CDMA Sparse Channel Estimation Using a GSIC/AM Algorithm for Radiolocation †

Sunwoo Kim, Timothy Pals, Ronald A. Iltis, and Hua Lee
Department of Electrical and Computer Engineering
University of California, Santa Barbara, CA 93106

Abstract

This paper considers channel parameter estimation in a sparse channel environment for radiolocation. The generalized successive interference cancellation (GSIC) algorithm is used to eliminate the multiple access interference (MAI). To adapt GSIC to sparse channels the alternating maximization (AM) algorithm is considered and the continuous time delay of each path is estimated. The time-of-arrival (TOA) of the first arrival ray is treated as the line-of-sight component. The TOAs from multiple reference nodes to a master node are then used to compute master node position.

I. Introduction

As wireless communication systems evolve into the third and the fourth generation (3G/4G), more wireless-based services become available. Among them is radiolocation in code division multiple access (CDMA) networks, which provides the absolute or the relative location information of the mobiles. Methods such as time-of-arrival (TOA), time-difference-of-arrival (TDOA), GPS-Assisted, and signal strength are currently under consideration to implement radiolocation [1].

In the TOA or TDOA methods, the quality of the radiolocation is directly related to the accuracy of the time delay estimates. However, the estimation of the time delay is a very challenging problem in CDMA where multiple users share a common frequency band. It is well known that multiple access interference (MAI) together with the near-far effect severely limits system performance in the absence of power control [2]. In the radiolocation application considered here, a master node determines round-trip travel times (RTTs) by transmitting a request-to-send (RTS) packet, and estimating TOAs of acknowledgment (ACK) packets transmitted by multiple reference nodes. Thus, power control is infeasible for this handshaking protocol [3], and the estimation of ACK packet TOA is complicated by the near-far effect.

The Generalized Successive Interference Cancellation (GSIC) algorithm [4] has been recently developed and shown to perform very well in the presence of a strong near-far effect. GSIC is computationally very efficient since its complexity is $\mathcal{O}((QK(K+1))/2)$ compared with $\mathcal{O}(Q^K)$ for a true maximum-likelihood solution, where Q is the number of the candidate delays and K is the number of reference nodes. However, the GSIC algorithm in [4] assumes that the multipath spread is a small fraction of the symbol duration, and hence this algorithm may not perform well for long channels. Furthermore, the GSIC approach assumes a tapped-delay line channel with Nyquist tap spacing, resulting in over-parameterization for a sparse multipath profile.

Delay spreads as large as 10μs [5] can be found in situations such as urban areas and macro-cell channels, and usually these channels are sparse. To accommodate such situations, a combination of GSIC and the alternating maximization algorithm (AM) [6] is considered and the continuous time delay of each multipath is estimated. Note that Cotter and Rao [7] have proposed a similar approach to sparse channel estimation for the single user case.

II. Signal and Channel Model

In the handshaking radiolocation protocol, a set of reference nodes with known positions transmits Direct Sequence (DS)-CDMA waveforms, upon receipt of a RTS signal from the master node. The resulting signal received at the master node is

$$r(t) = \sum_{k=1}^{K}\sum_{p=1}^{N_f}\sum_{m=0}^{M-1} f_{k,p} b_k(m) s_k(t - mT - T_{k,p}) + n(t), \qquad (1)$$

where

K	=	the number of reference nodes,
N_f	=	the number of multipaths,
M	=	the number of message symbols transmitted per packet.

The DS waveforms $s_k(t)$ have support $[0, T)$, and are composed of binary chips (± 1) with duration T_c. The

†This work was supported in part by DARPA under grant No. MDA972-00-1-0036 and by a grant from the International Foundation for Telemetering.

$b_k(m)$ represent the data (navigation message,) and $n(t) \in \mathcal{C}$ is additive circular white Gaussian noise with bilateral spectral density $2N_0$. The channel coefficients are denoted by $f_{k,p} \in \mathcal{C}$, representing the complex attenuation of the p-th path from the k-th reference node and $T_{k,p} \in \mathcal{R}$ is the continuous propagation delay corresponding to $f_{k,p}$. In the radiolocation problem considered here, the time delays satisfy $T_{k,p} \in [0, T]$ to avoid range ambiguities.

