The Mispricing Return Premium

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Abstract

We show that, when stock prices are subject to stochastic mispricing errors, expected rates of return may depend not only on the fundamental risk that is captured by a standard asset pricing model, but also on the type and degree of asset mispricing, even when the mispricing is zero on average. Empirically, the mispricing induced return premium, either estimated using a Kalman filter or proxied by the volatility and variance ratio of residual returns, is shown to be significantly associated with realized risk adjusted returns.

1 Introduction

In this paper we consider the implications of Jensen's inequality for stock prices and expected rates of return in markets which are less than perfectly efficient. In particular we show that, for securities which are subject to stochastic mispricing relative to a given equilibrium asset pricing model, it is likely that either their *prices* will fail to be unconditionally rational or their *returns* will fail to be unconditionally rational, or both. By *unconditionally rational prices* we mean prices whose unconditional expectations are consistent with the fundamental asset pricing model, and by *unconditionally rational returns* we mean returns whose unconditional expectations are consistent with the fundamental asset pricing model. For example, a stock that on average trades at 100% of its fundamental value, but whose price fluctuates about the fundamental value, will have an average return that is higher than that predicted by the model that determines the fundamental value. Our use of Jensen's inequality to impose restrictions on prices and returns follows earlier applications of Jensen's inequality to expectations based theories of asset pricing. For example, Siegel's (1972) paradox, which relies on the inequality, implies that if foreign exchange rates are stochastic and the forward exchange rate between currency A and currency B is equal to the expected future spot rate, then the forward rate of exchange between currency B and currency A cannot also be equal to the expected future spot rate: there must be a risk premium associated with at least one of the forward rates. Similarly, Cox *et al.* (1981) use Jensen's inequality to show pair-wise inconsistency between three traditional definitions of the expectations theory of the term structure of interest rates.

Our analysis follows immediately from Jensen's inequality because price is a non-linear function of expected return, so that if one variable is subject to random error then the expectation of the other variable will be biased. It is of course possible that neither prices nor expected returns are unconditionally rational. If prices are unconditionally rational, then the premium in unconditional expected returns due to mispricing is shown to depend on the volatility and first order autocorrelation of the mispricing.

Since the simple analysis underlying our empirical work relies only on Jensen's inequality, it is model-independent. We do not make specific assumptions about the *cause* of mispricing, so that our analysis applies to the models such as those of De Long *et al.* (1990), Madhavan and Smidt (1993), and Campbell and Kyle (2001), in which mispricing arises as the result of noise trader demands. Taking mispricing as given, we explore its empirical consequences for average returns. Since the mispricing is not directly observable, we use proxies for the mispricing return premium. Our empirical tests reveal that portfolios formed on the basis of these proxies have significantly different returns after adjusting for risk. This is consistent with at least a part of the Jensen's inequality effect being impounded in returns.

The remainder of the paper is organized as follows. In Section 2 we discuss how this paper is related to prior literature. Section 3 presents a simple one period example which shows that, in the presence of random security mispricing, unconditionally rational prices are inconsistent with unconditionally rational returns. Section 4 analyzes the return bias introduced by random mispricing in a general intertemporal context. Section 5 presents empirical results relating returns to proxies for the mispricing return premium, and Section 6 concludes.

2 Related Literature

This paper is at the intersection of the empirical market efficiency and the asset pricing literatures, which often overlap but to our knowledge have not been integrated. On the one hand, tests of classical asset pricing models such as the CAPM, CCAPM, or ICAPM implicitly rely on an assumption of market efficiency which permits the substitution of realized returns for expected returns. On the other hand, most tests of market efficiency rely on some model of equilibrium returns.[1] However, there is now extensive evidence that common stocks are mispriced relative to these models,[2] although the reasons for the pricing discrepancies remain in dispute. For example, de Bondt and Thaler (1985, 1987) find long run reversals of prior stock price changes which they interpret as corrections of prior over-reactions to news, while Jegadeesh and Titman (1993) among others find positive autocorrelation of individual stock returns at the 6-12 month horizon, which is consistent

[1] Cf. Fama (1991, p1575): 'Thus, market efficiency per se is not testable. It must be tested jointly with some model of equilibrium, an asset-pricing model.'

[2] French and Roll (1986) suggest that on average 4 to 12% of the daily return variance of common stock returns is due to mispricing.

with the slow adjustment to firm specific news documented in a large number of studies. Jegadeesh and Titman (1995) also find evidence that stock prices tend to over-react to firm specific information. Brennan *et al.* (1993) show that the speed of adjustment of prices to market wide information depends on firm size and analyst following. Lee and Swaminathan (2000) find that low (high) trading volume stocks tend to be under- (over-) valued by the market. There is also an extensive accounting literature demonstrating that stock market prices do not fully reflect the fundamental information contained in accounting statements.[3] Finally, recent research relates stock returns to non-fundamental information: Pastor and Stambaugh (2003), Acharya and Pedersen (2005) and Sadka (2006) show that stock returns are affected by (or at least covary with) the state of stock market liquidity, while Amihud (2002) shows that unanticipated increases in market illiquidity reduce the level of stock prices. Lee *et al.* (1991), Swaminathan (1996) (more circumspectly) and Baker and Wurgler (2006) argue that stock prices are affected by the state of 'sentiment'.

Our analysis is also related to an early literature on the implications of security mispricing for measuring rates of return, including Blume and Stambaugh (1983) and Roll (1983), who are concerned with the effects of daily auto-correlations and the bid-ask bounce on measured rates of return.[4] More recently, Liu and Strong (2006) analyze the effects of portfolio rebalancing assumptions on reported returns. Other recent contributions include Bessembinder and Kalcheva (2006), and Arnott *et. al.* (2006). Bessembinder and Kalcheva (2006) are concerned with the return bias induced by the bid-ask spread which causes *iid* pricing errors. We analyze a more general pricing error structure, and find empirically that the greatest return premium arises for stocks with positively correlated pricing errors which cannot be caused by simple bid-ask bounce, although it could be consistent with the dealer inventory behavior described by Madhavan and Smidt (1993); these authors show that the price set by the specialist departs from her best estimate of fundamental value by an amount that depends on the level of specialist inventories which follows a slow mean-reverting process [5]. Arnott *et. al.* (2006) use calibration to show that mispricing can potentially account for size and value effects in asset pricing. In contrast, we estimate the mispricing return

[3]For examples, see Sloan (1996) and Mohanram (2004).
[4]See also Canina *et al* (1998).
[5]See also Hendershott and Seasholes (2007)

premium directly and show that it is related to risk adjusted returns.

The analysis in this paper has implications for studies that find significant relations between stock returns and variables that may be proxies for the mispricing return premium we consider. Measures of the cost of transacting such as the bid ask spread or Kyle (1985) λ are likely to be positively associated with the magnitude of pricing errors since transactions costs impede arbitrage. This suggests that a part of the approximately 7% return differential between high and low liquidity portfolios documented in several studies[6] may be attributable to this mispricing return premium. The mispricing return premium also provides an alternative explanation for the excess returns found to be associated with idiosyncratic volatility by Malkiel and Xu(2006) and Spiegel and Wang (2006), since stocks with high idiosyncratic volatility tend to have a high mispricing return premium. Similarly, the sensitivity of stock returns to variables that have common effects on stock prices but not fundamental values, such as market liquidity or sentiment, will affect the volatility of mispricing and hence the mispricing return premium. Thus a part of the the annual return premium of around 7.5% between high and low liquidity beta portfolios reported by Pastor and Stambaugh (2003) may be attributable to the mispricing return premium. Finally, a return premium of the type that Hou and Moskowitz (2005) have found to be associated with slow adjustment to (market-wide) information is consistent with a model in which prices are unconditionally rational but adjust slowly to new information, since these stocks will offer a mispricing return premium.

3 A Simple Example

Consider an asset whose payoff at the end of one period is $\tilde{X}$, and denote its fundamental price by P^*. Then

$$P^* = \frac{E[\tilde{X}]}{1 + r^*} \tag{1}$$

where r^* is the equilibrium expected rate of return on the security according to some given pricing model. Let $P \equiv P^* \tilde{Z}$ denote the market price of the security, where $\tilde{Z}$ is a random variable which is independent of the payoff $\tilde{X}$. Then the mispricing of the security relative

[6]For an extensive survey of the research on liquidity and asset pricing see Amihud *et al.* (2005).

to the given model is written as $P^*(\tilde{Z}-1)$. If $\tilde{Z}$ has mean unity then we say that the price is unconditionally rational. Let $\tilde{R}$ denote the realized rate of return on the security. Then:

$$1+\tilde{R} = \frac{\tilde{X}}{P} = \frac{\tilde{X}}{P^*\tilde{Z}} = \frac{\tilde{X}(1+r^*)}{E[\tilde{X}]\tilde{Z}} \tag{2}$$

Taking expectations, we have:

$$E[1+\tilde{R}] = (1+r^*)E\left[\frac{1}{\tilde{Z}}\right] > 1+r^* \tag{3}$$

Thus the unconditional expected return on the security exceeds the equilibrium expected return, r^*, by what we have called a 'mispricing return premium' so long as the price is unconditionally rational so that $E[\tilde{Z}] = 1$. Conversely, if the returns are unconditionally rational so that $E[\tilde{R}] = r^*$, then $E[\tilde{Z}] > 1$ if $var(\tilde{Z}) > 0$ so that there is a positive mispricing price premium.

4 General Structure

Now consider an arbitrary multi-period setting in which the security pays a dividend of D_t at the end of period t and denote the *market* price at the end of the period by P_t. We contrast the market price with the *fundamental* price, P_t^*, which is the price according to some given asset pricing model. We write $P_t \equiv P_t^* Z_t$. We shall assume throughout that market prices are strictly positive and impose the further restriction that $z \equiv lnZ$, the log of the 'market pricing multiple', Z, is a time-homogeneous stationary process, so that the variance of relative mispricing is finite.

Then we can write $1+R_t$ the (gross) market return on the security in period t as

$$\begin{aligned} 1+R_t \equiv \frac{P_t^* Z_t + D_t}{P_{t-1}^* Z_{t-1}} &= \frac{P_t^*}{P_{t-1}^*}\frac{Z_t}{Z_{t-1}} + \frac{D_t}{P_{t-1}^* Z_{t-1}} \\ &\equiv (1+R_t^{*g})(1+\frac{\Delta Z_t}{Z_{t-1}}) + \delta_t^*(1/Z_{t-1}) \end{aligned} \tag{4}$$

where $\Delta Z_t \equiv Z_t - Z_{t-1}$, $R_t^{*g} \equiv (P_t^* - P_{t-1}^*)/P_{t-1}^*$ is the 'capital gain return' based on the fundamental price, and $\delta_t^* \equiv D_t/P_{t-1}^*$ is the dividend yield based on the fundamental price. Note that the return based on the fundamental price, R_t^* is equal to $R_t^{*g} + \delta_t^*$.

