature above the wall temperature. At this stage, the heat removal rate is equal to the energy generated by viscous dissipation. Where as, when $Br < 0$, the fluid gets

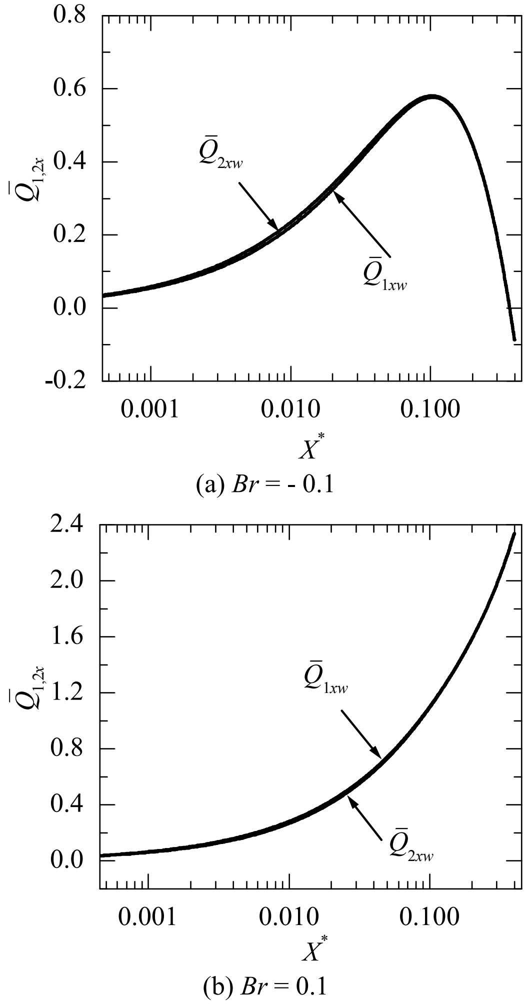


Figure 6 Variation of $\bar{Q}_{1,2x}$ with X^* for Cases 1 and 2

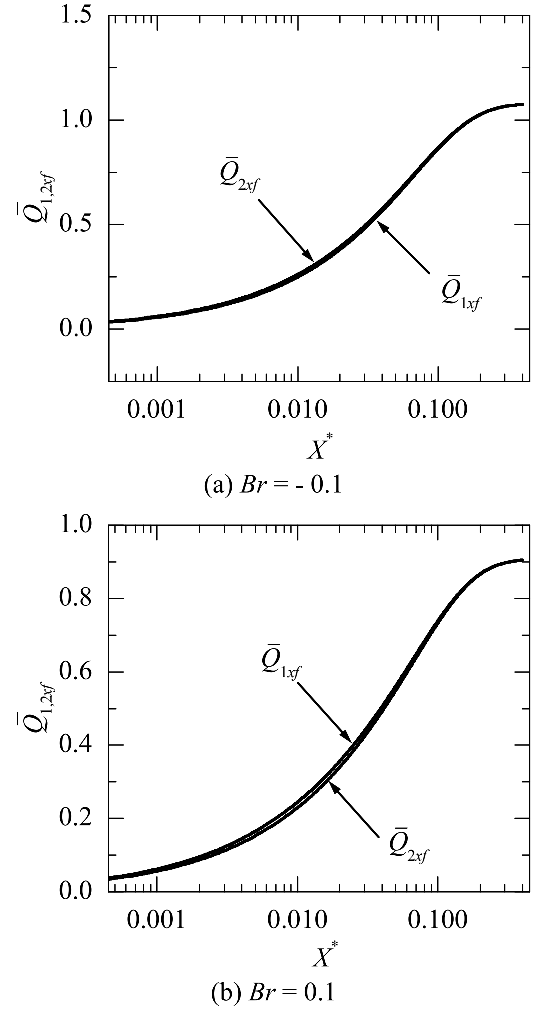


Figure 7 Variation of $\bar{Q}_{1,2xf}$ with X^* for Cases 1 and 2

heated to a limiting temperature higher than the wall temperature. This excess temperature depends on the energy generated due to dissipation. This amount of energy needs to be removed from the wall, to keep the wall at constant temperature. It is because of this change in the direction of heat flow, $\bar{Q}_{1,2xw}$ starts decreasing from some X^* when $Br < 0$. This X^* of course will be higher than X^*_{sw} where the Nusselt number displays an unbounded swing.