The received waveform $r(t)$ is sampled at the Nyquist rate $2/T_c = 1/T_s$ to form symbol-length vectors of $N_s = T/T_s$ samples, denoted by $\mathbf{r}(n) \in \mathcal{C}^{N_s}$ for n-th symbol. These vectors are defined by $\mathbf{r}(n) = [r((N_s-1)T_s + nT), r((N_s-2)T_s + nT), \ldots, r(nT)]^T$. For the n-th vector,

$$\mathbf{r}(n) = \sum_{k=1}^{K}\sum_{p=1}^{N_f}\sum_{q=0}^{1} f_{k,p} b_k(n-q)\mathbf{s}_k(T_{k,p} - qT) + \mathbf{n}(n), \tag{2}$$

where the signal matrix $\mathbf{s}_k(\tau) \in \mathcal{C}^{N_s}$ is defined by

$$\mathbf{s}_k(\tau) = [s_k((N_s-1)T_s - \tau), s_k((N_s-2)T_s - \tau), \ldots, s_k(-\tau)]^T \tag{3}$$

Stacking the vectors $\mathbf{r}(n)$ into $\mathbf{r} \in \mathcal{C}^{MN_s}$ yields

$$\mathbf{r} = \left[\mathbf{r}(M-1)^T, \mathbf{r}(M-2)^T, \ldots, \mathbf{r}(0)^T\right]^T \tag{4}$$

$$= \sum_{k=1}^{K}\sum_{p=1}^{N_f} f_{k,p}\mathbf{S}_k(T_{k,p})\mathbf{b}_k + \mathbf{n}, \tag{5}$$

where $\mathbf{b}_k = [b_k(M-1), b_k(M-2), \ldots, b_k(0)]^T$ and $\mathbf{S}_k(T_{k,p}) \in \mathcal{C}^{MN_s \times M}$ is a block Toeplitz signal matrix shown in (6).

III. Generalized Successive Interference Cancellation / Alternating Maximization

The generalized successive interference cancellation algorithm in [4] is a very effective technique to eliminate MAI and estimate the unknown channel parameters. By subtracting the strongest signal component existing in the received signal, the channel estimation becomes robust to the near-far effect. At every stage, the GSIC algorithm finds a "best fitting" signal and its corresponding TOA/channel estimates. The signal selected is reconstructed and subtracted from the received signal. The resulting cancelled signal is processed by subsequent stages. The geometric interpretation is given in Figure 1.

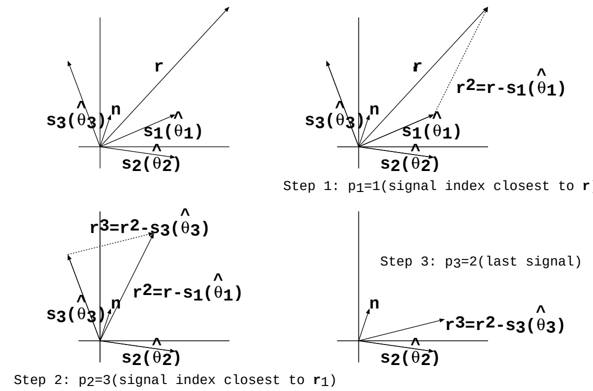


Fig. 1. Geometric interpretation of GSIC

The GSIC algorithm for the frequency-selective channel is formulated as

$$p_k = \underset{l \neq p_1,\ldots,p_{k-1}}{\arg\min} \left\{ \min_{\mathbf{T}_l, \mathbf{f}_l, \mathbf{b}_l} ||\mathbf{r}^k - \sum_{p=1}^{N_f} f_{l,p}\mathbf{S}_l(T_{l,p})\mathbf{b}_l||^2 \right\}, \tag{8}$$