Then the market return, R_t, is related to the fundamental return, R_t^*, by:

$$R_t = R_t^* + R_t^{*g}\frac{\Delta Z_t}{Z_{t-1}} + \frac{\Delta Z_t}{Z_{t-1}} - \delta_t^*(1 - 1/Z_{t-1}) \tag{5}$$

Assume for simplicity that the mispricing variable $\tilde{Z}_t$ is independent of the (fundamental) dividend yield δ_t^*. Then, taking expectations in (5), the expected market return is related to the expected fundamental rate of return by:

$$\begin{aligned} E[R_t] &= E[R_t^*] + E\left[\frac{\Delta Z_t}{Z_{t-1}}\right] + cov\left(R_t^{*g}, \frac{\Delta Z_t}{Z_{t-1}}\right) - E\left[\delta_t^*\right] E\left[1 - 1/Z_{t-1}\right] + E\left[R_t^{*g}\right] E\left[\frac{\Delta Z_t}{Z_{t-1}}\right] \\ &\equiv E[R_t^*] + B_1 + B_2 + B_3 + B_4 \end{aligned} \tag{6}$$

where $B_1 \equiv E\left[\frac{\Delta Z_t}{Z_{t-1}}\right]$, etc., and $B \equiv B_1 + B_2 + B_3 + B_4$ denotes the premium in the unconditional expected return associated with mispricing relative to the asset pricing model that determines the fundamental price.

Consider first the premium element B_1. Note that we can write $\Delta Z/Z = e^{\Delta z} - 1$, where $z \equiv lnZ$. Then it follows from the convexity of the exponential function and the assumed time-homogeneity and stationarity of z that $B_1 \equiv E[\Delta Z/Z] > 0$. Assume next that the unconditional distribution of $\Delta z \equiv lnZ_t - lnZ_{t-1}$ is normal with parameters $(\mu_{\Delta z}, \sigma_{\Delta z})$. Since z is a stationary random variable, $\mu_{\Delta z} = 0$. Then

$$B_1 = e^{\frac{1}{2}\sigma^2_{\Delta z}} - 1$$

Now we can always write:

$$\Delta z_t \equiv z_t - z_{t-1} = (\rho_1^z - 1)z_{t-1} + \eta_t$$

where the first order autocorrelation of z, $\rho_1^z < 1$ since z is stationary, and η_t is a zero mean normally distributed error term that is independent of z_{t-1}. This allows us to write:

$$B_1 = e^{(1-\rho_1^z)\sigma_z^2} - 1. \tag{7}$$

where σ_z^2 is the unconditional variance of z. Thus the mispricing return premium is decreasing in the first order autocorrelation of z; *ceteris paribus,* mispricing that is rapidly eliminated or even reversed will lead to a higher premium. The premium is also increasing in the unconditional variance of the mispricing.

If $B_2 \equiv cov\left(R_t^{*g}, \frac{\Delta Z_t}{Z_{t-1}}\right) > 0$, the mispricing is associated with *over-reaction* since the pricing error tends to increase when fundamentals improve and to decrease when they deteriorate. If $B_2 < 0$, the mispricing is associated with *slow adjustment* since an increase (decrease) in the fundamental price is accompanied on average by a smaller proportional change in the market price. Daniel *et al.* (1998) demonstrate that both over-reaction and under-reaction to information can occur as a result of investor overconfidence about the precision of private information and biased self-attribution. If $B_2 = 0$, the mispricing is *unrelated to fundamentals.*

Then, approximating the fourth term in (6)by a Taylor expansion,

$$B_3 \approx -E\left[\delta_t^*\right]\left[1 - \frac{1}{E[Z]}\left(1 + \frac{\sigma_Z^2}{E[Z]^2}\right)\right]. \tag{8}$$

B_3 reflects the impact of mispricing on the dividend yield. It will be zero if the fundamental dividend yield is zero, and under the unconditional rational pricing assumption that $E[Z] = 1$, B_3 is likely to be small. When $E[Z] = 1$, $B_3 = E[\delta_t^*]\sigma_Z^2 > 0$, so that if the standard deviation of the mispricing is 0.3 and $\sigma_Z^2 = 0.09$, then B_3 will be of the order of 0.2% for a stock with a 2% (fundamental) dividend yield. B_3 will also be positive when there is average underpricing so that $E[Z_t] < 1$. However, if there is overpricing on average, then B_3 can be negative, although it is bounded below by the (negative of the) fundamental dividend yield. For example, if there is 10% overpricing ($E[Z_t] = 1.1$), $B_3 \approx -4$ bp per year under the same assumptions for σ_Z and $E[\delta_t^*]$.

$B_4 \equiv E\left[R_t^{*g}\right] E\left[\frac{\Delta Z_t}{Z_{t-1}}\right] = E\left[R_t^{*g}\right] B_1$. Since the expected capital gain is of the order of 10% per year, B_4 is an order of magnitude smaller than B_1, and for simplicity we ignore it. Then we can write the mispricing return premium as $B \approx B_1 + B_2 + B_3$. Moreover, in order to achieve statistical identification of mispricing it is necessary to assume that mispricing is uncorrelated with fundamentals so that $B_2 = 0$ and $B \approx B_1 + B_3$. Since B_3 is small, the mispricing return premium can be expected to be positive so long as the (log of) mispricing, z, follows a time-invariant stationary process, as we have assumed. It is worth considering also the possibility that z follows a process that depends on time or other state variables. It is possible for example that young firms tend to have a high volatility of mispricing because of the difficulty of valuing them; and that this high volatility is accompanied by a high average level of mispricing, and that both the mean and volatility of mispricing tend to decline as the firm matures. In this case we might assume that $\Delta z(t) \equiv lnZ_t - lnZ_{t-1}$ is normal with parameters $(\mu_{\Delta z}(t), \sigma_{\Delta z}(t))$, where t denotes calendar time. Now it is quite possible to choose these parameters so that $B_1 = 0$, and for the mispricing return premium to be zero (unconditional rational returns).[7] Hence the existence of a mispricing return premium is essentially an empirical question which we investigate in the following section by constructing portfolios using ex-ante estimates of mispricing return premium, $B = B_1 + B_3$ that are derived under the assumption of a time-homogeneous process for z.

5 Empirical Analysis

In this section we present evidence that risk adjusted returns are related to estimates for the mispricing return premium, B. Our empirical analysis, which is based on the general model of mispricing presented in Section 4, imposes the assumption that innovations in mispricing are uncorrelated with fundamental returns. As discussed by Watson (1986) and Harvey (1989), such an orthogonality assumption is necessary for identification in the Kalman filter estimation of mispricing. The assumption implies that $B_2 = 0$. Then $B = B_1 + B_3$.

We use two approaches to proxy for B_1. First, we use a Kalman filter, assuming that individual security mispricing follows a simple AR1 process which is uncorrelated with fundamentals. The AR1 assumption is restrictive, and does not allow for positive short term autocorrelation in returns: as a result, our estimation algorithm did not converge for a significant number of stocks.[8] Therefore our second approach uses the volatility and variance ratio of residual returns to proxy for mispricing. The variance ratio, which is approximately a linear combination of sample autocorrelations (Cochrane, 1988), does not require us impose any structure on the mispricing process. B_3, which is calculated using Equation (8) based on estimates of the dividend yield and mispricing parameters, turns out

[7] Pastor and Veronesi (2003) develop a model in which the ratio of the market price to the book value declines with firm maturity as a result of learning about firm profitability. Mispricing with unconditional rational returns would accentuate this decline.

[8] We also developed an estimate of the premium based on an AR2 process for the mispricing. See Khil and Lee (2002). The empirical results for this model are qualitatively similar to those for the AR1 process. However, they are less significant, which is probably due to the difficulty of identifying the parameters of the more complex model.

to be negligibly small.

5.1 Data

The primary data that we use are the monthly returns and dividends on all stocks registered on the NYSE, AMEX and NASDAQ from January 1962 to December 2004, which are taken from CRSP. We include only common shares, and exclude preferred stocks, ADR's, REIT's, etc. To alleviate the potential influence of 'stale prices', we include only observations with positive trading volume and with valid month-end closing prices. We also filter out penny stocks. We use as risk factors monthly returns on the 3 Fama-French factors, and the momentum factor of Carhart (1997); these, together with 1-month Tbill returns, are taken from Ken French's website.[9] We use data on book values from COMPUSTAT, and on prices, market capitalization and share turnover from CRSP.[10] Finally, in some of our regressions we use data on analyst following and the dispersion of analysts' forecasts of earnings 1 to 2 quarters ahead, which are taken from IBES.

5.2 AR1 Estimates of Mispricing

In order to identify mispricing, the 'fundamental return', R^*, was assumed to follow an ex-post version of the Fama-French (1993) 3-factor model (FF3):

$$R^*_{i,t} - R_{F,t} = b_i(R_{M,t} - R_{F,t}) + c_i SMB_t + d_i HML_t + \epsilon_{i,t} \tag{9}$$

where $R^*_{i,t}$ is the fundamental return on stock i in month t, $R_{F,t}$ is the riskless interest rate, and $R_{M,t}, SMB_t, HML_t$ are the Fama-French factors. Then, to a first order approximation, the market return, $R_{i,t}$ is given by:

$$R_{i,t} - R_{F,t} = \alpha_i + b_i(R_{M,t} - R_{F,t}) + c_i SMB_t + d_i HML_t + e_{i,t} \tag{10}$$

where $e_{i,t} = z_{i,t} - z_{i,t-1} + \epsilon_{i,t}$, and $z_{i,t}$ is (approximately) the log of the mispricing factor at time t, and α_i corresponds to the mispricing return premium effect.

[9] $http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html$

[10] To address the issue of inter-dealer trading in OTC markets, we multiply the NASDAQ trading volume by 0.6, following Atkins and Dyl (1997).

