The Relativistic kinetics of gravitational waves collisional damping in hot Universe

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Abstract

The article is a translation of authors paper [1] printed earlier in the inaccessible edition and summarizing the results of research of gravitational waves damping problem in the cosmologic plasma due to the different interactions of elementary particles.

Introduction

The evolution of the cosmological gravitational waves (GW) in the Universe, in case that perfect fluid or a collisionless gas is considered as a material medium model, is researched sufficiently [2] - [6]. Let us note basic results:

- perfect fluid doesn't change a vacuum character of the propagation of the gravitational radiation;
- dispersion is essential only for super-density objects.

There were made an attempts to provide an analysis of evolution for dissipative mediums (see, for example [7],[8]). In authors works [6], [9] — [11] the damping of gravitational radiation during the propagation in the collisionless gas was considered in the context of kinetic approach. Given paper may be considered as a concluding in the whole cycle of researches.

1 Self-consistent model of weak gravitational waves evolution in the Friedmann Universe

In the spatially flat Friedmann world with a metric:

$$d\overset{o}{s}{}^2 = \overset{o}{g}_{ik}\, dx^i dx^k =$$

$$= a^2(\eta)[d\eta^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2] \qquad (1)$$

gravitational waves are described by metric perturbation:

$$h_{ik} = g_{ik} - \overset{o}{g}_{ik} = \tilde{h}_{ik}(\eta) e^{-in_\alpha x^\alpha}; \qquad \alpha, \beta = \overline{1,3}, \qquad (2)$$

where $n_\alpha = \text{const}$ - wave vector GW[1]; it's reference components, measuring by synchronic observer in metric are (1),

$$k_{(\alpha)} = \frac{n_\alpha}{a}; \qquad k = \frac{n}{a},$$

where $n = \sqrt{n_1^2 + n_2^2 + n_3^2}$. It is obvious that in the flat Friedmann world waves of any length λ can be counted like short:

$$\lambda \ll R \to \infty \tag{3}$$

where R —the radius of background curvature. Then on the h_{ik} additional conditions [2] of graduation and tracelessity can be imposed:

$$h_{i4} = 0, \quad h = 0, \quad h^{ik}_{\ ,k} = 0 \to h^\beta_\alpha n_\beta = 0; \tag{4}$$

here and further the indexes lift and lower by the background metric $\overset{o}{g}_{ik}$, $h = g_{ik}h^{ik}$. How it will be seen from the further, the perturbation of the momentum-energy tensor (MET) of medium, determining by the GW appearance is representable in the form:

$$\delta T_{ik} = \tau(\eta) h_{ik}. \tag{5}$$

Then the system of linearized by h Einstein equations can be written in the form

$$\psi'' + n^2\psi - 2\left(\frac{a'}{a}\right)^2 \psi - \left(\frac{a''}{a}\right)\psi + 16\pi\psi a^2\tau = 0,$$

where in further we will set:

$$\tilde{h}_{\alpha\beta}(\eta) = a(\eta) S_{\alpha\beta}\psi(\eta), \tag{6}$$

at that $S_{\alpha\beta} = \text{const}$. Taking into account the background Einstein equations

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi\varepsilon a^2}{3}, \ \frac{a''}{a} = -\frac{4\pi}{3}a^2(\varepsilon + 3p) - \frac{8\pi}{3}a^2\varepsilon, \tag{7}$$

finally we will receive the evolution of *GW conform amplitude* equation $\psi(\eta)$:

$$\psi'' + n^2\psi + 8\pi a^{-2}\psi\left[-\frac{1}{6}(\overline{\varepsilon} - 3\overline{p}) + 2\overline{p} + 2\overline{\tau}\right] = 0, \tag{8}$$

where for convenience we have proceed to conserving on the ultrarelativistic stage conform densities of energy and pressure $\overline{\varepsilon} = a^4(\eta)\varepsilon$, $\overline{p} = a(\eta)^4 p$ etc.