for $k = 1, 2, \ldots, K$, and $\mathbf{T}_l = [T_{l,1}, T_{l,2}, \ldots, T_{l,N_f}]$. To simplify the problem of jointly estimating $f_{l,p}$ and $\mathbf{b}_l$ we treat their product as a single vector

$$\mathbf{b}'_{l,p} = f_{l,p}\mathbf{b}_l. \tag{9}$$

Then, the 2-norm in (8) is simplified as

$$||\mathbf{r}^k - \Sigma_l(\mathbf{T}_l)\mathbf{b}'_l||^2, \tag{10}$$

where

$$\Sigma_l(\mathbf{T}_l) = [\mathbf{S}_l(T_{l,1}), \mathbf{S}_l(T_{l,2}), \ldots, \mathbf{S}_l(T_{l,N_f})] \in \mathcal{C}^{MN_s \times MN_f},$$
$$\mathbf{b}'_l = [\mathbf{b}'^T_{l,1}, \mathbf{b}'^T_{l,2}, \ldots, \mathbf{b}'^T_{l,N_f}]^T \in \mathcal{C}^{MN_f}. \tag{11}$$

Define the projection matrix $\mathbf{P}_{\Sigma_l(\mathbf{T}_l)} \in \mathcal{C}^{MN_s \times MN_s}$ as

$$\mathbf{P}_{\Sigma_l(\mathbf{T}_l)} = \Sigma_l(\mathbf{T}_l)(\Sigma_l^H(\mathbf{T}_l)\Sigma_l(\mathbf{T}_l))^{-1}\Sigma_l^H(\mathbf{T}_l). \tag{12}$$

Following [6] the projection matrix $\mathbf{P}_{\Sigma_l(\mathbf{T}_l)}$ can be decomposed as

$$\mathbf{P}_{\Sigma_l(\mathbf{T}_l)} = \mathbf{P}_{\mathbf{S}_l(T_{l,i})_{\Sigma_{l,i}(\mathbf{T}_l)}} + \mathbf{P}_{\Sigma_{l,i}(\mathbf{T}_l)}, \tag{13}$$

where $\Sigma_{l,i}(\mathbf{T}_l)$ is equivalent to the matrix (11) after removing the block $\mathbf{S}_{l,i}(T_{l,i})$ and

$$\mathbf{S}_l(T_{l,i})_{\Sigma_{l,i}(\mathbf{T}_l)} = (\mathbf{I} - \mathbf{P}_{\Sigma_{l,i}(\mathbf{T}_l)})\mathbf{S}_l(T_{l,i}). \tag{14}$$

By substituting $\mathbf{b}'_l$ in (10) with its linear least-squares solution $\hat{\mathbf{b}}'_l$, the following equivalent maximization problem is obtained [6].

$$\hat{T}_{l,i} = \max_{T_{l,i}} \mathbf{r}^{k^H}\mathbf{P}_{\mathbf{S}_l(T_{l,i})_{\Sigma_{l,i}(\mathbf{T}_l)}}\mathbf{r}^k. \tag{15}$$

$$\mathbf{S}_k(T_{k,p}) = \begin{bmatrix} \mathbf{s}_k(T_{k,p}) & \mathbf{s}_k(T_{k,p}-T) & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{s}_k(T_{k,p}) & \mathbf{s}_k(T_{k,p}-T) & \ldots & \mathbf{0} \\ & \ddots & \ddots & \mathbf{s}_k(T_{k,p}) & \mathbf{s}_k(T_{k,p}-T) \\ \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} & \mathbf{s}_k(T_{k,p}) \end{bmatrix}. \quad (6)$$

$$\mathbf{P}_{\mathbf{S}_l(T_{l,i})_{\Sigma_{l,i}(\mathbf{T}_l)}} = (\mathbf{I} - \mathbf{P}_{\Sigma_{l,i}(\mathbf{T}_l)})\mathbf{S}_l(T_{l,i}) \left\{\mathbf{S}_l(T_{l,i})^H(\mathbf{I} - \mathbf{P}_{\Sigma_{l,i}(\mathbf{T}_l)})\mathbf{S}_l(T_{l,i})\right\}^{-1} \mathbf{S}_l(T_{l,i})^H(\mathbf{I} - \mathbf{P}_{\Sigma_{l,i}(\mathbf{T}_l)})^H. \quad (7)$$

where $\mathbf{P}_{\mathbf{S}_l(T_{l,i})_{\Sigma_{l,i}(\mathbf{T}_l)}}$ is shown in (7).