Following Poterba and Summers (1988), the log of the mispricing was assumed to follow the AR(1) process:

$$z_t = \phi_1 z_{t-1} + \eta_t \tag{11}$$

Then, following Khil and Lee (2002), a Kalman filter was used to estimate the logarithm of the mispricing factor, z_t, and the parameters of the mispricing process, ϕ_1 and σ_η, for each security, from the FF3 residual returns, $e_{i,t}$. The observation equation for the Kalman filter is:

$$e_t = z_t - z_{t-1} + \epsilon_t \tag{12}$$

and the transition equation is $z_t = \phi_1 z_{t-1} + \eta_t$. Details of the Kalman filter algorithm are given in the Appendix. The parameters of the mispricing process, ϕ_1 and σ_z, were estimated at the beginning of January of each year from 1967 to 2004 for all stocks with at least 36 monthly returns using the FF3 residual returns estimated over the previous 60 months as available. Then B_1 was calculated from the relation

$$B_1 \approx (1 - \phi_1)\sigma_z^2 \tag{13}$$

.

B_3 was also estimated at the beginning of January each year. The expected dividend yield, $E[\delta^*]$, was calculated by dividing sum of the dividends paid over the previous 12 months by the year end stock price. $E[Z]$ and σ_Z^2 were calculated using a lognormal assumption with $E[z] = 0$.

5.2.1 Mispricing and Returns

Although the focus of this paper is on the mispricing return premium, it is useful to confirm the validity of our estimates of mispricing, $z_{i,t}$, yielded by the Kalman filter. For this purpose the (log) mispricing was estimated *every month* for all securities with at least 36 monthly returns using the FF3 residual returns estimated over the previous 60 months as available so that there is no 'look-ahead' bias in the estimation. Then securities were assigned to one of ten portfolios in month t on the basis of z_{t-1} and the equally weighted return on the portfolio was calculated. In addition, the spread between the returns on the high and low z portfolios was computed. Panel A of Table 1 reports for each portfolio the average value of $z_{i,t-1}$, the average monthly return, and the intercept from a regression of the portfolio excess returns on the three Fama-French factors (FF3 α). The average value of z_{t-1} is -6.37 for the low z portfolio and 7.44 for the high z portfolio, so that estimated mispricing tends to fall in the range of $\pm 6-7\%$. Both the average returns and the FF3 α's vary monotonically and inversely with z_{t-1} which is to be expected as market prices revert towards fundamental values. FF3 α for the low(high) z portfolio is 0.87% (-0.67%) per month, and the t-statistic on the spread between these returns is close to 9. Thus z_{t-1} seems to capture a significant element of mispricing. To see whether this is merely another version of the short run reversal effect noted by Jegadeesh (1990), the analysis was repeated. This time the securities were sorted into 25 portfolios, first into quintiles on the basis of the lagged monthly return, R_{t-1}, and then within each quintile into a further five portfolios based on z_{t-1}. Panel B of Table 1 shows that z carries significant information about future returns beyond the reversal effect captured by R_{t-1}: the return associated with z is of the order of $47 - 121bp$ per month and the t-statistics on the spread between high and low z portfolios are in the range $3 - 9$ for the different R_{t-1} quintiles.

5.2.2 Mispricing Characteristics

Panel A of Table 2 reports time series average of statistics from the annual cross-sectional distributions of Kalman filter parameter estimates. The mean (median) AR1 coefficient is 0.07 (0.14). Note that if the mispricing were due solely to an *iid* bid-ask spread, ϕ_1 would be equal to zero. The mean (median) volatility of mispricing is 5.59%(4.09%), and the inter-quartile range is from 2.39% to 6.90%. The mean(median) value of B_1 is 4.18%(1.79%)*per annum*. B_3, which is reported in *bp per annum*, is close to zero, so that $B \approx B_1$. The inter-quartile range of the estimated mispricing return premium, B, is about 3.5% *per annum*, which suggests that the premium may be economically significant.

To confirm that our *ex-ante* estimates of the mispricing return premium are related to fundamental firm characteristics, we regress the Kalman filter estimates of the first order autocorrelation, ϕ_1, the variance of the mispricing variable, σ_z^2, B_1, and B_3 on firm characteristics that may be expected to influence mispricing. At the end of each year from 1976 to 2003, we regress the estimates obtained using residual returns over the previous 5 years and the estimated dividend yields on the year-end characteristics: firm size, book-to-market ratio, share price, share turnover, the logarithm of the number of analysts following the firm, and the dispersion of the analyst earnings forecasts for the next 1 to 2 quarters. Only firms that are followed by at least two analysts are included in the regressions. Panel B of Table 2 reports the time series average of the coefficients, and HAC adjusted t-statistics.

Intuition suggests that small firms will be more subject to mispricing, and we find a strong negative relation between the variance of mispricing and firm size. Moreover, for small firms the mispricing tends to be more transient (low ϕ_1), which also increases B_1. The net effect is that B_1 is significantly negatively related to firm size. Book-to-market ratio is negatively associated with the variance of mispricing, and positively associated with ϕ_1, the persistence of mispricing. As a result, growth firms tend to have more transient and volatile mispricing, and therefore a higher value of B_1. A lower stock price is associated with more volatile and more transient mispricing, and therefore with a higher value of B_1. The effect of turnover is to reduce the persistence of mispricing but to increase the variance of the mispricing, and the net effect is to increase B_1. We had expected the number of analysts following a stock to reduce the variability of mispricing, and the dispersion of analysts forecast to increase the variability of mispricing. While the sign of the coefficient on the number of analysts is consistent with the expectations, the coefficient is not significant, and the net effect of the number of analysts on the return premium is insignificant. On the other hand, the dispersion of analysts' forecast is marginally significantly associated with variability of the mispricing and therefore with B_1, and the direction of the effect is consistent with prior intuition. Our results are generally consistent with the findings of Kumar and Lee(2006) that the returns on small firms with low stock prices have higher loadings on a measure of retail investor sentiment, which induces transient mispricing.

The estimated value of B_3 loads positively on firm size and B/M,which is consistent with large value firms having higher dividend yields. The small magnitude of B_3 means that the regression coefficients for $B \equiv B_1 + B_3$ are virtually identical to those for B_1.

5.2.3 Estimated Mispricing Return Premium and Realized Returns

In order to assess the relation between our estimates of the mispricing return premium, B, and risk-adjusted returns, stocks were assigned in January of each year from 1967 to 2004 to one of ten equal size portfolios according to the previous month end mispricing return premium estimate, B. An equal investment was assumed to made in each stock in the portfolios at the beginning of the year and no rebalancing was assumed within the year.[11] The first portfolio allocation occurs at the end of December 1966, and the last at the end of December 2003. The post-ranking returns were then linked across time, yielding a time series of returns for each decile from January 1967 to December 2004. On average, there are about 200 stocks within each portfolio, and at no time is the number of stocks in a portfolio less than 110.

Panel A of Table 3 reports the characteristics of the portfolios. Portfolio 'NCV' contains stocks for which the Kalman Filter did not converge. Betas of the portfolios were obtained by regressing excess returns on the portfolios on the three FF factors and the momentum factor of Carhart (1997). The high premium portfolios tend to have higher loadings on the market and on SMB, and lower loadings on HML. The estimated mispricing return premium is strongly related to firm size as we found in Table 2, which is also consistent with the pattern of loadings on SMB: the relation is almost perfectly monotonic and the firms in the high premium portfolio are less than 1/20th of the size of firms in the low premium portfolio. The lower loadings of high premium portfolios on HML are also consistent with the finding in Table 2 that the return premium is higher for growth firms. The size composition of decile portfolios is consistent with the relation between firm size and the return premium reported in Table 2. There is no relation between the loadings on MOM and the premium. The average firm in Portfolio NCV has characteristics that are close to the average of all firms, except for β_{HML}, which is close to that of the high premium portfolio.

The average estimated AR1 coefficient, ϕ_1, increases from -0.13 for the low premium

[11] Since mispricing is most likely to found among small stocks, we use an equal weighting scheme to compensate for the over-representation of large, liquid, and closely followed stocks that are less likely to be subject to mispricing. Liu and Strong (2006) show that monthly rebalancing can lead to significant biases in average returns, especially for small, low price, value and loser stocks.

portfolio to 0.32 for the high premium portfolio; the latter corresponds to a half life for mispricing innovation of 0.6 months, or about 12 trading days.[12] The average volatility of mispricing, σ_z, is monotonic, ranging from 1.08% for the low premium portfolio to 16.70% for the high premium portfolio. Finally, the estimated volatility of the fundamental return, σ_ϵ, is almost monotonically increasing across portfolios, so that the firms with the most (fundamental) idiosyncratic risk tend to be those most subject to mispricing. Note that expression (13) implies that B_1, which is the dominant component of the mispricing return premium, is non-negative. The estimated annualized premium runs from 14 bp to over 16%. For the first six portfolios the premium estimates are moderate, reaching 1.98% for portfolio 6. However, they increase rapidly for the last four portfolios, more than quadrupling between portfolios 8 and 10.

Panel B of Table 3 contains our basic results. The 10-1 spread corresponds to a zero-investment portfolio that is long in the high premium portfolio and short in the low premium portfolio. The average returns of the portfolios are almost monotonically increasing with the estimated mispricing return premium, and the spread between the high and low premium portfolios is almost 1% per month. The excess returns on the decile portfolios were regressed in turn on the excess market returns, on the 3 FF factors, and on the 4 Carhart factors. The intercepts from these regressions provide estimates for risk-adjusted returns. There is a clear tendency for the risk-adjusted returns to increase with the premium estimates for all three risk adjustment benchmarks. The correlations between the return premium estimates and both the FF3 and the FF4 adjusted returns are over 0.96. The spread in FF3 adjusted return between the high and low premium portfolios is 72 bp per month, or 8.64% per year, and is highly significant. The average estimated MRP corresponds well to the average risk adjusted returns on the 10 portfolios: the average MRP across the 10 portfolio is 3.48% per annum, compared with the average annualized values of FF3(FF4) alphas of 2.92%(3.31%). These results provide strong evidence that realized stock returns contain an economically and statistically significant mispricing return premium.

