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A Note on Sparse Networks Tolerating Random Faults for Cycles

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1 Introduction

An $\mathcal{O}(n)$-vertex graph $G^*(n,p)$ is called a random-fault-tolerant (RFT) graph for an n-vertex graph G_n if $G^*(n,p)$ contains G_n as a subgraph with probability $\mathrm{Prob}(G_n,G^*(n,p))$ converging to 1, as $n \to \infty$, even after deleting each vertex from $G^*(n,p)$ independently with constant probability p. The construction of RFT graphs for various graphs has been extensively studied in the literature[1, 3]. The purpose of this paper is to show a proof of the following theorem mentioned in [1].

Theorem I *An n-vertex cycle C_n has an RFT graph with $\mathcal{O}(n)$ edges.*

2 Sketch of the Proof of Theorem I

The proof of the theorem is based on the following lemma shown in [2].

Lemma II *There exist constants c and q_t, $0 < c, q_t \leq 1$, such that a $\lceil\sqrt{m}\,\rceil \times \lceil\sqrt{m}\,\rceil$ grid has a connected component of size at least cm with probability converging to 1, as $m \to \infty$, even after deleting each vertex from the grid independently with constant probability q, if $q < q_t$.* □

The following lemmas can be proved by the same arguments in [1]. Let $G(n)$ be a $2\lceil\sqrt{\lceil n/4\rceil/c}\,\rceil \times 2\lceil\sqrt{\lceil n/4\rceil/c}\,\rceil$ grid with one direction diagonals.

Lemma 1 *If $p < p_t = 1-(1-q_t)^{1/4}$, $G(n)$ is an RFT graph for C_n.* □

Let $H(n,p)$ be a graph obtained from $G(n)$ by replacing each vertex in $G(n)$ by k vertices, and each edge (x,y) by k^2 edges forming a complete bipartite graph between the vertices representing x and the vertices representing y, where k is the smallest integer such that $1-(1-p^k)^4 < q_t$.

Lemma 2 *If $p \geq p_t$, $H(n,p)$ is an RFT graph for C_n.* □

3 Estimate of c

Since it has been known that the largest value of q_t is close to 0.4 [2], the largest value of p_t is close to 0.12. On the other hand, no estimate has been known for c. Figure 1 shows simulation results estimating c. The results suggest that the largest value of c is around 0.25, which means that the size of $G(n)$ is practical. Figure 2 shows simulation results for $\mathrm{Prob}(C_n,G(n))$ when $c = 0.25$ and $n = 10^2$,10^4, and 10^5. The results suggest that $G(n)$ contains C_n with high probability even for practical values of n and p.

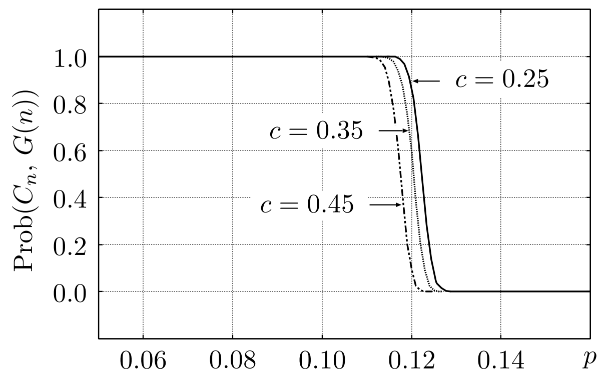

Figure 1: $\mathrm{Prob}(C_n,G(n))$ for $n = 10^5$, and $c = 0.25$,0.35, and 0.45.

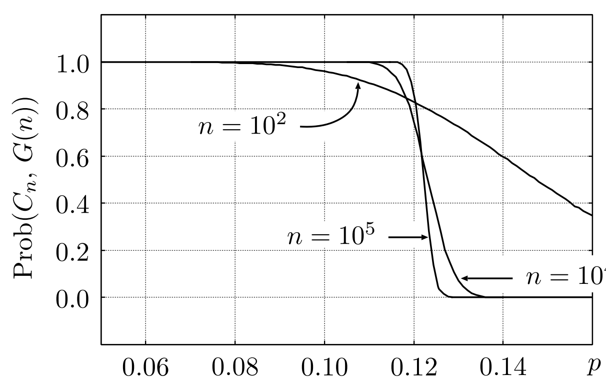

Figure 2: $\mathrm{Prob}(C_n,G(n))$ for $c = 0.25$, and $n = 10^2$,10^4, and 10^5.

References

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