As shown in [6] AM yields a non-decreasing likelihood at every iteration, but is not guaranteed to converge to the global maximum. GSIC guarantees a non-decreasing likelihood also [4] and hopefully achieves a stationary point different from the AM solution. Therefore, GSIC may provide a good starting point (i.e. initialization) for AM.

In (8) $\sum_{p=1}^{N_f} f_{l,p}\mathbf{S}_l(T_{l,p})\mathbf{b}_l$ can be viewed as $K = N_f$ flat fading virtual users having the same spreading sequence and different time delays. The GSIC flat fading algorithm in [4] can be used to estimate $T_{l,p}$ and $\mathbf{b}'_l = f_{l,p}\mathbf{b}_l$. Then, the $\hat{T}_{l,p}$ are used as the initial values for AM. Note that there is a constraint on delays, i.e. $T_{l,i} \neq T_{l,j}$ for $i \neq j$ and the final $\hat{T}_{k,i}$ should be reordered such that $\hat{T}_{k,1} < \hat{T}_{k,2} < \ldots < \hat{T}_{k,N_f}$. $\hat{T}_{k,1}$ is used to compute the range between the master and the k-th reference node.

The final algorithm is summarized in Table I

IV. Multipath Channel Simulation and Radiolocation

The received signal model in (1) is approximate since N_f is fixed for every reference node. Next, a more accurate propagation and received signal model is introduced for simulations by relaxing the constraint on N_f and setting the number of multipaths for the k-th reference node as P_k.

The signal transmitted by the k-th reference node is $\mathrm{Re}[u_k(t)e^{j2\pi f_c t}]$, where f_c is the carrier frequency. The complex baseband transmitted signal is given as

$$u_k(t) = \sum_{m=0}^{M-1} b_k(m)s_k(t - mT), \quad (16)$$

The complex baseband signal received at the master node from K reference nodes is

$$v(t) = \sum_{k=1}^{K}\sum_{p=1}^{P_k} a_{k,p}e^{-j2\pi f_c T_{k,p}} u_k(t - T_{k,p}). \quad (17)$$

In a user-specified two-dimensional geometry, $\{a_{k,p}\} \in \mathcal{R}$ and $\{T_{k,p}\} \in \mathcal{R}$ are generated by a ray-tracing program [8].

In a two-dimensional radiolocation scenario, at least three reference nodes communicating with a master node are required to determine x and y coordinates. Having three reference nodes with their absolute position information, the GSIC algorithm estimates their TOAs. A linearized positioning solution for terrestrial radiolocation derived in [8] is used after the TOAs are obtained from the GSIC.

V. Simulations and Results

The simulation has been conducted with two channel models. First, the GSIC/AM is tested under a random channel model in which 3 multipaths per user are assumed and their TOAs are uniformly chosen between 0 to T in every ensemble run. Specifically, $T_{k,1} \in [0, 10T_c)$, $T_{k,2} \in [10T_c, 20T_c)$, and $T_{k,3} \in [20T_c, T)$. 31-chips Gold sequences are assigned to each user. The powers for users are set differently so as to simulate a near-far effect. The nominal SNR (E_b/N_0) is varying from 6 dB to 16 dB. User 1's power equals the nominal SNR. The powers of User 2 and 3 are 5 dB and 10 dB higher than that of user 1 respectively. The number of users is $K = 3$, with a packet length of $M = 5$ symbols. Acquisition is defined as the event that $|\tilde{T}_k - T_k| < T_c/2$, and the probability of acquisition (P_{acq}) is computed after 50 ensemble runs. Figure 2 shows P_{acq} versus nominal SNR.