The results in Panel B are based on equally weighted portfolio returns. For completeness,

[12]This compares with a half life of 2-22 trading days for dealer inventory innovations estimated by Madhavan and Smidt (1993): in their model, the deviation between price and fundamental value is proportional to the dealer inventory.

we also calculated risk adjusted returns on value weighted portfolios. The difference between the risk adjusted returns on an equally weighted and a value weighted portfolio is equal to -cov(w_i, α_i), where w_i is the weight of stock i and α_i is the risk adjusted return. Therefore, if the covariance between MRP and firm size is negative as the evidence in Tables 2 and 3 suggests, then value weighted risk adjusted returns will be less than the equally weighted returns. In addition, value weighting creates an asymmetric bias effect on the returns of high and low MRP portfolios, due to errors in the classification variable: this asymmetric misclassification bias will reduce the ex post association between portfolio returns and the sorting variable. The asymmetric bias arises because a large firm is likely to enter the high MRP portfolio only because it has *positive* measurement error in MRP: such a firm will have a low risk adjusted return, and value weighting will assign greater importance to this misclassified firm, magnifying the downward bias on the portfolio returns. On the other hand, value weighting reduces the bias due to small firms misclassified into the low MRP portfolio.

These effects are illustrated in Panel C of Table 3, which reports FF3 adjusted returns. The first line repeats the equally weighted returns from Panel B. The second line reports the value weighted returns. The last three lines report returns on value weighted portfolios after winsorizing away the largest 10%, 20% and 30% of firms from each portfolio. Comparing the first two lines, we see that for every portfolio, the value weighted return is less than the corresponding equally weighted return, and the 10-1 return difference falls from 72 bp per month to -30 bp, which is no longer significant. Elimination of the largest 10% of firms in each portfolios has little effect on the low MRP portfolios, but substantially increases the risk adjusted returns on the high MRP portfolios as the misclassification bias is reduced. The resulting 10-1 spread rises to 27 bp. As more large firms are removed from each portfolio, the return spread increases and becomes statistically significant. When the 30% largest firms are removed, the value weighted return spread is 62 bp, which is comparable to the 72 bp spread reported for the equally weighted portfolios.

Because of the bias due to the negative covariance between firm size and MRP and the measurement errors in MRP, we focus our remaining analysis on the equally weighted portfolios.

5.2.4 Robustness Tests

We now consider some robustness tests of our basic results. Figure 1A shows that the positive association between the mispricing return premium estimates and the realized FF3 adjusted returns is dominated by the returns on the high premium portfolio. To ensure that our results are not driven by this extreme portfolio, we repeated the analysis forming portfolios after winsorizing away the stocks in this top decile. Figure 1B shows that the positive association between risk adjusted returns and premium estimates is maintained in the reduced sample. Panel A of Table 4 shows that the effect of winsorization is to halve the average return spread and to reduce the FF3 risk adjusted spread from 72 bp to 28 bp per month. Both spreads remain statistically significant.

As a further robustness test, the analysis was repeated leaving one month between the end of the Kalman filter estimation and the portfolio formation: the Kalman filter was estimated using data ending in November of each year, and the portfolios were formed at the end of December each year. The result reported in Panel B of Table 4 is to *increase* the FF3 risk adjusted return spreads by 14 to 86 bp per month.

We found in Table 2 that the mispricing return premium was related to firm size and book-to-market ratio. Therefore, to take account of the possibility that the FF3 model does not adequately capture difference in returns attributable to differences in size and book-to-market ratio, we adopted a *characteristic adjusted return* approach. At each year end, 25 equally-weighted benchmark portfolios were formed by first assigning the sample firms into size quintiles based on NYSE quintile breakpoints; then within each size quintile, assigning firms to book-to-market quintiles based on the corresponding NYSE book-to-market quintile breakpoints. The characteristic adjusted stock returns are the difference between the average returns and the returns on the corresponding benchmark portfolio. The first line of Panel C of Table 4 reports the characteristic adjusted returns. The spread between the high and the low premium portfolios is now 48 bp per month with a $t-$statistic of 2.60. The characteristic adjusted returns were further adjusted for risk using the 3 risk models and the results are also reported in Panel C. This further adjustment of the returns does not change the results. It is not surprising to find that the return spread between the high and low premium portfolios is smaller for size and book-to-market characteristic adjusted returns, since the benchmark portfolios used to adjust returns are themselves affected by the mispricing return premium.

It has been suggested to us that our results may be due to the use of relatively high frequency, monthly, returns and that it would disappear if a longer period were used to compute returns. To explore this, we calculate quarterly returns on our portfolios and regress them on quarterly returns on the FF factors.[13] The results, which are reported in Panel D of Table 4, basically confirm our previous findings. The FF3 α's continue to increase monotonically across the return premium portfolios and the spread between the FF3 α's on high and low premium portfolios is 2.45% per quarter with a t-statistic of 2.49. Only for the momentum adjusted FF4 α' is there a hint of lack of robustness. The α's are no longer quite monotonic, and while the spread in α's between between the high and low premium portfolios is a healthy 1.58% per quarter, the corresponding t-statistic is only 1.51. Of course, this reduction in the t-statistic is in part due to the smaller number of observations when quarterly returns are used. Although these quarterly return results are less compelling, it is important to recognize that the overwhelming majority of asset pricing tests have been conducted at the monthly frequency where our evidence of the mispricing return premium is unambiguous.

Malkiel and Xu (2006) and Spiegel and Wang (2006) have found a positive association between risk adjusted returns and idiosyncratic volatility,[14] and we saw in Table 3 that the mispricing return premium is positively related to idiosyncratic volatility. This raises the possibility that our findings are due to a positive premium for idiosyncratic volatility. Therefore, in order to determine whether the mispricing return premium provides incremental explanatory power for risk adjusted returns relative to idiosyncratic volatility, securities were sorted into quintiles based on idiosyncratic volatility, then within each quintile, further sorted into 5 portfolios on the basis of MRP. In results not reported here, the spread in risk adjusted returns between the high and low mispricing return premium portfolios was found

[13] The quarterly returns are calculated by compounding the monthly returns already calculated.

[14] Ang et al. (2006) report a negative return premium for idiosyncratic volatility. However, their study is not comparable to this or the cited studies, since their estimates of idiosyncratic volatility are derived from daily rather than monthly returns.

to be positive and significant at the 10% level or better for all but the lowest idiosyncratic risk quintile - for the highest idiosyncratic risk quintile, the spread is 60 *bp* per month with a t-statistic of 2.91. On the other hand, when the order of the sorts was reversed, the spread in risk adjusted return between the high and low idiosyncratic volatility portfolios was significant only for the highest mispricing return premium quintile (t-statistic of 2.56). Thus while there is strong evidence for the mispricing return premium, the evidence for an independent idiosyncratic volatility premium is weak at best.

5.3 Variance Ratio and Volatility of Residual Returns

Equation (7) shows that the major component of mispricing return premium, B_1, is increasing in the volatility of mispricing σ_z, and decreasing in the first order autocorrelation ρ_1^z. Unfortunately, as we have seen, the mispricing variable, z, is not directly observable and therefore these parameters can be inferred only by making strong assumptions about the stochastic process of the mispricing variable. These assumptions are unlikely to be satisfied in practice. Therefore, in this section we consider a more informal approach to proxying for the mispricing return premium.

Define the k-month variance ratio for (FF3) residual returns by $VR(k) \equiv \left(var(e^k)/k\right)/var(e^1)$, where e^k is the cumulative residual return over k months. First, we observe that in the absence of mispricing, the FF3 residuals will be serially independent, so that $VR(k) = 1$. To the extent that there is transient mispricing, the variance ratio will be less than 1, and the stronger is the mean reversion in mispricing, the lower will the ratio. This suggests using the variance ratio of residual returns as a proxy for ρ_1^z. The volatility of residual returns, σ_e, depends on both the idiosyncratic volatility of the fundamental returns (σ_ϵ) and the variability of mispricing (σ_z). Therefore we can think of residual return volatility (σ_e) as a noisy signal of the volatility of mispricing (σ_z). Hence we expect the mispricing return premium to decrease in the variance ratio and to increase in residual return volatility.

Our analysis is based on the 24-month variance ratio of the FF3 residuals.[15] 10 portfolios were formed each year based on the 24-month residual variance ratio estimated over the

[15]Results obtained using 12 and 36 month variance ratios were similar.

previous 60 months, $VR(24)$. Only stocks with at least 36 monthly returns within the past 5 years were considered. For any 24 month period, an unbiased estimator of variance ratio is provided by the ratio of the squared sum of residuals over that period divided by the sample variance of 1-month residuals. Our estimate of the variance ratio is obtained by averaging the estimators for each 24-month period over the last 60 months, and is therefore unbiased. Securities were then assigned to one of 10 portfolios according to the variance ratio estimate.

The characteristics of the portfolios are reported in Panel A of Table 5. The estimates of the variance ratios for individual stocks are very noisy, so that the sample selection premium involved in sorting on this variable causes the spread of portfolio average variance ratio estimates to be very wide, ranging from 0.12 to 2.04 as compared with the value of unity implied by the *iid* assumption. Nevertheless, it is striking that for 8 out of the 10 portfolios the estimate of $VR(24)$ is less than unity; this, together with the fact that the average residual autocorrelation is negative for 9 portfolios, suggests a widespread tendency for residual returns to reverse themselves, which is consistent with transient mispricing. There is relatively little difference in the average residual variances of the stocks in the portfolios except for the high VR portfolio whose residual variance is about 33% higher than those of the other portfolios. Firm size tends to decrease with the variance ratio, but the variation in firm size across decile portfolios is not as marked as it was for the previous two portfolio formation methods. As conjectured, the (FF3) risk adjusted returns reported in Panel B are decreasing in the variance ratio; the correlation between the risk adjusted returns and the variance ratio is -0.89. This pattern of returns is consistent with that reported in Table 3: low VR portfolios that we expect to have a higher return premium have higher risk adjusted returns. The (FF3) risk adjusted spread is 4.08% per year when sorting on $VR(24)$, as compared to a spread of over 8% when sorting on the Kalman filter based estimates of the return premium; however, the t-statistic on the spread is now 3.91, as compared with 3.25 reported in Table 3.

As a robustness check on these variance ratio results, the analysis was repeated using a *characteristic adjusted return* approach as in Section 5.2.4. The return spread falls from 34 bp per month when adjusted for risk using the FF3 model as reported in Panel B of Table

5, to 23 bp per month using the characteristic adjusted returns, as reported in Panel B of Table 6. This reduction in the return spread when using characteristic adjusted returns is consistent with our previous observation that the benchmark returns are likely to be affected by the mispricing return premium. As seen in Panel C, further adjustment for risk of the characteristic adjusted returns does not change the results.

Equation (7) shows that the major component of mispricing return premium, B_1, is the product of the variance of mispricing, σ_z^2, and $(1 - \rho_1^z)$. Since the variance ratio is a proxy for ρ_1^z, and the residual variance of returns a proxy for σ_z^2, we should expect their effects on the mispricing return premium to be multiplicative: for low (high) residual variance, the effect of the variance ratio should be small(large); and for high (low) variance ratio, the effect of residual variance should also be small (large).