For calculation of $\overline{\tau}(\eta)$ we will advert to relativistic kinetic equations

$$[H, f] = J[f]. \tag{9}$$

The appearance of GW leads to the distortion of mass surface, thereof it is necessary to produce a renormalization of momentum [3]-[4]. It is simpler to

[1] Here and further the Greek letters run the values from 1 to 3; Latin - from 1 to 4.

make an indicated renormalization by transformation (9) of momentum variables p_i to the momentum variables $\mathbb{P}_i$

$$\mathbb{P}_i = p_i - \frac{1}{2}h_i^k p_k,\; p_i = \mathbb{P}_i + \frac{1}{2}h_i^k \mathbb{P}_k + O(h^2). \tag{10}$$

The Jacobian of this transformation in case of traceless perturbations of metric in linear by h approach is equal to one. In new variables the Hamilton function coincide with the unperturbed value accurate to members $O(h^2)$:

$$H(x,p) = \frac{1}{2}g^{ik}p_i p_k = \frac{1}{2}\overset{o}{g}{}^{ik}\,\mathbb{P}_i\mathbb{P}_k = \overset{o}{H}(x,p). \tag{11}$$

Therefore $g = \overset{o}{g} + O(h^2)$, differential of volume of momentum space is invariant with respect to transformation (10)

$$d\pi' = \frac{2s+1}{(2\pi)^3}\,\frac{\alpha^4 p}{\sqrt{-g}}\delta(H - \frac{1}{2}m^2) =$$

$$= \frac{2s+1}{(2\pi)^3}\,\frac{\alpha^4 \mathbb{P}}{\sqrt{-g}}\delta(\overset{o}{H} - \frac{1}{2}m) = d\,\overset{o}{\pi}\,. \tag{12}$$

Taking into account the fact, that momentum variables can be contained in invariant scattering amplitude M_{if} by means of all possible contractions of type (p,p'), it can be strictly shown, that collision integral accurate to $O(h^2)$ is invariant with respect to transformations (10). Thus, representing a distribution function $f(x,p)$ in form

$$f(x^i,p_i) = f_0(\eta,\mathbb{P}_4) + \delta f(x^i,\mathbb{P}_k), \tag{13}$$

where $f_0(\eta,\mathbb{P}_4)$ - isotropic solution of kinetic equations (9), and integrating the kinetic equations (9), we will obtain linearized kinetic equations for distributions deviations δf:

$$\mathbb{P}^i\frac{\partial \delta f}{\partial x^i} + \frac{1}{2}\frac{\partial f_0}{\partial \mathbb{P}_0}\mathbb{P}_\beta\mathbb{P}^\alpha h'^\beta_\alpha = J^1[f_0,\delta f]. \tag{14}$$

The isotropic part of distribution function f_0 satisfies to zero approximation of equation (9):

$$\mathbb{P}^0\frac{\partial f_0}{\partial \eta} + \frac{a'}{a}(\mathbb{P}_0\mathbb{P})\frac{\partial f_0}{\partial \mathbb{P}_0} = J^0[f_0]. \tag{15}$$

Calculating the MET perturbation with the account of transformation (10), we'll find:

$$\delta T_{ik} = -p h_{ik} + \sum\int \mathbb{P}_i\mathbb{P}_k\delta f d\pi, \tag{16}$$

where summation is carried out by all kinds of particles, participating in reactions. From linear equations (14) is clear, that $\delta f \sim S_{\alpha\beta}\mathbb{P}^\alpha\mathbb{P}^\beta$. Actually, supposing such dependence we arrive at conclusion that collision integral J^1 in consequence of it's invariance can be only a linear combination of contractions of type $S_{\alpha\beta}\mathbb{P}^{\alpha}\mathbb{P}^{\beta}$, $S_{\alpha\beta}n^{\alpha}\mathbb{P}^{\beta}$, $S_{\alpha\beta}n^{\alpha}n^{\beta}$, $S_{\alpha\beta}\delta^{\alpha\beta}$, from which only the first is different from zero. And integrals of type (16) are defined by the only selected spatial direction, n_{α}. Therefore these integrals become the linear combination of contractions:

$$S_{\gamma\delta}n^{\gamma}n^{\delta},\ S_{\gamma\delta}\delta^{(\gamma\alpha}n^{\beta}n^{\gamma)},\ S_{\gamma\delta}\delta^{(\alpha\gamma}\delta^{\beta\gamma)},$$

from which only the last is different from zero and equal to $S_{\alpha\beta}$.