In the second simulation, the ray-tracing technique is used to provide the channel parameters in a user-defined geometry. For convenience, the nominal SNR, 16 dB was set to correspond to a distance of 200 m between the master and a reference node, with no obstacles in between. Figure 4 shows the radiolocation results using the GSIC/AM and the frequency selective GSIC (FS-GSIC) in [4]. In each figure 20 estimated positions of the master node are marked by dots. It is clear that GSIC/AM outperforms FS-GSIC in positioning as more dots are located around the true position of the master node in Figure 4(c) and 4(d). In these cases, GSIC/AM gives better TOA estimates for the sparse channel (Figure 3), which results when a reference node is blocked by a structure and far away from the master node.

TABLE I

GSIC/AM ALGORITHM

For $k = 1, 2, \ldots, K$

Form the cancelled signal

$\mathbf{r}^k = \mathbf{r} - \sum_{l=1}^{k-1} \Sigma_{p_l}(\hat{\mathbf{T}}_{p_l})\hat{\mathbf{b}}'_{p_l}$

For $l \neq p_1, p_2, \ldots, p_{k-1}$

1.Initialize $\hat{\mathbf{T}}_l$ by flat fading GSIC

For $i = 1, 2, \ldots, N_f$

Form the cancelled signal

$\mathbf{r}^{k,i} = \mathbf{r}^k - \sum_{n=1}^{i-1} \mathbf{S}_l(\hat{T}_{l,n})\hat{\mathbf{b}}'_{l,n}$

$\hat{T}_{l,i} = \arg\max_{T_{l,i} \neq \hat{T}_{l,1},\ldots,\hat{T}_{l,i-1}}$
$\sum_{m=0}^{M-1} || \sum_{r=0}^{1} \mathbf{r}^{k,i}(m+r)^H \mathbf{s}_l(T_{l,i} - rT)||^2$

For $m = 0, 1, \ldots, M-1$

$\hat{b}'_{l,i}(m) = \frac{\mathbf{s}_l(\hat{T}_{l,i}-T)^H \mathbf{r}^{k,i}(m+1) + \mathbf{s}_l(\hat{T}_{l,i})^H \mathbf{r}^{k,i}(m)}{||\mathbf{s}_l(\hat{T}_{l,i})||^2 + ||\mathbf{s}_l(\hat{T}_{l,i}-T)||^2}$

Next i

2.Do AM while $||\hat{\mathbf{T}}_l - \tilde{\mathbf{T}}_l|| > \delta$

$\tilde{\mathbf{T}}_l = \hat{\mathbf{T}}_l$

For $i = 1, 2, \ldots, N_f$

$\hat{T}_{l,i} = \arg\max_{T_{l,i} \neq \hat{T}_{l,1},\ldots,\hat{T}_{l,i-1}}$
$\mathbf{r}^{k^H} \mathbf{P}_{\mathbf{S}_l(T_{l,i})_{\Sigma_{l,i}(\tilde{\mathbf{T}}_l)}} \mathbf{r}^k$

save $\hat{\mathbf{T}}_l$

Next l

$p_k = \arg\min_{l \neq p_1,\ldots,p_{k-1}}$
$\left\{ \min_{\hat{\mathbf{T}}_l, \hat{\mathbf{b}}'_l} ||\mathbf{r}^k - \Sigma_l(\hat{\mathbf{T}}_l)\hat{\mathbf{b}}'_l||^2 \right\}$

$\hat{\mathbf{b}}'_{p_k} = (\Sigma_{p_k}^H(\tilde{\mathbf{T}}_{p_k})\Sigma_{p_k}(\tilde{\mathbf{T}}_{p_k}))^{-1}\Sigma_{p_k}^H(\tilde{\mathbf{T}}_{p_k})\mathbf{r}^k$