In order to determine the effect of the residual variance on the relation between risk adjusted returns and the variance ratio, 25 portfolios were formed each year. First the stocks were sorted into quintiles based on the estimated variance of the residual returns. Then within each variance quintile the stocks were further sorted into quintiles based on $VR(24)$.[16] The time series of returns on the resulting 25 portfolios were calculated as in the previous case assuming equal investments in each stock and annual rebalancing. Risk-adjusted returns were then calculated as before and the results are reported in Table 7. We focus on the results for returns that are adjusted for risk using the 3 and 4 factor models, which are quite similar.

The relation between risk adjusted returns and the variance ratio depends strongly on the residual variance, as expected. As the residual variance increases, the relation, which is negative and insignificant for the lowest residual variance quintile, becomes positive and is highly significant for the two highest residual variance quintiles. The spread in the FF3 risk adjusted returns for the highest residual variance quintile is 9.12% per year with a t-statistic of 4.51.

In order to determine the effect of the variance ratio on the relation between the residual variance and risk adjusted returns, 25 portfolios were reformed by sorting first on the

[16]Similar results were obtained when sorting simultaneously on these two variables.

variance ratio and then on the residual variance. The results are reported in Table 8. As we conjectured, only for the lower VR quintiles is there any evidence that residual variance is associated with risk adjusted returns after controlling for VR. For the lowest VR quintile the (FF3) risk adjusted return increases monotonically in the residual variance, and the spread between the risk-adjusted returns on the high and low σ_e^2 portfolios is 9.36% per year with a t-statistic of 2.87.[17]

In summary, risk adjusted returns are increasing in residual variance, but only when the variance ratio is low; and risk adjusted returns are decreasing in the variance ratio, but only when the residual variance is high. This is consistent with the hypothesis that the variance ratio of residual returns is a good proxy for the first order autocorrelation of mispricing and that the volatility of residual returns is a proxy for the volatility of mispricing.

6 Conclusion

In this paper we have shown that when market prices differ from fundamental prices because of stochastic pricing errors which are zero on average, a premium in average returns is created as a result of Jensen's inequality. The premium is decreasing in the persistence of the pricing errors and increasing in their volatility. A second premium is created by the covariance of the fundamental return with the innovation in the proportional mispricing, and there is a third premium due to the effect of average mispricing on the dividend yield. Additional return effects are created if the mispricing is time-dependent. In this paper we have assumed that the second premium is zero, and formed portfolios based on estimates of the sum of the pure Jensen's inequality effect and the dividend yield effect. We find that the former effect is an order of magnitude greater than the latter. We test whether risk-adjusted returns reflect the mispricing return premium we have calculated, and find strong evidence that they do.

In order to estimate the mispricing return premium, we assume that fundamental returns follow the Fama-French three-factor model, and estimate the log of mispricing, $z_{i,t}$,

[17] While Malkiel and Xu (2006), Spiegel and Wang (2006), and Ang *et al.* (2006a,b) find significant associations between risk adjusted returns and idiosyncratic volatility, none of them consider the variance ratio.

by applying a Kalman filter to the residuals from a regression of stock returns on the Fama-French factors under the assumption that mispricing follows an AR(1) process whose innovations are orthogonal to fundamentals. We confirm that the $z_{i,t}$ estimates correspond to mispricing by showing that they predict future risk-adjusted returns; we also find that they have incremental explanatory power relative to the lagged return. The mispricing premium depends only on the parameters of the AR(1) process of the pricing errors (and the dividend yields). We find that the premium is larger for small, low price, growth firms with high share turnover and dispersion in analyst earnings estimates; it seems reasonable that such firms should be most subject to pricing errors. Each year we form ten equally weighted portfolios on the basis of mispricing return premium estimates derived from data over the previous five years. For the high premium portfolio the average volatility of mispricing, σ_z, is approximately 17% as compared with 1% for the low premium portfolio. The average estimated autocorrelation of mispricing is 0.32 for the high premium portfolio. This is inconsistent with the mispricing we find being simple *iid* bid-ask bounce. However, it falls within the range of dealer inventory and price adjustment parameters found by Madhavan and Smidt (1993).

The returns on the 10 portfolios are linked over time assuming no rebalancing except at year end. When the returns are regressed on the the three Fama-French (1993) factors, the intercepts or risk adjusted returns have a correlation of 0.96 with the estimated mispricing return premium, and the annualized spread in the risk adjusted returns between the highest and lowest premium portfolios is 8.64% with t-statistic of 3.25. Moreover, the average estimated mispricing return premium across the decile portfolios is 3.48% per annum, close to the average annualized risk adjusted returns of 2.92% (FF3) and 3.31% (FF4) for these portfolios. When returns are adjusted for risk by subtracting the returns on size and book-to-market portfolios, the annualized return spread is 5.76% per year with t-statistic of 2.60. Thus there is significant evidence that risk(characteristic) adjusted returns are affected by the mispricing return premium that we have analyzed.

We also form portfolios based on the 24-month variance ratio of the residuals from the Fama-French 3-factor model. Consistent with our hypothesis, we find that risk-adjusted and characteristic adjusted returns are significantly higher on low variance ratio portfolios, and when we sort first on residual return volatility and then on variance ratio we find that the effect is more pronounced for the high residual volatility groups of portfolios. Moreover, when we form portfolios first on the variance ratio and then on residual volatility, we find that the relation between risk adjusted returns and the residual variance is the strongest for the low variance ratio quintiles. These findings are consistent with the hypothesis that the variance ratio of residual returns is a good proxy for the first order autocorrelation of mispricing and that the volatility of residual returns is a proxy for the volatility of mispricing.

We have provided strong empirical evidence for the effect of the mispricing return premium on the average level and the cross-section of security returns. This has implications for the conduct and interpretation of empirical tests of the effects on risk-adjusted returns of variables such as liquidity and idiosyncratic volatility. The return effect that we have found associated with the random mispricing may also be interpreted as premia associated with these variables.

Appendix: Kalman Filter Algorithm for an AR1 z process

Assume that the logarithm of the mispricing component, z_t, follows a AR1 process, the transition equation can then be denoted as

$$\alpha_t = T_t \alpha_{t-1} + w_t \text{ with } w_t \sim N(0, Q_t) \tag{A1}$$

where $\alpha_t = [z_t]$, $T_t = [\phi_1]$, $w_t = [\eta_t]$,$Q_t = \left[\sigma_\eta^2\right]$.

The observation equation is based on the FF3 risk adjusted returns,e_t, $t = 1, 2, ...T$, and is given

$$e_t = \mu + s_t' \alpha_t + \epsilon_t \tag{A2}$$

with $s_t = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. The assumption that innovations in mispricing are uncorrelated with innovations in fundamentals implies that ϵ and w are uncorrelated.

At the end of each year from 1967 to 2003, for each stock, a Kalman filter is fitted to the FF3 adjusted returns over the past 60 months, following a 2-stage iteration process. The first stage is the prediction stage. At time $t-1$, the optimal predictor, $\alpha_{t|t-1}$, and the associated covariance, $P_{t|t-1}$, are given by

$$\alpha_{t|t-1} = T_t \alpha_{t-1|t-1} \tag{A3}$$

$$P_{t|t-1} = T_t P_{t|t-1} T_t' + Q_t \tag{A4}$$

The second stage is the updating stage. When e_t becomes observable at time t, we can calculate the prediction errors given by the predicted parameter in the first stage, $v_t = e_r - e_{t|t-1} = s_t'(\alpha_t - \alpha_{t|t-1}) + \epsilon_t$, with mean of 0, and variance of $s' P_{t|t-1} s_t + \sigma_\epsilon^2$. Define, $f_t \equiv s_t' P_{t|t-1} s_t + \sigma_\epsilon^2$, the parameters then are updated as follows:

$$\alpha_{t|t} = \alpha_{t|t-1} + P_{t|t-1} s_t (e_t - s_t' \alpha_{t|t-1}) f_t^{-1} \tag{A5}$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} s_t s_t' P_{t|t-1} f_t^{-1} \tag{A6}$$

and $P_{t|t-1} s_t f_t^{-1}$ is also known as "Kalman Gain". The log likelihood function of the observations (e_1, e_2, ... e_t) can be calculated as

$$L = -\frac{T}{2} log(2\pi) - \frac{1}{2} \sum_{t=1}^{T} log(f_t) - \frac{1}{2} \sum_{t-1}^{T} \frac{v_t^2}{f_t} \tag{A7}$$

Following Campbell (1989), initial guesses of α_0 and P_0 are set to zero and $\frac{\sigma_\eta^2}{1-\phi_1^2}$, respectively. We maximize the above log likelihood function to get the final parameter estimates. Based on the estimated parameters, the premium is calculated by

$$B_1 = e^{(1-\phi_1)\sigma_z^2} - 1 = e^{\frac{\sigma_\eta^2}{1+\phi_1}} - 1 \tag{A8}$$

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Table 1: Returns on Equally-Weighted Decile Portfolios formed each month t, from January 1967 to December 2004, on Kalman Filter Estimates of the logarithm of mispricing, z_{t-1}

z, the logarithm of mispricing, is estimated each month from the Fama-French 3-factor model residuals estimated over the previous 60 months. The estimation uses a Kalman filter and assumes an AR(1) process for z. Securities are allocated to portfolios on the basis of the z estimates for the previous month. Equally weighted excess returns on the portfolios are then regressed on the 3 Fama-French factors. The intercept, α, from these regressions is reported in Panel A. Panel B reports returns on portfolios formed each month by sorting on R_{t-1}, the previous month return, and then on z_{t-1}. The returns and α's are in *per cent* per month, and the t-statistics are adjusted for autocorrelation and heteroscedasticity.