Thus, finally we'll obtain:

$$\delta\overline{T}_{ik} = -\overline{h}_{ik}p + \overline{h}_{ik}\overline{\tau}_f, \tag{17}$$

where

$$\overline{\tau}_f = \frac{1}{S^2}\sum\int d\pi\mathbb{P}_{\alpha}\mathbb{P}_{\beta}S_{\alpha\beta}\delta f$$

and the designation $S^2 = S_{\alpha\beta}S_{\alpha\beta}$ is incorporated, - summation is carried out by repeating indexes.

Thus, equation (8) takes form:

$$\psi'' + \eta^2\psi + 8\pi a^{-2}\psi\left[-\frac{1}{6}(\overline{\varepsilon} - 3\overline{p}) + 2\overline{\tau}_f\right] = 0. \tag{18}$$

2 Calculation of collision integral

Let's turn to calculation of $\overline{\tau}_f$. In case of equilibrium distributions f_0 an integral J^1 can be represented in form [11]:

$$J^1[f_0\delta f] = \frac{\delta f}{1 \pm f_0}\mathbf{K}(\mathbb{P}, \eta), \tag{19}$$

where

$$\mathbf{K}(\mathbb{P}, \eta) =$$

$$= \sum\int\prod_{f,i}{}' d\pi\delta(\mathbb{P}_f - \mathbb{P}_i)W_{if}\prod_f{}' f_0\prod_f{}'(1 \pm f_0).f_0)' \tag{20}$$

Here summation is carrying out by all initial,(i),and final, (f), states of given sort particles, participating in reactions; $\prod'_{f,i} d\pi$ means the product of momentum volumes of all particles, except given, W_{if} - invariant scattering matrix (details see in Ref. [13]).

Let's illustrate a statement (19) for four-particle reactions $ab \longleftrightarrow cd$. In that case collision integral takes form:

$$J_{ab\longleftrightarrow cd} = \frac{1}{2(2S_a+1)(2S_b+1)}\times$$

$$\times\int\frac{d\pi_b}{2}\frac{d\pi_c}{2}\frac{d\pi_d}{2}(2\pi)^4\delta(\mathbb{P}_a + \mathbb{P}_b - \mathbb{P}_c - \mathbb{P}_d)\overline{|M_{if}|}^2\times$$

$$\times \{f_c f_d(1 \pm f_a)(1 \pm f_b) - f_a f_b(1 \pm f_c)(1 \pm f_d)\} \,. \tag{21}$$

Multipliers $(2S_a+1)^{-1}$, $(2S_i+1)^{-1}$ (21) correspond to the average by polarized states of particles [14], and coefficients 1/2 respond to the selected normalization of colliding particles wave functions [15]. Substituting $f = f_0 + \delta f$, where f_0 — isotropic equilibrium distribution function, and $\delta f \sim S_{\alpha\beta}\mathbb{P}^\alpha\mathbb{P}^\beta$, and linearizing an integral (21) by δf, it is easy to make sure that addends in the figure brackets of type $\delta f_c f_d^0(1 \pm f_a^0)(1 \pm f_b^0)$ during integration by $d\pi$ turn to zero. Actually, directing an axis $\mathbb{P}^c$ along the wave vector n_α we'll achieve, that δf_c will be proportional to expressions $(\mathbb{P}_{c2}^2 - \mathbb{P}_{c3}^2)$ and $\mathbb{P}_{c2}\mathbb{P}_{c3}$. Since multipliers at δf_c are invariant relatively to transformations $\mathbb{P}_2 \to \mathbb{P}_3$ and $\mathbb{P}_3 \to \mathbb{P}_2$ (for all particles simultaneously), and δf_c at that change a sign, then during integration by momentums in infinite limits, this added turns to zero. Non-zero contribution in J^1 will give only members, containing δf_a, since integration doesn't carry out by momentums $\mathbb{P}_a$. Thus, discarding non-sufficient for hot model statistical multipliers, we'll find for ultrarelativistic particles:

$$\mathbf{K}(\eta, p) = \nu(p, \eta)p, \tag{22}$$

where effective frequency of collisions $\nu(p, \eta)$ is equal:

$$\nu(p,\eta) =$$

$$= \frac{(2S_c+1)(2S_d+1)}{16\pi^2(2S_a+1)p^2} \int\limits_0^\infty f_0(q)dq \int\limits_0^{4pq} s\sigma_{tot}(s)ds; \tag{23}$$

p - absolute value of physical momentum,

$$\sigma_{t_0t}(s) = \frac{1}{16\pi S}\int\limits_0^1 dx|\overline{M(s,x)}|^2$$

- total scattering cross-section,

$$x = -t/s, \quad s = (p_a + p_b)^2, \quad t = (p_a - p_c)^2$$

- kinematic invariants.