reorder $\hat{\mathbf{T}}_{p_k}$ and $\hat{\mathbf{b}}'_{p_k}$

such that $\hat{T}_{p_k,1} < \hat{T}_{p_k,2} < \ldots < \hat{T}_{p_k,N_f}$

save $\hat{\mathbf{T}}_{p_k}, \hat{\mathbf{b}}'_{p_k}$

Next k

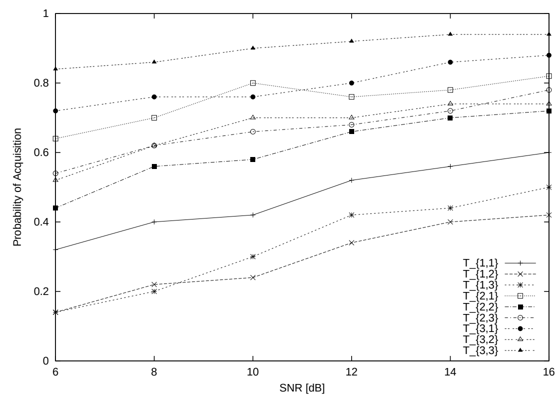


Fig. 2. Probability of Acquisition

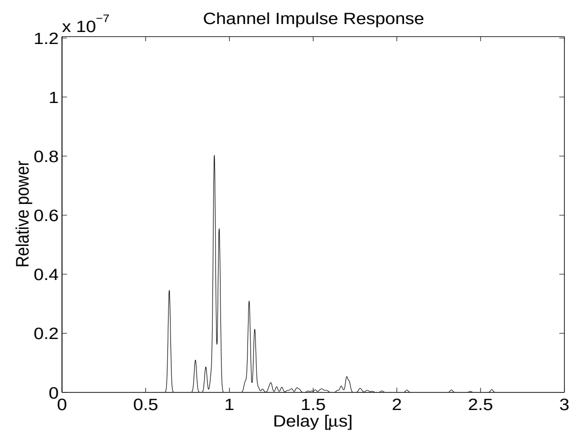


Fig. 3. Sparse Channel

VI. Conclusions

A new GSIC/AM algorithm has been derived to estimate the continuous TOAs of multipaths in CDMA systems, and its performance in radiolocation has been verified through the simulations. GSIC/AM is a computationally effective approach when the multipath channel is long and sparse. The flat fading GSIC algorithm in [4] provides a good starting point for the AM algorithm. One possible drawback of this algorithm is that computing the projection matrix in (15) becomes expensive as M and N_f increase. However, the overall GSIC/AM approach is still far more efficient than true maximum likelihood.

References

[1] J. J. Caffery, Jr., *Wireless location in CDMA cellular radio systems*, Kluwer Academic Pulishers, Norwell, MA, 2000.

[2] S. Verdú, *Multiuser Detection*, Cambridge University Press, New York, 1998.

[3] M. Joa-Ng and I-T. Lu, "A novel spread spectrum-based synchronization and location determination method for wireless system," *IEEE Commun. Lett.*, vol. 3, no. 6, pp. 177–179, Jun. 1999.

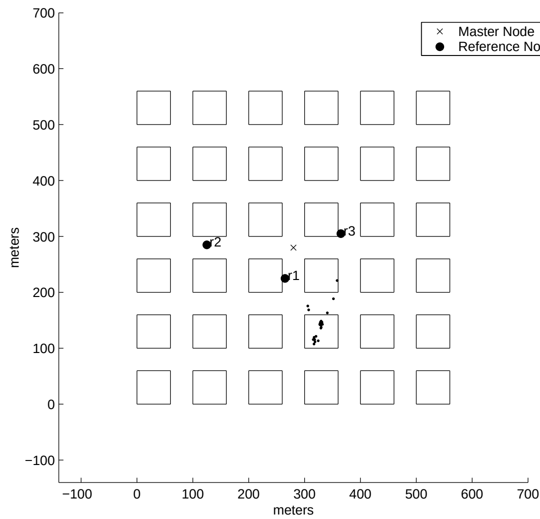
[4] R. A. Iltis and S. Kim, "Geometric derivation of expectation-maximization and generalized successive interference cancellation algorithms with applications to CDMA channel estimation," submitted to IEEE Trans. Signal Processing.

[5] T. S. Rappaport, S. Y. Seidel, and R. Singh, "900-MHz multipath propagation measurements for U.S. digital cellular radiotelephone," *IEEE Trans. Veh. Technol.*, vol. 39, no. 2, pp. 132–139, May 1990.