Panel A: Portfolios formed on z_{t-1}

	Portfolio Low z_{t-1}	2	3	4	5	6	7	8	9	Portfolio High z_{t-1}	Spread High-Low
Average z_{t-1}	-6.37	-2.03	-1.04	-0.51	-0.17	0.07	0.38	0.92	2.02	7.44	
Average Return	2.24	1.89	1.72	1.63	1.43	1.19	0.87	0.85	0.72	0.57	-1.67
t-statistic	7.16	6.82	6.57	6.55	5.96	4.96	3.60	3.37	2.86	1.98	-10.17
FF3 α	0.87	0.55	0.39	0.35	0.19	-0.04	-0.35	-0.42	-0.52	-0.67	-1.54
t-statistic	8.22	6.36	5.11	5.44	3.11	-0.63	-5.53	-6.99	-7.58	-5.68	-8.94

Panel B: Portfolios formed first on R_{t-1}, then on z_{t-1}

	FF3 α (%)						t-statistic					
	Low z_{t-1}	2	3	4	High z_{t-1}	Spread (5-1)	Low z	2	3	4	High z_{t-1}	Spread (5-1)
High R_{t-1}	-0.61	-0.44	-0.59	-0.79	-1.08	-0.47	-4.86	-4.78	-6.39	-5.56	-6.51	-3.18
2	0.09	0.05	-0.19	-0.33	-0.35	-0.45	1.00	0.57	-2.22	-4.27	-4.29	-3.82
3	0.38	0.33	0.24	-0.23	-0.35	-0.73	4.27	3.87	2.87	-2.32	-3.48	-6.35
4	0.58	0.48	0.35	0.01	-0.23	-0.81	5.69	5.06	4.18	0.08	-2.33	-6.86
Low R_{t-1}	1.36	1.01	0.62	0.44	0.14	-1.21	9.16	7.45	5.24	3.13	1.13	-9.62

Table 2: Mispricing Return Premium and Firm Characteristics
Panel A reports time series averages from the annual cross-sectional distributions of Kalman filter parameter estimates. Panel B reports time series averages of coefficients from Fama-MacBeth cross-sectional regressions of security mispricing return premium estimates on firm characteristics. $t-$statistics are computed using standard errors computed from the time series of the coefficients and take account of heteroscedasticity and autocorrelation using a Newey-West adjustment with 4 lags.
The AR1 coefficient, ϕ_1, the variance of mispricing, σ_z^2 and components of mispricing return premium, B_1 and B_3 are estimated from the residuals from Fama-French 3-factor regressions using a Kalman filter and assuming an AR1 process for mispricing. Only firms with at least 2 analysts are included in the regressions. *Size* is the market value of equity at the end of the year. *B/M* is the book-to-market ratio calculated from the year end market value and the most recent book equity that is available at least two quarters before the year end. *Price* is the share price at the end of the year. *Turnover* is the ratio of the average number of shares traded per month to the number of shares outstanding during the last quarter of the year. NASDAQ turnover is multiplied by 0.6. *#Analysts* is the number of investment analysts following the firm at the end of the year. *DISP* is the dispersion of analysts' earnings forecasts for the next 1-2 quarters.

Panel A: Summary Statistics

	ϕ_1	σ_z (%)	B_1 (%p.a).	B_3 (bp.p.a).	$B_1 + B_3$ (%p.a).
Mean	0.07	5.59	4.18	0.31	4.18
25%	-0.20	2.39	0.73	0.00	0.73
Median	0.14	4.09	1.79	0.05	1.79
75%	0.35	6.90	4.29	0.21	4.29
Std. Dev.	0.43	5.42	10.74	3.19	10.75

Panel B: Fama-MacBeth Regression: 1976-2003

	ϕ_1(%)		σ_z^2(%^{2})		B_1(%p.a.)		B_3(bp.p.a.)		$B_1 + B_3$(%p.a.)	
	coeff	t-stat	coeff	t-stat	coeff	t-stat	coeff	t-stat	coeff	t-stat
Const	-0.85	-0.57	85.76	4.66	6.15	4.97	0.01	0.07	6.15	4.98
log(Size)	0.75	2.20	-8.52	-3.67	-0.65	-4.07	0.05	2.61	-0.65	-4.07
B/M	1.74	3.20	-5.10	-3.24	-0.36	-3.92	0.12	3.62	-0.36	-3.90
Price	0.04	2.02	-0.09	-3.18	-0.01	-4.86	0.00	-1.60	-0.01	-4.86
Turnover	-0.35	-4.32	2.14	5.70	0.12	4.44	0.00	0.91	0.12	4.44
log(#Analysts)	1.83	2.70	-1.18	-0.64	-0.04	-0.29	-0.06	-2.22	-0.05	-0.30
DISP	0.19	0.12	9.59	1.88	0.45	2.07	-0.01	-0.21	0.45	2.07
$AdjR^2$ (%)	0.88		5.68		14.25		2.71		14.23	

Table 3: Properties of Decile Portfolios formed each year from January 1967 to December 2004 on Kalman Filter Estimates of the Mispricing Return Premium

Panel A reports the summary statistics for equally-weighted portfolio characteristics. *Mispricing Return Premium (MRP)*, which is annualized and in *per cent*, is equal to $B_1 + B_3$. The $\beta' s$ are the loadings of the portfolio returns on the 3 Fama-French and the Carhart Momentum factors. *Size* is the time series mean portfolio size in billion \$. σ_z and σ_ϵ, the volatility of the (log) mispricing variable (z) and the monthly volatility of the fundamental return (ϵ), are quoted in *per cent*. σ_e is the average volatility of residuals of Fama-French 3-factor regression over the previous 60 month, quote in %. Panel B reports average returns and the intercepts (α) from regressions of excess returns on the market excess returns (CAPM), the 3 Fama-French factors (FF3), and the 3 Fama-French factors plus the Carhart Momentum factor (FF4). The returns and $\alpha' s$ are in *per cent* per month, and the t-statistics are adjusted for autocorrelation and heteroscedasticity. Stocks for which the Kalman filter fails to converge are included in Portfolio NCV.

	Portfolio Low Bias	2	3	4	5	6	7	8	9	Portfolio High Bias		Portfolio NCV
Panel A: Portfolio Characteristics												
β_{mkt}	0.90	0.93	0.93	0.98	1.02	1.04	1.02	1.06	1.03	1.03		1.01
t-statistic	43.14	45.19	44.37	45.34	47.08	44.22	38.17	38.81	31.32	20.54		53.83
β_{SMB}	0.46	0.48	0.55	0.58	0.69	0.76	0.92	1.13	1.38	1.66		0.84
t-statistic	14.55	14.88	15.16	12.56	17.37	14.40	20.46	23.19	27.00	18.50		23.78
β_{HML}	0.43	0.40	0.39	0.41	0.40	0.40	0.31	0.34	0.25	0.18		0.25
t-statistic	10.61	11.12	8.68	9.56	9.18	8.45	6.50	5.15	3.54	1.30		8.21
β_{MOM}	-0.03	-0.03	-0.04	-0.04	0.01	-0.04	-0.01	-0.03	-0.04	-0.06		-0.07
t-statistic	-1.04	-1.11	-1.15	-1.29	0.19	-1.19	-0.22	-0.81	-0.76	-0.49		-1.77
Size (\$ B)	2.13	2.05	1.61	1.30	1.03	0.82	0.52	0.31	0.17	0.09		1.01
ϕ_1	-0.13	-0.11	-0.06	-0.01	0.05	0.09	0.13	0.15	0.21	0.32		
σ_z (%)	1.08	1.91	2.59	3.28	4.05	4.83	5.87	7.11	9.25	16.70		
σ_ϵ (%)	7.79	7.66	7.85	8.07	8.29	8.54	9.10	10.02	10.87	13.09		
σ_e (%)	7.94	8.09	8.56	9.08	9.63	10.27	11.30	12.80	14.73	20.71		
Mispricing Return Premium (% p.a.)	0.14	0.41	0.71	1.05	1.46	1.99	2.73	3.89	6.07	16.34		
Number of Stocks												
Mean	200	200	200	199	199	198	198	197	196	196		
Mininum	114	115	116	116	111	114	113	111	114	111		

	Portfolio Low Bias	2	3	4	5	6	7	8	9	Portfolio High Bias	Spread $10-1$	Portfolio NCV
Panel B: Returns (*per cent* per month)												
Raw Return	1.29	1.33	1.40	1.47	1.55	1.47	1.66	1.66	1.78	2.28	0.99	1.43
t-statistic	5.58	5.77	5.70	5.67	5.51	5.16	5.39	4.85	4.59	5.04	3.24	4.67
Capm α	0.38	0.40	0.46	0.50	0.55	0.46	0.63	0.59	0.69	1.14	0.77	0.40
t-statistic	2.84	3.23	3.35	3.62	3.73	3.02	3.68	2.92	2.92	3.66	2.82	2.46
FF3 α	0.06	0.09	0.15	0.18	0.22	0.10	0.31	0.22	0.33	0.78	0.72	0.12
t-statistic	0.75	1.39	1.92	2.53	3.28	1.59	4.37	2.23	3.11	3.74	3.25	1.63
FF4 α	0.09	0.12	0.19	0.22	0.22	0.14	0.32	0.25	0.37	0.84	0.75	0.19
t-statistic	1.27	2.01	2.64	3.10	3.11	2.15	3.99	2.31	2.86	3.32	2.97	2.06

Table 3 (Continued): Properties of Decile Portfolios formed each year from January 1967 to December 2004 on Kalman Filter Estimates of the Mispricing Return Premium

Panel C reports the intercepts (α) from regressions of equally and value weighted excess returns on the 3 Fama-French factors (FF3). It also reports the value-weighted portfolio α's when the top 10%, 20%, and 30% largest firms are winsorized away from each decile portfolio. The $\alpha's$ are in *per cent* per month, and the t-statistics are adjusted for autocorrelation and heteroscedasticity.

	Portfolio Low Bias	2	3	4	5	6	7	8	9	Portfolio High Bias	Spread $10-1$
Panel C: FF3 α (*per cent* per month)											
EW	0.06	0.09	0.15	0.18	0.22	0.10	0.31	0.22	0.33	0.78	0.72
t-statistic	0.75	1.39	1.92	2.53	3.28	1.59	4.37	2.23	3.11	3.74	3.25
VW	0.04	0.02	-0.02	0.08	-0.03	0.01	0.14	0.09	-0.25	-0.27	-0.30
t-statistic	0.59	0.38	-0.21	1.18	-0.38	0.08	1.41	0.75	-2.11	-1.47	-1.53
VW with 10% winsorization	-0.10	-0.04	0.03	0.06	0.21	-0.01	0.17	0.05	0.02	0.18	0.27
t-statistic	-1.19	-0.52	0.41	0.93	1.65	-0.08	2.20	0.75	0.22	0.97	1.24
VW with 20% winsorization	-0.12	-0.02	0.04	0.06	0.11	0.00	0.12	0.18	0.07	0.34	0.46
t-statistic	-1.51	-0.27	0.48	0.82	1.34	-0.06	1.69	1.71	0.71	1.90	2.17
VW with 30% winsorization	0.01	0.02	0.04	0.11	0.14	0.03	0.10	0.16	0.32	0.63	0.62
t-statistic	0.16	0.22	0.42	1.35	1.61	0.42	1.23	1.49	2.46	3.11	2.63

Table 4: Robustness Tests for Equally Weighted Portfolio Sorting Based on Kalman Filter Estimates of the Mispricing Return Premium

Four robustness tests on average, risk adjusted, and size and B/M adjusted returns for the chosen portfolios and the spread between the high and low mispricing premium portfolios. Under 10% *Winsorization*, stocks with the highest 10% of mispricing premium estimates are filtered out. Under $1 - Month\ Lag$, portfolios are constructed annually on Kalman filter estimated mispricing premium with 1-month lag. Under *Size and B/M Adjusted*, the average returns and risk adjusted α's are based on size and book-to-market adjusted returns. Under *Quarterly Returns* returns and α's are computed using quarterly returns on the portfolios and risk factors. Except for *Quarterly Returns* the returns and $\alpha's$ are in *per cent* per month, and the t-statistics are adjusted for autocorrelation and heteroscedasticity.