Though, it is straight forward to calculate $\bar{Q}_{1,2xf}$ {Eqs. (30) and (31)} from the values of θ_1^* and θ_2^* which can be obtained from Figure 4, variation of Q_{1xf} and Q_{2xf} with X^*, is shown in Figure 7 for a) $Br = -0.1$ and b) $Br = 0.1$.

$\bar{Q}_{1xf}$ and $\bar{Q}_{2xf}$ also replicate the qualitative differences shown by $\bar{Q}_{1xw}$ and $\bar{Q}_{2xw}$ in Figure 6. $\bar{Q}_{1xf} < \bar{Q}_{2xf}$ when $Br < 0$ and $\bar{Q}_{1xf} > \bar{Q}_{2xf}$ when $Br > 0$. Unlike $\bar{Q}_{1,2xw}$, $\bar{Q}_{1,2xf}$ monotonically increases with X^* until the fluid reaches the limiting temperature beyond which $\bar{Q}_{1,2xf}$ remain unchanged. By comparing Figure 6 (a) and (b) with Figure 7 (a) and (b) respectively, it can be noted that $\bar{Q}_{1,2xf} > \bar{Q}_{1,2xw}$ when $Br < 0$ whereas, $\bar{Q}_{1,2xw} > \bar{Q}_{1,2xf}$ for $Br > 0$. This implies that the fluid gets heated more than the energy transferred from the wall due to viscous dissipation, whereas, the fluid gets less cooled than the energy removed from the wall due to viscous dissipation.

CONCLUSION

The influence of dissipative entry temperature {the temperature attained in an adiabatic preparatory zone as described by Barletta and Magyari [22]} on temperature profiles, local Nusselt numbers has been examined. In addition, the plots presented in the present study for the wall heat transfer and energy gained or lost by the fluid can be conveniently used for calculating the same instead of an average Nusselt number.

Dissipative entry temperature that depends on the radial coordinate can be replaced by a uniform entry temperature equal to the bulk mean temperature of the dissipative entry temperature. This retains the validity and simplicity of assuming the entry temperature to be uniform. However, the Brinkman number needs to be based on the bulk mean temperature of the dissipative entry temperature.

REFERENCES

[1] Graetz, L., Uber die wrmeleitungsfähigkeit von flüssigkeiten, *Ann. Phys. Chem.,* Vol. 18, 1883, pp. 79-94.

[2] Graetz, L., Uber die wärmeleitungsfähigkeit von flüssigkeiten, *Ann. Phys. Chem.,* Vol. 25, 1885, pp. 337-357.

[3] Nusselt, W., Die abhängigkeit der wäreübergangszahl von der rohrlänge, *VDIZ*, Vol. 54, 1910, pp. 1154-1158.

[4] Shah, R. K., and London, A. L., Laminar Flow Forced Convection in Ducts, *Advances in Heat Transfer*, Academic New York, 1978.

[5] Kakac, S., Shah, R. K., and Aung, W., Handbook of Single-Phase Convective Heat Transfer, *Wiley*, New York, 1987.

[6] Brinkman, H. C., Heat effect in capillary flow I, *Apl. sci. Res.*, Vol. A2, 1951, pp. 120-124.

[7] Ou, J. W., and Cheng, K. C., Viscous dissipation effects on thermal entrance region heat transfer in pipes with uniform wall heat flux, *Apl. sci. Res.*, Vol. 28, 1973, pp. 289-301.

[8] Collins, M. W., and Keynejad, M., Effects of temperature dependence of viscous and viscous dissipation on laminar flow heat transfer in circular tubes, *Int. J. Heat Fluid Flow*, Vol. 4, 1983, pp. 33-42.

[9] Quaresma, J. N. N., and Cotta, R. M., Exact solutions for thermally developing tube flow with variable wall heat flux, *Int. Comm. Heat Mass Transfer*, Vol. 21, 1994, pp. 729-742.

[10] Stefano Piva, An analytical approach to fully developed heating of laminar flows in circular pipes, *Int. Comm. Heat Mass Transfer*, Vol. 22, 1995, pp. 815-824.

[11] Barletta, A., On forced convection in a circular duct with slug flow and viscous dissipation, *Int. Comm. Heat Mass Transfer*, Vol. 23, 1996, pp. 69-78.