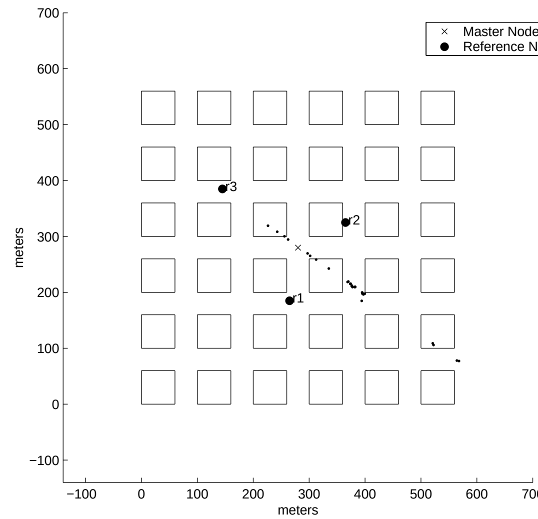
[6] I. Ziskind and M. Wax, "Maximum likelihood localization of multiple sources by alternating projection," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 36, pp. 1553–1560, Oct. 1988.

[7] S. F. Cotter and B. D. Rao, "Sparse channel estimation via matching pursuit with application to equalization," *IEEE Trans. Commun.*, vol. 50, pp. 374–377, Mar. 2002.

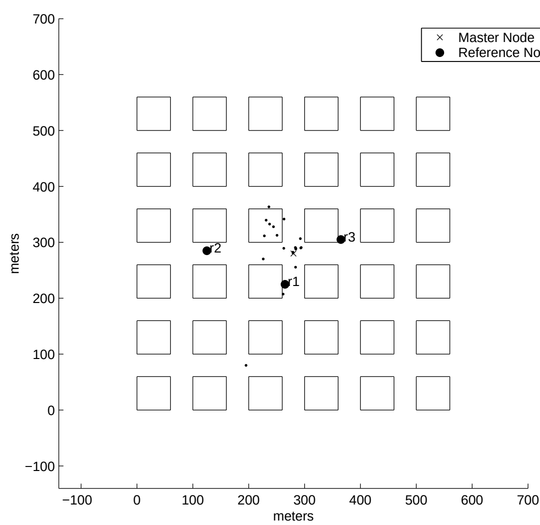
[8] S. Kim, T. Pal, R. A. Iltis, and H. Lee, "CDMA multipath channel estimation using generalized successive interference cancellation algorithm for radiolocation," in *Proc. 37th Annual Conference on Information Sciences and Systems*, Princeton, NJ, Mar. 2002.



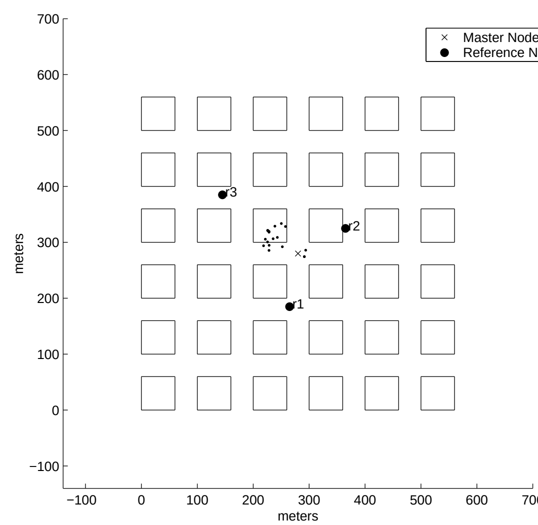
(a) $N_f = 5, M = 5, N_r = 20$ at 16dB by FS GSIC



(b) $N_f = 5, M = 5, N_r = 20$ at 16dB by FS GSIC



(c) $N_f = 5, M = 5, N_r = 20$ at 16dB by GSIC/AM



(d) $N_f = 5, M = 5, N_r = 20$ at 16dB by GSIC/AM

Fig. 4. GSIC vs. GSIC/AM positioning