	Portfolio Low premium	2	5	9	Portfolio High premium	Spread 10 − 1
Panel A: 10% Winsorization: 1967.01-2004.12						
Average Return	1.29	1.29	1.49	1.69	1.79	0.50
t-statistic	5.59	5.53	5.50	4.88	4.57	2.26
Capm α	0.37	0.36	0.51	0.61	0.69	0.32
t-statistic	2.81	2.95	3.38	2.98	2.88	1.69
FF3 α	0.05	0.06	0.15	0.24	0.33	0.28
t-statistic	0.66	0.90	2.03	2.40	3.06	2.17
FF4 α	0.08	0.09	0.19	0.28	0.37	0.29
t-statistic	1.09	1.58	2.56	2.48	2.77	2.11
Panel B: 1-Month Lag: 1967.01-2004.12						
Average Return	1.34	1.32	1.57	1.93	2.51	1.17
t-statistic	5.61	5.48	5.98	5.22	5.45	4.03
Capm α	0.40	0.38	0.59	0.84	1.38	0.98
t-statistic	3.25	3.14	4.18	3.71	4.29	3.71
FF3 α	0.12	0.08	0.27	0.48	0.98	0.86
t-statistic	1.79	1.23	3.75	4.36	4.57	4.19
FF4 α	0.13	0.13	0.32	0.57	1.10	0.98
t-statistic	1.79	2.18	3.80	3.88	3.95	3.78
Panel C: Size and B/M Matched: 1969.01-2004.12						
Average Return	-0.18	-0.26	-0.13	-0.13	0.30	0.48
t-statistic	-2.92	-4.44	-2.65	-1.91	1.93	2.60
Capm α	-0.14	-0.22	-0.11	-0.16	0.22	0.37
t-statistic	-2.39	-4.21	-2.41	-2.41	1.51	2.10
FF3 α	-0.12	-0.23	-0.09	-0.08	0.31	0.43
t-statistic	-1.98	-4.64	-1.82	-1.26	2.11	2.57
FF4 α	-0.23	-0.29	-0.17	-0.14	0.25	0.48
t-statistic	-3.86	-5.59	-3.23	-2.17	1.58	2.51
Panel D: Quarterly Returns: 1969:I-2004:IV. (% per quarter)						
Average Return	4.03	4.14	5.02	5.84	7.45	3.42
t-statistic	5.79	6.09	5.80	4.74	5.08	3.22
Capm α	1.14	1.19	1.77	2.11	3.53	2.39
t-statistic	2.12	2.42	2.91	2.39	3.09	2.60
FF3 α	0.22	0.29	0.77	1.18	2.67	2.45
t-statistic	0.78	1.34	3.02	2.69	2.90	2.49
FF4 α	0.24	0.20	0.33	0.63	1.82	1.58
t-statistic	1.05	1.13	1.45	1.53	1.73	1.51

Table 5: Properties of Equally-Weighted Decile Portfolios formed each year from January 1967 to December 2004 on the Variance Ratio of Residual Returns

$VR(24)$ is the average of the 24-month variance ratios which are estimated from Fama-French 3-factor residuals over the previous 60 months. The $\beta's$ are the loadings of the portfolio returns on the 3 Fama-French and the Carhart Momentum factors. *Size* is the time series mean portfolio size in billion \$. σ_e^2 is the average variance of the residuals from Fama-French 3-factor regressions over the previous 60 months. Panel B reports average returns and the intercepts (α) from regressions of excess returns on the market excess returns (CAPM), the 3 Fama-French factors (FF3), and the 3 Fama-French factors plus the Carhart Momentum factor (FF4). The *t*-statistics are adjusted for autocorrelation and heteroscedasticity.

	High VR	2	3	4	5	6	7	8	9	Low VR	Spread $10-1$
Panel A: Portfolio Characteristics											
β_{mkt}	1.05	1.02	1.02	1.00	0.98	1.01	0.98	1.00	1.00	0.94	
t-statistic	43.50	45.99	49.79	42.21	44.85	49.06	53.91	45.45	39.36	33.65	
β_{SMB}	0.99	0.90	0.87	0.90	0.82	0.84	0.84	0.83	0.78	0.95	
t-statistic	26.10	25.22	22.94	22.06	18.27	32.03	27.82	22.73	16.83	24.85	
β_{HML}	0.18	0.28	0.31	0.30	0.31	0.35	0.33	0.36	0.35	0.34	
t-statistic	5.07	6.97	8.55	6.82	7.96	9.65	9.59	8.89	7.46	7.14	
β_{MOM}	-0.05	-0.08	-0.07	-0.05	-0.06	-0.04	-0.02	0.00	-0.02	0.03	
t-statistic	-1.70	-1.74	-1.79	-1.06	-1.79	-0.98	-0.56	-0.15	-0.47	0.74	
Size (\$ B)	0.79	0.86	0.96	1.00	1.06	1.09	1.09	1.04	1.17	1.03	
VR(24)	2.04	1.13	0.83	0.65	0.52	0.41	0.33	0.26	0.19	0.12	
σ_e^2 ($\%^2$)	225.25	163.59	153.00	151.95	145.69	140.27	137.75	135.20	134.84	135.48	
Number of Stock											
Mean	287	287	288	288	289	288	289	289	289	289	
Mininum	159	158	161	164	158	160	161	163	159	158	
Panel B: Returns *per cent* per month											
Average Return	1.38	1.42	1.49	1.57	1.56	1.55	1.55	1.61	1.67	1.71	0.32
t-statistic	4.16	4.59	4.88	5.18	5.39	5.33	5.33	5.42	5.68	5.78	3.47
Capm α	0.31	0.39	0.46	0.54	0.56	0.53	0.55	0.61	0.67	0.72	0.41
t-statistic	1.70	2.36	2.82	3.20	3.33	3.38	3.47	3.77	4.19	4.10	4.89
FF3 α	0.04	0.08	0.14	0.23	0.25	0.20	0.23	0.28	0.35	0.38	0.34
t-statistic	0.47	1.03	2.00	2.82	2.90	2.94	3.23	4.01	4.42	4.31	3.91
FF4 α	0.10	0.16	0.21	0.28	0.31	0.24	0.25	0.28	0.36	0.35	0.26
t-statistic	0.97	1.68	2.37	2.71	3.36	2.86	3.09	3.56	4.13	3.79	2.93

Table 6: Size and Book-to-market adjusted returns for Equally-Weighted Decile Portfolios formed each year from January 1967 to December 2004 on the Variance Ratio of Residual Returns
$VR(24)$ is the average of the 24-month variance ratios which are estimated from Fama-French 3-factor residuals over the previous 60 months. The $\beta's$ are the loadings of the portfolio returns on the 3 Fama-French and the Carhart Momentum factors. *Size* is the time series mean portfolio size in billion \$. Panel B reports size and book-to-market adjusted returns. Panel C reports the intercepts (α) from regressions of size and book-to-market adjusted returns on the market excess returns (CAPM), the 3 Fama-French factors (FF3), and the 3 Fama-French factors plus the Carhart Momentum factor (FF4). The returns and $\alpha's$ are in *per cent* per month, and the t-statistics are adjusted for autocorrelation and heteroscedasticity.

	High VR	2	5	9	Low VR	Spread $10-1$
Panel A: Portfolio Characteristics						
β_{mkt}	0.04	0.01	-0.02	0.00	-0.04	
t-statistic	2.12	0.44	-1.60	0.24	-2.70	
β_{SMB}	0.11	0.02	-0.03	-0.04	0.05	
t-statistic	3.69	1.10	-1.45	-1.92	1.67	
β_{HML}	-0.12	-0.06	-0.02	0.01	0.01	
t-statistic	-3.47	-2.53	-0.63	0.34	0.28	
β_{MOM}	0.08	0.03	0.06	0.09	0.11	
t-statistic	3.63	1.72	4.20	3.97	4.52	
Size(\$B)	0.81	0.89	1.11	1.22	1.08	
VR(24)	1.94	1.07	0.49	0.18	0.11	
BM	0.91	0.88	0.88	0.87	0.89	
Number of Stocks						
Mean	247	248	249	250	250	
Panel B: Size and B/M Matched Returns, *per cent* **per month**						
Adjusted Return	-0.19	-0.20	-0.15	0.00	0.04	0.23
t-statistic	-2.09	-3.75	-2.73	-0.06	0.70	2.18
Panel C: Size and B/M Matched Risk Adjusted Returns, *per cent* **per month**						
CAPM α	-0.23	-0.21	-0.13	0.01	0.07	0.30
t-statistic	-2.66	-4.01	-2.52	0.11	1.15	2.95
FF3 α	-0.17	-0.17	-0.11	0.02	0.07	0.24
t-statistic	-2.02	-3.19	-1.95	0.33	1.10	2.49
FF4 α	-0.25	-0.20	-0.17	-0.07	-0.04	0.21
t-statistic	-3.11	-3.40	-2.98	-1.19	-0.77	2.19

Table 7: Properties of Equally-Weighted 5 by 5 Portfolios Sorted on σ_e^2 and Variance Ratios from January 1967 to December 2004.
At the end of each year from 1966 to 2003, stocks are sorted into σ_e^2 quintile portfolios, where σ_e^2 is the variance of the residuals from Fama-French 3-factor model regressions estimated over the previous 60 months. Within each σ_e^2 quintile, they are further sorted into 5 variance ratio (VR) portfolios, where VR is defined as the ratio between the actual and the implied (assuming zero autocorrelation) variances of FF3 residuals over a 24-month period. The table reports average returns and the intercepts ($\alpha's$) from regressions of excess returns on the market excess returns (CAPM), the 3 Fama-French factors (FF3), and the 3 Fama-French factors plus the Carhart Momentum factor (FF4). The returns and $\alpha's$ are in *per cent* per month, and the t-statistics are adjusted for autocorrelation and heteroscedasticity.