[12] Antonio Barletta, and Enzo Zanchini, Forced convection in the thermal entrance region of a circular duct with slug flow and viscous dissipation, *Int. J. Heat Mass Transfer*, Vol. 40, 1997, pp. 1181-1190.

[13] Basu, T., and Roy, D. N., Laminar heat transfer in a tube with viscous dissipation, *Int. J. Heat Mass Transfer*, Vol. 28, 1985, pp. 699-701.

[14] Lin, T. F., Hawks, K. H., and Leidenfrost, W., Analysis of viscous dissipation effect on thermal entrance heat transfer in laminar pipe flows with convective boundary conditions, *Wärme-und Stoffübertragung*, Vol. 17, 1983, pp. 97-105.

[15] Antonio Barletta, Slug flow heat transfer in circular ducts with viscous dissipation and convective boundary conditions, *Int. J. Heat Mass Transfer*, Vol. 40, 1997, pp. 4219-4228.

[16] Alan Jones, S., Extension to the solutions of the Graetz problem, *Int. J. Heat Mass Transfer*, Vol. 14, 1971, pp. 619-623.

[17] Hiroyuki Nagasue, Steady state heat transfer with axial conduction in laminar flow in a circular tube a specified temperature or heat flux wall, *Int. J. Heat Mass Transfer*, Vol. 24, 1981, pp. 1823-1832.

[18] Orhan Aydin, Effects of viscous dissipation on the heat transfer in forced pipe flow. Part 2: Thermally developing flow, *Energy Conservation Management*, Vol. 46, 2005, pp. 3091-3102.

[19] Orhan Aydin, and Mete Avci, Viscous dissipation effects on heat transfer in a Poiseuille flow, *Applied Energy*, Vol. 83, 2006, pp. 495-512.

[20] Gokturk Tunc, and Yildiz Bayazitoglu, Heat transfer in micro tubes with viscous dissipation, *Int. J. Heat Mass Transfer*, Vol. 44, 2001, pp. 2395-2403.

[21] Jambal, O., Shigechi, T., Davva, G., and Momoki, S., Effect of viscous dissipation and fluid axial heat conduction on heat transfer for non-Newtonian fluids in ducts with uniform wall temperature. Part I: Parallel plates and circular ducts, *Int. Comm. Heat Mass Transfer*, Vol. 32, 2005, pp. 1165-1173.

[22] Barletta, A., and Magyari, E., Thermal entrance heat transfer of adiabatically prepared fluid with viscous dissipation in a tube with isothermal wall, *ASME J Heat Transfer*, Vol. 128, 2006, pp. 1185-1193.

[23] Barletta, A., and Magyari, E., Forced convection with viscous dissipation in the thermal entrance region of a circular duct with prescribed wall heat flux, *Int. J. Heat Mass Transfer*, Vol. 50, 2007, pp. 26-35.

[24] Satyamurty, V. V., Successive Accelerated Replacement scheme applied to study of natural convection heat transfer in porous cryogenic insulations, ASME paper no. 84-WA/HT-37, 1984

[25] Marpu, D. R., and Satyamurty, V. V., Influence of variable fluid density on free convectin in rectangular porous media, *ASME Journal of Energy Resources Technology*, Vol. 111, 1989, pp. 214 – 220.

[26] Satyamurty, V. V., Marpu, D. R., Relative effects of variable fluid properties and non-Darcy flow on convection in porous media, *ASME HTD*, Vol. 96, 1988, pp. 613-621.

[27] Satyamurty, V. V., and Bhargavi, D., 2010, "Forced Convection in Thermally Developing Region of a Channel Partially Filled with a Porous Material and Optimal Porous Fraction," Int. J. Thermal Sciences, **49**, pp. 319-332.

[28] Ramjee Repaka and Satyamurty, V. V., 2010, "Local and Average Heat Transfer in the Thermally Developing Region of an Asymmetrically Heated Channel," Int. J. Heat Mass Transfer, **53**, pp. 1654-1665.

[29] Antia, H. M., Numerical methods for scientists and engineers, *Tata McGraw Hill*, 1991

[30] Dellinger, T. C., Computations on non-equilibrium merged shock layer by Successive Accelerated Replacement scheme, *AIAA Journal*, Vol. 9, 1971, pp. 262-269.