When all interacting particles are ultrarelativistic, kinetic equations (14) take form:

$$\frac{\partial \delta f}{\partial \eta} + a\nu\delta f - \frac{\mathbb{P}_\alpha}{\mathbb{P}}\frac{\partial \delta f}{\partial x^\alpha} =$$

$$= -\frac{1}{2}\frac{\partial f_0}{\partial \mathbb{P}}\frac{S_{\alpha\beta}\mathbb{P}_\alpha\mathbb{P}_\beta}{\mathbb{P}}\left(\frac{\psi}{a}\right)' e^{-i\eta_\alpha x^\alpha}. \tag{24}$$

It should be pointed out, that here $\mathbb{P} \equiv \mathbb{P}_0$ differs from physical momentum by multiplier ($\mathbb{P} = a(\eta)p_{phys}$). It is obvious, that at $\nu > 0$ the second member in equation, conditioned by interparticle collisions, leads to the decrease of δf, i.e., to the relaxation towards the equilibrium distribution. Let's write down the solution of equation (24), turning to zero at $S_{\alpha\beta}=0$ and owning the structure of collisionless equation at $\eta_0 \to 0$:

$$\delta f = -\frac{1}{2}\frac{\partial f_0}{\partial \mathbb{P}}\frac{S_{\alpha\beta}\mathbb{P}_\alpha\mathbb{P}_\beta}{\mathbb{P}}e^{-i\eta_\alpha x^\alpha - i\pi_\alpha - \gamma(\eta)} \times \tag{25}$$
$$\times \lim_{\eta_0\to 0}\left\{\left(\frac{\psi}{a}\right)e^{i\pi_\alpha n_\alpha \eta_0} + \int\limits_{\eta_0}^{\eta}(\psi/a)' e^{\gamma(\eta')+i\pi_\alpha n_\alpha \eta'}d\eta'\right\},$$

where unit vector $\pi_\alpha = \mathbb{P}_\alpha/\mathbb{P}$ and *damping decrement* are incorporated:

$$\gamma(\eta,\mathbb{P}) = \int\limits_{\eta_0}^{\eta}\nu(\eta,\mathbb{P})a(\eta)d\eta = \int\limits_{t_0}^{t}\nu(t,\mathbb{P})dt. \tag{26}$$

It is not too hard to calculate the MET perturbation, conditioned by δf,

$$\delta T_{\alpha\beta} = \sum_a \int \mathbb{P}_\alpha\mathbb{P}_\beta \delta f d\pi =$$
$$= -\frac{\pi S_{\alpha\beta}}{4a^2}e^{-in_\alpha x^\alpha}\sum_a \frac{2S_a+1}{(2\pi)^3}\int\limits_0^\infty d\mathbb{P}\mathbb{P}^4\frac{\partial f_0}{\partial \mathbb{P}}e^{-\gamma(\eta,\mathbb{P})}\times$$
$$\times \lim_{\eta_0\to 0}\left\{\left(\frac{\psi}{a}\right)_{\eta_0} J[\eta(\eta-\eta_0)] + \right.$$
$$\left. + \int\limits_{\eta_0}^{\eta} e^{\gamma(\eta',\mathbb{P})}\left(\frac{\psi}{a}\right)^1 J[n(\eta-\eta')]d\eta'\right\}. \tag{27}$$

The summation in (27) is carried out by sorts of particles a, participating in reactions, and incorporated function:

$$J(x) = \frac{8}{x^2}\left[\frac{\sin x}{x}\left(\frac{3}{x^2}-1\right) - \frac{3\cos x}{x^2}\right], \tag{28}$$

having the asymptotics

$$\underset{x\to\infty}{J(x)\simeq} -\frac{8\sin x}{x^3}, \quad J(0) = \frac{16}{15}. \tag{29}$$

Let's consider now the relationships, that are correct on the ultrarelativistic stage of Universe evolution (see., for example, [16]):

$$a = a_1\eta,$$
$$\overline{T}^4 = (aT)^4 = \frac{45}{4\pi^3 N}a_1^2, \tag{30}$$

N - the statistical factor of particles number:

$$N = \sum_B (2S+1) + \tfrac{7}{8}\sum_F (2S+1).$$

Then we'll obtain:

$$16\pi\overline{a}^2\overline{\tau}_f =$$

$$\frac{45}{8\pi^4 N\eta}\sum_a(2S+1)\int\limits_0^\infty dzz^4\frac{\partial f_0}{\partial z}e^{-\gamma(z,\eta)}\times$$

$$\times\lim_{\eta_0\to 0}\left\{\left(\frac{\psi}{\eta}\right)_{\eta_0} J[\eta(\eta-\eta_0)]+\right.$$

$$\left.+\int\limits_{\eta_0}^{\eta} e^{\gamma(z,\nu_0')}\left(\frac{\psi}{\eta'}\right)^1 J[\eta(\eta-\eta')]d\eta'\right\}, \tag{31}$$

where $z = \mathbb{P}/\overline{T} \equiv p/T$.

Thus, finally we have - *the evolution of cosmological GW in isotropic ultra-relativistic gas* ($\varepsilon = 3p$) *defines by equation:*

$$\psi'' + \eta^2\psi + 16\pi a^{-2}\overline{\tau}_f\psi = 0, \tag{32}$$

where $\overline{\tau}_f$ *is described by expression (31).*

3 Extreme cases

Let's at first consider the case of fast relaxation

$$\gamma \gg 1. \tag{33}$$

Producing the asymptotical estimation of integral (31) by Fourier method, we'll reduce an equation (32) to the form

$$\psi'' + \eta^2\psi + \frac{8}{5}\left(\frac{\psi}{\eta}\right)^1\frac{1}{a\nu_{eff}\eta} = 0, \tag{34}$$

where

$$\frac{1}{\nu_{eff}} = -\frac{\sum(2S+1)\int\limits_0^\infty dz\ z^4\frac{\partial f_0}{\partial z}\frac{1}{\nu(z,\eta)}}{4\sum(2S+1)\int\limits_0^\infty dz\ z^3 f_0}. \tag{35}$$

In consequence of condition (33) the last member in the left part of equation (34) is small in comparison with the first two members - it responds to the weak damping of GW vacuum oscillations:

$$h^\alpha_\beta = -\frac{1}{a}\exp\left(-\int\limits_0^{\eta_0}\frac{4d\eta}{5a\eta^2\nu_{eff}}\right)\times$$

$$\times \left\{ S^{+}_{\alpha\beta} e^{-i(n\eta - n_\alpha x^\alpha)} + S^{-}_{\alpha\beta} e^{i(n\eta - n_\alpha x^\alpha)} \right\}. \quad (36)$$

At $\nu_{eff} t \to \infty$ the damping of GW vanishes: in the ultrarelativistic fluid GW propagate as well as in vacuum. This fact was mentioned in the beginning of the article.

Let now $\gamma \ll 1$. In the collisionless approximation an equation (31) can be simplified:

$$16\pi \overline{a}^2 \overline{\tau}_f = \frac{3}{2\eta} \lim_{\eta_0 \to 0} \left\{ \left(\frac{\psi}{\eta} \right)_{\eta_0} J[\eta(\eta - \eta_0)] + \right.$$

$$\left. + \int\limits_{\eta_0}^{\eta} \left(\frac{\psi}{\eta'} \right)^{1} J[\eta(\eta - \eta')] d\eta' \right\}. \quad (37)$$

This expression have two asymptotics. In the long-wavelength limit ($n\eta \ll 1$) we'll find

$$16\pi a^{-2} \overline{\tau}_f \simeq \frac{8}{5} \frac{\psi}{\eta^2}. \quad (38)$$

Substituting (38) in the equation 32, we'll obtain researched in [5] oscillations, as a solution:

$$\psi = \sqrt{\eta} \times$$

$$\left[C_+ \cos \left(\frac{3\sqrt{3}}{2} \ln n\eta \right) + C_- \sin \left(\frac{3\sqrt{3}}{2} \ln n\eta \right) \right]. \quad (39)$$

Such behavior, however, is forming only in the case that collisionless situation occurred from the very beginning at $\eta = 0$. And if strong-collision phase preceded the collisionless phase (down to $\eta = \eta_0$), then instead of (38) we have now:

$$16\pi a^{-2} \overline{\tau}_f \simeq \frac{8}{5} \left[\frac{\psi(\eta)}{\eta^2} - \frac{\psi(\eta_0)}{\eta_0 \eta} \right], \quad (40)$$

and equation (32) becomes inhomogeneous

$$\psi'' + \frac{8}{5} \frac{\psi}{\eta^2} = \frac{8}{5} \frac{\psi(\eta_0)}{\eta_0 \eta}. \quad (41)$$

It's solution, satisfying to the sewing condition at the moment $\eta = \eta_0$, has a form:

$$\psi(\eta) = \psi(\eta_0) \frac{\eta}{\eta_0} + A \sqrt{\frac{\eta}{\eta_0}} \sin \left(\frac{3\sqrt{3}}{2} \ln \frac{\eta}{\eta_0} \right). \quad (42)$$

Main part of GW amplitude h^α_β at that remains constant. Thus, a solution for long GW's, obtained in Ref. [5], is true only in the case that Universe started from the collisionless phase, and is false in the case of existing of initial hydrodynamic stage. At the last case GW stores it's final condition on hydrodynamic stage.

4 The evolution of short waves

Let's consider the evolution of short ($n\eta \gg 1$) GW's at arbitrary $\gamma(\eta, \mathbb{P})$. In this case it is convenient to lay:

$$\psi = \widetilde{\psi}(\eta)^0 \exp\left(-i\int \Omega d\eta\right), \tag{43}$$

where $\Omega(\eta)$ - is a large value, $\widetilde{\psi}(\eta)$ - slowly changing function. At $\eta \to 0$ GW's with any η are long, therefore it is necessary to redefine the solution (25) at a point of time η_0, when GW's become short: $\eta_0 \gtrsim \eta^{-1}$. Carrying out necessary calculations, we'll find in the case of ultrarelativistic particles:

$$16\pi a^{-2}\overline{\tau}_f = -\frac{3}{4}\frac{\Omega}{n^4\eta^2}\sum\frac{2s+1}{2\pi^2}\int\limits_0^\infty dz z^4\frac{\partial f_0}{\partial z}\Omega'\left\{\frac{2}{3}n^2 - \right.$$

$$\left. -(\Omega'^2 - n^2) + \frac{\Omega'^2 - n^2}{2\Omega' n}\ln\left|\frac{\Omega' + n}{\Omega' - n}\right|\right\}, \tag{44}$$

where $\Omega' = \Omega + i\nu(\eta, p)a(\eta)$. For the weak-collision plasma $\Omega \approx n$, holding in Ref.(44) members $\Omega^2 - n^2$, we'll reduce an equation (32) to the form:

$$\widetilde{\psi}'' - 2i\Omega\widetilde{\psi}' - (\Omega^2 - n^2)\widetilde{\psi} - 4\frac{ia\nu_{eff}}{n\eta^2} = 0, \tag{45}$$

where ν_{eff} defines analogically by (35). In mentioned approach the solution of equation (36) is:

$$\begin{aligned} &\Omega^2 \simeq n^2 + \tfrac{2}{\eta^2}, \\ &\widetilde{\psi} \simeq k^{1/2}(\eta) = \exp\left(-\tfrac{1}{2}\gamma_G(\eta)\right), \end{aligned} \tag{46}$$

where

$$\gamma_G(\eta) = \frac{4}{\eta^2}\int\limits_{\eta_0}^{\eta}\frac{a\nu_{eff}}{\eta^2}d\eta = \int\limits_{t_0}^{t}\frac{\nu_{eff}dt}{k^2t^2}. \tag{47}$$

Let's point an inquisitive fact: neglecting damping of GW and introducing value $\Phi = \psi/a$, we'll obtain from (45) an equation for the mass scalar field

$$\Delta_2\Phi - \frac{R}{6}\Phi + m_G^2\Phi = 0,$$

with the effective mass of graviton

$$m_G = \frac{1}{\sqrt{2}t}.$$

Let's clarify the question about GW evolution, which make sense only in the approximation of geometrical optics. Averaged pseudotensor of momentum energy is equal to

$$< t^{ik} >= \frac{1}{32\pi} < h_q^{n,i}\overline{h}\,_n^{q,k} > . \tag{48}$$

Substituting here (6), we'll obtain MET of perfect ultrarelativistic fluid with energy density

$$\varepsilon_G = \frac{s^2 n^2}{a^4} \exp[-\gamma_G(\eta)], \tag{49}$$

i.e., the value γ_G (or $k(\eta)$) can be named as a decrement of GW energy damping.

5 An example of GW energy damping decrement calculation in the reactions of type $ee^+ \to$ hadrons

For illustration we'll carry out a calculation of GW damping according to stated above scheme for reactions of type $ee^+ \to$ hadrons, the cross-sections of which have a scaling behavior [17]

$$\sigma_{t_0 t}(s) = \frac{4\pi\alpha^2}{5s} \sum e_i^2 \equiv \frac{4\pi\alpha^2 Q^2(s)}{3s}, \tag{50}$$

where e_i- fundamental charges; their number, and, therefore the value Q, weakly depend from s. After calculations we'll find

$$\nu_{eff}(s) = \frac{45\zeta(3)N\alpha^2}{2\pi^3} T Q^2(T), \tag{51}$$

$$\gamma_G(\infty) = \frac{4(a\nu_{eff})\eta_0}{n^2\eta_0} \sim \frac{1}{\eta_0}. \tag{52}$$

As it was mentioned above, the GW damping is possible during synchronous fulfillment of conditions $n\eta > 1$ (short GW's) and $e_{eff}a < n$ (weak-collision gas). In the opposite case we have either long GW's, or GW's in the perfect liquid; in the both cases damping is absent. Therefore, regarding the fulfillment of these conditions in the initial point of time η_0, we arrive to the conclusion, that $\gamma_G(\infty) \simeq 4\beta^2$, where $\beta < 1$ - unknown factor, guaranteing the severity of made approximations, moreover all GW's, having the wavelength

$$\lambda \gtrsim \frac{\lambda_\gamma \beta 2\pi^3}{45\zeta(3)Q^2(t_0)N\alpha^2} \sim 10^4 \lambda_\gamma, \tag{53}$$

on the actual point of time, damp with the same decrement (where λ_γ - wavelength of relict photons).

6 The results of GW damping decrement calculation in the different reactions

Let's diagram the evolution of GW in hot Universe as a dependence from the behavior of total cross-section of scattering. Let's isolate following typical situations:

A: $n\eta \ll 1, \quad a\nu\eta \gg 1$ - long waves in the liquid,

B: $n\eta \gg 1, \quad a\nu\eta \gg 1$ - short waves in the liquid,

C: $n\eta \ll 1, \quad a\nu\eta < 1$ - long waves in the weak-collision gas,

D: $n\eta \gg 1, \quad a\nu\eta < 1$ - short waves in the weak-collision gas.

$\sigma_{t_0t}(s) \sim \frac{\alpha^2}{s}$, $n_0 \sim \alpha^2$:
- $n > n_0 \quad C \to$ (D)
- $n < n_0 \quad C \to A \quad \to B$

$\sigma_{t_0t} \sim \dfrac{\alpha^2}{m\sqrt{s}}$:
- $\alpha^2 < m \quad C \to$ (D)
- $\alpha^2 > m \quad A \longrightarrow \quad B \quad \longrightarrow$ (D)

$\sigma_{t_0t} \sim \frac{\alpha^2}{m^2}\left(\frac{s}{m^2}\right)^\gamma$, $n_0 \sim \frac{m^2}{\alpha^2}\left(\frac{m}{\alpha^2}\right)^{\frac{2}{2\gamma+1}}$:
- $n > n_0 \quad A \to B \quad \to$ (D)
- $n < n_0 \quad A \to$ [C] $\to$ (D)

Situations, where collisional GW damping occur are circled; the domain of GW amplitude fixation are squared. The problem of damping effectiveness requires the realization of calculations for concrete reactions.

The results of such calculations, fulfilled in papers [9]-[11], are represented in the Table 1. In two last columns of the Table represented the results, summarized by all reactions of given type. G - Fermi constant,

$$\sigma = \frac{0,1\alpha_X N^2 m_{p1}}{m_X},$$

m_{p1} - Planck mass, m_X - mass of X - bosons, α_X - interaction constant, N_X - the number of particles participating in reactions.

Summing the results, we'll note, that collision damping of cosmological GW's in the all conceivable reactions is not great. However, the reason of weakness of GW damping isn't the weakness of interparticle interaction, but the circumstance, that exactly in the medium with intense interparticle interactions, i.e. in the perfect fluid, GW's don't damp. In order to provide an appreciable GW damping, collisions don't need to be very frequent, that, in turn, leads to the weakness of damping.

Table 1. The damping decrement of cosmological gravitational waves in the early hot universe for different processes in cosmological plasma

Process	Total cross-section of scattering $\sigma_{t_0t}(s)$	The region of modern wavelengths, where damping is maximal (in cm)	Maximal value γ_G
$e\gamma^+ \leftrightarrow e\gamma$ 1) $ee^+ \leftrightarrow \gamma\gamma$	$\frac{2\pi\alpha^2}{s}\left(\ln\frac{s}{m^2}+\frac{1}{2}\right)$	~ 100	$0,01$
$ee^+ \leftrightarrow \mu\mu^+$ 2) $ee^+ \leftrightarrow$	$\frac{4\pi\alpha^2}{3s}Q^2(s)$	$\lambda > \frac{10^4\beta^2\lambda_\gamma}{Q^2N'} \sim 10^3$	$4\beta^2$
$e\nu \leftrightarrow e\nu$ 3) $ee^+ \leftrightarrow \nu\overline{\nu}$ $e\overline{\nu} \leftrightarrow e\overline{\nu}$ $\nu_\mu e \leftrightarrow \nu_e\mu$ $\nu_\mu\overline{\nu}_e \leftrightarrow e_+\mu^-$ $\nu_\mu\mu_+ \leftrightarrow e_+\nu_e$	$G_F^2 s(g_L^2+\frac{1}{3}g_R^2)/\pi$ $G_F^2 s(\frac{1}{3}g_L^2+\frac{1}{3}g_R^2)/\pi$ $G_F^2 s(\frac{1}{3}g_L^2+g_R^2)/\pi$ $G_F^2 s/\pi$ $G_F^2 s/3\pi$ $G_F^2 s/3\pi$	$\sim 2\cdot 10^{20}$	$0,74$
$X \leftrightarrow \overline{q}\,\overline{q}$ 4) $X \leftrightarrow ql$	$\lvert M\rvert^2 = 8\pi\alpha_x m_x^2 N$ $\sigma > 1$	$\frac{\sigma^{1/3}}{\alpha_X N}$	$0,29\frac{N_X}{N} \sim 3\cdot 10^{-2}$

1) By data of Ref.[12]; 2) by data of Ref.[9],[10]; 3) by data of Ref. [12]; 4) by data of Ref. [12],[18].

The unique sufficiently effective (in order of GW damping) interactions are the interactions with scaling behavior of cross-section. Exactly scale-invariant interactions in consequence of identical time law of collision frequency with GW frequency can infinitely long influence on GW and thereby apply to the sufficient damping of GW. However at attempt to calculate a GW damping decrement in the most effective region $\nu \sim k$ we'll fall into long-length collisionless phase, in which the GW energy determination is difficult to define unambiguously. The calculations of damping decrement during electro-weak interactions generally confirmed the Hawking's estimation [7] and rather improve it. Realized calculations point that the height of energetic spectrum of cosmological GW's can be decreased approximately in 1,5 - 2 times at lengths $\lambda \gtrsim 10^4\beta^2/N'Q^2$cm and additional to this the same effect at $\lambda \sim 2 \times 10^{20}$cm.

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