	Average Returns (%)						t-statistic					
	Hi VR	2	3	4	Lo VR	Spread (5-1)	Hi VR	2	3	4	Lo VR	Spread (5-1)
Lo σ_e^2	1.23	1.27	1.23	1.27	1.21	-0.03	6.42	6.80	6.70	6.68	6.09	-0.38
2	1.34	1.33	1.33	1.41	1.43	0.09	5.43	5.36	5.41	5.94	5.86	1.26
3	1.41	1.41	1.52	1.50	1.61	0.20	4.75	4.78	5.25	5.02	5.40	2.54
4	1.57	1.65	1.73	1.65	1.86	0.29	4.14	4.37	4.82	4.48	5.07	3.04
Hi σ_e^2	1.52	1.62	2.09	2.06	2.30	0.78	3.39	3.53	4.58	4.31	4.78	4.95
	CAPM α (%)						t-statistic					
	Hi VR	2	3	4	Lo VR	Spread (5-1)	Hi VR	2	3	4	Lo VR	Spread (5-1)
Lo σ_e^2	0.41	0.45	0.41	0.43	0.36	-0.05	3.07	3.54	3.45	3.60	2.81	-0.65
2	0.40	0.40	0.40	0.48	0.50	0.10	2.70	2.80	2.80	3.58	3.59	1.36
3	0.39	0.39	0.52	0.50	0.61	0.22	2.37	2.48	2.96	2.76	3.48	2.87
4	0.45	0.53	0.63	0.56	0.79	0.34	2.04	2.42	2.95	2.59	3.45	3.64
Hi σ_e^2	0.32	0.43	0.92	0.88	1.16	0.84	1.18	1.50	2.90	2.69	3.43	5.52
	FF3 α (%)						t-statistic					
	Hi VR	2	3	4	Lo VR	Spread (5-1)	Hi VR	2	3	4	Lo VR	Spread (5-1)
Lo σ_e^2	0.12	0.15	0.13	0.16	0.07	-0.05	1.24	1.73	1.59	1.92	0.82	-0.64
2	0.07	0.07	0.04	0.16	0.13	0.06	0.75	0.72	0.44	1.83	1.56	0.81
3	0.07	0.02	0.13	0.13	0.23	0.16	0.90	0.24	1.55	1.35	2.90	2.06
4	0.08	0.18	0.34	0.21	0.44	0.36	0.72	1.88	3.33	1.83	3.77	3.67
Hi σ_e^2	0.05	0.12	0.60	0.54	0.82	0.76	0.30	0.67	2.90	2.36	3.41	4.51
	FF4 α (%)						t-statistic					
	Hi VR	2	3	4	Lo VR	Spread (5-1)	Hi VR	2	3	4	Lo VR	Spread (5-1)
Lo σ_e^2	0.16	0.18	0.14	0.17	0.10	-0.06	1.94	2.27	2.02	2.32	1.19	-0.75
2	0.11	0.13	0.09	0.19	0.20	0.09	1.34	1.53	1.09	2.58	2.57	1.12
3	0.13	0.10	0.20	0.20	0.24	0.11	1.40	1.21	2.25	2.23	2.76	1.32
4	0.12	0.23	0.38	0.23	0.48	0.36	0.94	1.84	3.15	1.74	3.35	3.72
Hi σ_e^2	0.24	0.34	0.71	0.61	0.84	0.60	1.20	1.44	2.78	2.29	3.33	3.83

Table 8: Properties of Equally-Weighted 5 by 5 Portfolios Sorted on Variance Ratios and σ_e^2 from January 1967 to December 2004.
At the end of each year from 1966 to 2003, stocks are sorted into variance ratio (VR) quintile portfolios, where VR is defined as the ratio between the actual and the implied (assuming zero autocorrelation) variances of the residuals from Fama-French 3-factor model regressions over a 24 month period estimated over the previous 60 months. Within each VR quintile, they are further sorted into 5 σ_e^2 portfolios, where σ_e^2 is the variance of FF3 residuals estimated over the previous 60 months. The table reports average returns and the intercepts ($\alpha's$) from regressions of excess returns on the market excess returns (CAPM), the 3 Fama-French factors (FF3), and the 3 Fama-French factors plus the Carhart Momentum factor (FF4). The returns and $\alpha's$ are in *per cent* per month, and the t-statistics are adjusted for autocorrelation and heteroscedasticity.

	Average Returns (%)						t-statistic					
	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)
Hi VR	1.22	1.29	1.55	1.49	1.43	0.21	5.98	4.91	4.68	3.80	3.08	0.59
2	1.29	1.33	1.47	1.71	1.84	0.55	6.91	5.35	4.85	4.50	4.02	1.49
3	1.22	1.29	1.48	1.61	2.06	0.84	6.63	5.30	5.33	4.55	4.57	2.23
4	1.26	1.41	1.56	1.64	2.01	0.76	6.79	5.87	5.33	4.62	4.34	1.93
Lo VR	1.21	1.40	1.59	1.77	2.34	1.13	6.21	6.03	5.69	4.96	4.90	2.79
	CAPM α (%)						t-statistic					
	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)
Hi VR	0.37	0.33	0.49	0.34	0.22	-0.15	2.75	2.18	2.49	1.53	0.78	-0.49
2	0.47	0.40	0.44	0.58	0.67	0.20	3.71	2.69	2.75	2.59	2.21	0.66
3	0.40	0.36	0.49	0.52	0.90	0.50	3.25	2.65	2.87	2.48	2.87	1.53
4	0.43	0.49	0.56	0.57	0.84	0.42	3.58	3.54	3.16	2.77	2.78	1.26
Lo VR	0.38	0.48	0.62	0.72	1.20	0.82	2.90	3.55	3.89	3.19	3.56	2.32
	FF3 α (%)						t-statistic					
	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)
Hi VR	0.09	0.00	0.18	0.04	-0.05	-0.14	0.89	0.04	1.95	0.36	-0.25	-0.66
2	0.17	0.07	0.08	0.23	0.35	0.18	1.96	0.74	1.09	2.11	1.72	0.78
3	0.11	0.01	0.10	0.19	0.58	0.47	1.32	0.08	1.09	1.92	2.78	1.96
4	0.14	0.16	0.18	0.23	0.49	0.35	1.71	1.74	1.83	2.35	2.42	1.42
Lo VR	0.08	0.15	0.21	0.34	0.86	0.78	0.94	1.69	2.46	2.94	3.62	2.87
	FF4 α (%)						t-statistic					
	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)
Hi VR	0.13	0.05	0.20	0.09	0.21	0.08	1.51	0.57	1.82	0.70	0.98	0.37
2	0.22	0.11	0.18	0.29	0.51	0.29	2.87	1.41	1.96	2.08	1.96	1.09
3	0.12	0.07	0.17	0.26	0.71	0.60	1.50	0.89	1.83	2.32	2.69	2.07
4	0.17	0.18	0.25	0.27	0.53	0.36	2.28	2.38	2.76	2.11	2.38	1.43
Lo VR	0.10	0.21	0.23	0.35	0.88	0.78	1.16	2.53	2.63	2.59	3.51	2.87

Figure 1: Mispricing Return Premium and Realized FF3 Alpha

Figure 1A plots annualized FF3 alphas against the estimated mispricing return premium for decile portfolios formed using the whole sample. For Figure 1B, the 10% of the securities with the highest premium estimates are removed from the sample before forming portfolios.

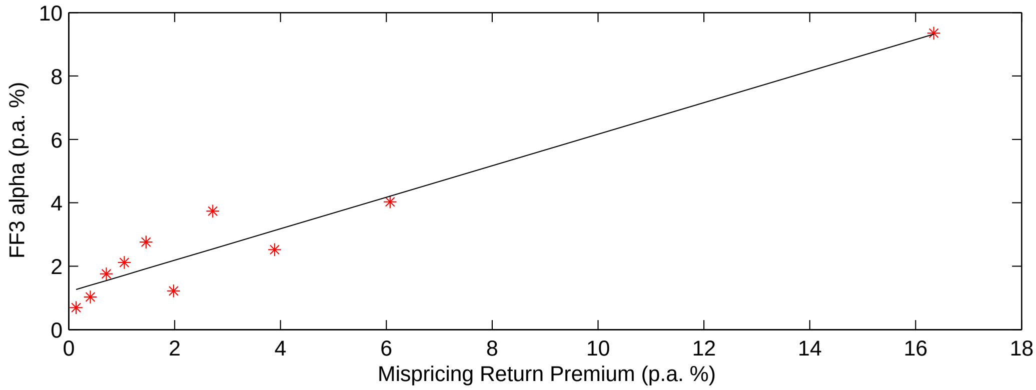

Figure 1A: Whole Sample

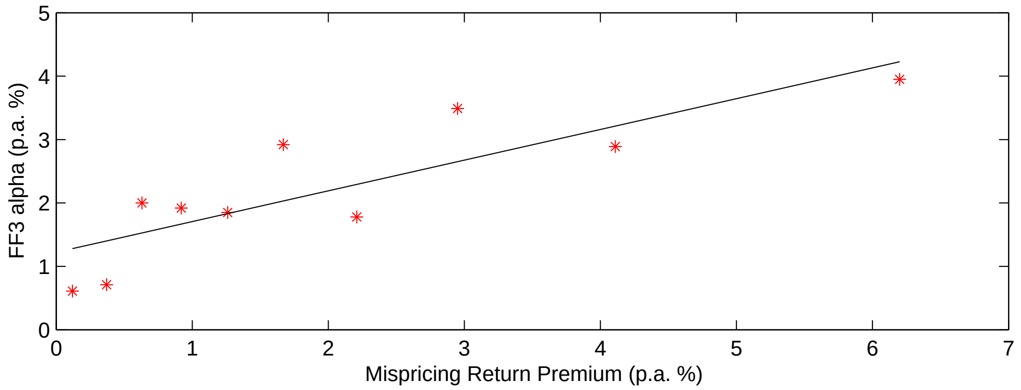

Figure 1B: 10% winsorization: