Layered Sources Transmission With Multiple Antennas

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***Abstract*—This paper studies the optimal transmission of layered multimedia sources, which require unequal target error rates in their bitstream, over open-loop multiple antenna systems. First, we analyze the behavior of the crossover point of the outage probability curves for vertical Bell Labs space-time architecture with a minimum mean-square error receiver and orthogonal space-time block codes. It is proven that, as we increase the number of transmit antennas, which is set to be equal to that of receive antennas, the crossover point for the outage probability monotonically decreases. Next, we show that those results can be exploited to simplify the computational complexity involved with the optimal space-time coding of a layered bitstream.**

I. Introduction

The demand for multimedia services has invoked much research on cross-layer design. In this paper, we study the optimal design of a low-complex open-loop multiple-input multiple-output (MIMO) system for the transmission of layered (or progressive) multimedia sources. First, we compare the outage probabilities of vertical Bell Labs space-time architecture (V-BLAST) with a minimum mean-square error (MMSE) receiver and orthogonal space-time block codes (OSTBC). In [1], the authors analyzed how the crossover point of the outage probabilities of the two space-time codes behaves as spectral efficiency increases. On the other hand, in this paper, we analyze the behavior from the viewpoint of the number of antennas. More specifically, it is shown that as we increase the number of transmit antennas which is set to be equal to that of receive antennas, the crossover point of the outage probability curves monotonically decreases. The results hold for an arbitrarily given spectral efficiency, and propagation channels such as spatially correlated Rayleigh or Rician fading channels in addition to i.i.d. Rayleigh fading channels. By exploiting the results, we derive the optimization method for the space-time coding of progressive packets in a system with a large number of antennas.

II. Preliminaries

Consider a MIMO system with N_t transmit and N_r receive antennas. A space-time codeword, $\mathbf{S} = [\mathbf{s}_1 \cdots \mathbf{s}_T]$ of size $N_t \times T$ is transmitted over T symbol durations. The baseband equivalent model, at the kth time symbol duration ($k = 1, \cdots, T$), is given by

$$\mathbf{y}_k = \mathbf{H}\mathbf{s}_k + \mathbf{n}_k \tag{1}$$

where $\mathbf{s}_k$ is an $N_t \times 1$ transmitted signal vector, $\mathbf{y}_k$ is an $N_r \times 1$ received signal vector, and $\mathbf{n}_k$ is an $N_r \times 1$ zero-mean complex AWGN vector with $\mathcal{E}\left[\mathbf{n}_k\mathbf{n}_l^H\right]=\sigma_n^2\mathbf{I}_{N_r}\delta(k-l)$, where $(\cdot)^H$ denotes Hermitian operation. In (1), $\mathbf{H}$ denotes the $N_r \times N_t$ channel matrix, whose entries are i.i.d. $\sim \mathcal{CN}(0,1)$. It is assumed that $\mathbf{H}$ is random, but constant over T symbol durations. Let γ_s denote SNR per symbol, which is defined as $\gamma_s := \mathcal{E}\left[|(\mathbf{s}_k)_i|^2\right]/\sigma_n^2$ where $(\mathbf{s}_k)_i$ is the ith component of $\mathbf{s}_k$. Let N_s denote the number of symbols packed within a space-time codeword $\mathbf{S}$. The spatial multiplexing rate is defined as N_s/T. We assume that $\mathbf{H}$ is known at the receiver, but not known at the transmitter.

Next, we briefly present the outage probability expression of the space-time code, which is derived in [2], for any given piecewise-linear diversity-multiplexing tradeoff (DMT) function [3]. Consider a space-time code whose DMT characteristic function is given by

$$d(r) = v - ur, \quad \text{for} \;\; \alpha \le r \le \beta \;\; (\alpha > 0) \tag{2}$$

where $d(r) \ge 0$, and $u \ge 0$ and $v \ge 0$ are real constants. Let $P_{out}(\gamma_s)$ denote the outage probability for the space-time code whose DMT is given by (2). In [2], it is shown that, as $\gamma_s \to \infty$, $P_{out}(\gamma_s)$ can be expressed as

$$P_{out}(\gamma_s) = k_d \left(\frac{2^R}{k_r}\right)^u \frac{1}{\gamma_s^v} \;\; \text{for} \left(\frac{2^R}{k_r}\right)^{\frac{1}{\beta}} \le \gamma_s \le \left(\frac{2^R}{k_r}\right)^{\frac{1}{\alpha}} \tag{3}$$

where R is the spectral efficiency (bits/s/Hz), and k_r and k_d are arbitrary positive constants.

Consider two space-time codes which have linear DMT characteristics as follows:

$$d_1(r) = v_1 - u_1 r \quad \text{and} \quad d_2(r) = v_2 - u_2 r, \\ \text{for} \;\; \alpha \le r \le \beta \;\; (\alpha > 0) \tag{4}$$

where

$$u_i > 0 \;\; \text{and} \;\; v_i > 0 \quad (i = 1, 2)$$

$$v_1 - u_1\alpha < v_2 - u_2\alpha \;\; \text{and} \;\; v_1 - u_1\beta > v_2 - u_2\beta.$$

That is, there exists a crossover in the range $\alpha < r < \beta$ for the two DMT functions. Let $P_{out,1}(\gamma_s)$ and $P_{out,2}(\gamma_s)$ denote the outage probabilities of the space-time codes whose DMT functions are given by $d_1(r)$ and $d_2(r)$, respectively. Then, from (3), as $\gamma_s \to \infty$, we have

$$P_{out,i}(\gamma_s) = k_d \left(\frac{2^R}{k_r}\right)^{u_i} \frac{1}{\gamma_s^{v_i}} \quad (i = 1, 2) \tag{5}$$

for $(2^R/k_r)^{1/\beta} \le \gamma_s \le (2^R/k_r)^{1/\alpha}$. From (5), for a given spectral efficiency R, we find the outage probability, P_{out}^*, for which $P_{out,1}(\gamma_s)$ and $P_{out,2}(\gamma_s)$ are identical. In [2], it is shown that P_{out}^* is given by

$$P_{out}^* = k_d \left(\frac{2^R}{k_r}\right)^{\frac{u_1 v_2 - u_2 v_1}{v_2 - v_1}} \quad (6)$$

The DMT characteristics of V-BLAST with an MMSE receiver and OSTBC, denoted by $d_V(r)$ and $d_O(r)$, respectively, are given by [4]

$$d_V(r) = \begin{cases} N_r - N_t + 1 - \frac{1}{N_t}(N_r - N_t + 1)r, \\ \quad \text{for } 0 \le r \le N_t \\ 0, \quad \text{for } N_t \le r < \infty \end{cases} \quad (7)$$

$$d_O(r) = \begin{cases} N_r N_t - N_r N_t\, r/r_s, & \text{for } 0 \le r \le r_s \\ 0, \quad \text{for } 1 \le r < \infty \end{cases} \quad (8)$$

where r_s denotes the spatial multiplexing rate of the OSTBC. Note that for $N_t = 2$, the Alamouti scheme achieves $r_s = 1$, whereas $r_s = 3/4$ is the maximum achievable rate for $N_t = 3$ or 4 in the complex OSTBC. For $N_t > 4$, $r_s = 1/2$ is the maximum rate. To compare the above codes, we assume that $N_r \ge N_t \ge 2$.

III. The Behavior of the Crossover Point

From (6), (7), and (8), it can be shown that the crossover point of the outage probabilities for V-BLAST and OSTBC, P_{out}^*, is given by

$$P_{out}^* = k_d \left(\frac{2^R}{k_r}\right)^{\frac{(N_r - N_t + 1)(1 - N_t/r_s)N_r}{(N_t - 1)(N_r + 1)}} \quad (9)$$

If we let $N_t = N_r = n$ in (9), we have

$$P_{out}^* = k_d \left(\frac{2^R}{k_r}\right)^{\frac{n - n^2/r_s}{n^2 - 1}} \quad (10)$$

We will prove that P_{out}^*, given by (10), is a strictly decreasing function in $n \ge 2$. We define function $f(n)$ as

$$f(n) = \frac{n - n^2/r_s}{n^2 - 1} \quad (11)$$

Assuming that $n \ge 2$ is a real number, $df(n)/dn$ can be expressed as

$$\frac{df(n)}{dn} = \frac{-r_s n^2 + 2n - r_s}{r_s(n^2 - 1)^2} \quad (12)$$

Let $g(n)$ be the numerator of $df(n)/dn$. Then, it can be shown that $g(n)$ is a monotonically decreasing function in $n \ge 1/r_s$.

To begin, suppose that $n \ge 4$. Then, from $1/2 \le r_s \le 3/4$ (i.e., $4/3 \le 1/r_s \le 2$), we have

$$g(n) \le g(4) = -17r_s + 8 < 0. \quad (13)$$

From (12) and (13), it follows that

$$\frac{df(n)}{dn} < 0 \quad \text{for } n \ge 4. \quad (14)$$

If we substitute N_t = 2, 3 and 4 along with the corresponding multiplexing rates (i.e., r_s = 1, 3/4 and 1/2, respectively) into (11), we have

$$f(2) = -2/3, \;\; f(3) = -15/8, \;\; \text{and } f(4) = -28/15. \quad (15)$$

From (14) and the inequality of $f(2) > f(3) > f(4)$, we have $df(n)/dn < 0$ for $n \ge 2$. From this, (10) and (11), it is seen that as the number of antennas, $n(\ge 2)$, increases, P_{out}^* strictly decreases, regardless of a spectral efficiency R.

Let $P_{out,1}^*$ denote the crossover point when the number of antennas is n_1 (i.e., $n_1 \times n_1$ MIMO systems), and $P_{out,2}^*$ denote the crossover point when the number of antennas is n_2 is used (i.e., $n_2 \times n_2$ systems). Suppose that $n_1 < n_2$. Then, from the given results, we have

$$P_{out,1}^* > P_{out,2}^*. \quad (16)$$

Based on (16), the outage probabilities of V-BLAST and OSTBC are qualitatively depicted in Fig. 1. Suppose that a target outage probability, $P_{out,T}$, is smaller than $P_{out,1}^*$ but greater than $P_{out,2}^*$. Then, from Fig. 1, it is seen that OSTBC is preferable to V-BLAST for an $n_1 \times n_1$ MIMO system, whereas V-BLAST is preferable for an $n_2 \times n_2$ system ($n_1 < n_2$).

Note that DMT characteristics are not influenced by spatial correlation or line-of-sight (LOS) signal components [5], [6]. In other words, the DMT function for spatially correlated Rayleigh fading or Rician fading is identical to that for i.i.d. Rayleigh fading. From this, it follows that the analysis of the crossover points is also valid over those channels at high SNR.

IV. Space-Time Coding of a Layered Bitstream

We exploit the analysis in the previous section to design a MIMO system for the transmission of the applications which need unequal target error rates in their bitstream. To begin, we briefly present the transmission of layered multimedia sources. Progressive encoders employ a mode of transmission so that encoded data have gradual differences of importance in their bitstreams. Suppose that the system takes the bitstream from the progressive source encoder, and transforms it into a sequence of N_P packets. Each of these N_P progressive packets can be encoded with different transmission data rates, as well as different MIMO techniques, so as to yield the best end-to-end performance. The error probability of an earlier packet

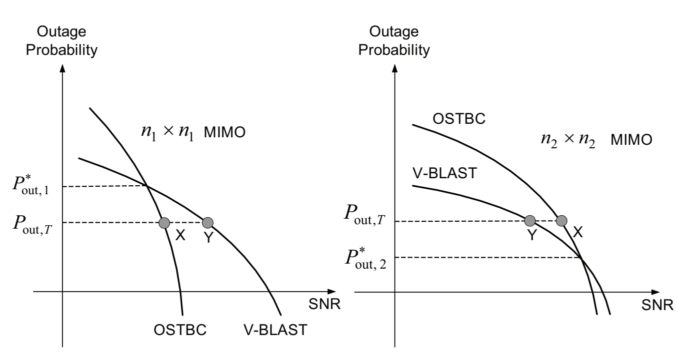

Fig. 1. Outage probabilities of V-BLAST and OSTBC for $n_1 < n_2$.

needs to be lower than or equal to that of a later packet, due to the gradually decreasing importance in the progressive bitstream.

Let N_R denote the number of candidate transmission data rates employed by a system. The number of possible assignments of N_R data rates to N_P packets would exponentially grow as N_P increases. Further, in a MIMO system, if each packet can be encoded with different space-time codes (e.g., V-BLAST or OSTBC), the assignment of space-time codes as well as data rates to N_P packets yields a more complicated optimization problem. Note that each source, e.g., an image, has its inherent rate-distortion characteristic, from which the performance of the expected distortion is computed. Hence, for example, when a series of images is transmitted, the above optimization should be addressed in a real-time manner, considering which specific image (i.e., rate-distortion characteristic) is transmitted in the current time slot.

We use the analytical results in the previous section to optimize the assignment of space-time codes to progressive packets for a MIMO system with a large number of antennas. Suppose that the progressive bitstream has been optimized for an $n_1 \times n_1$ MIMO system, and the result is such that the k_1th, k_2th, ..., k_jth packets in a sequence of N_P packets are encoded with V-BLAST. Then, our analysis indicates that the above packets also should be encoded with V-BLAST rather than with OSTBC for an $n_2 \times n_2$ system $(n_1 < n_2)$. This is because we have proven that, when V-BLAST is preferable for a packet in an $n_1 \times n_1$ MIMO system, the same packet in an $n_2 \times n_2$ system $(n_1 < n_2)$ also should be encoded with V-BLAST, as long as the target error rate of the latter is the same as or higher than that of the former (see Fig. 1). As a result, it can be shown that the number of possible assignments of space-time codes to N_P packets in an $n_2 \times n_2$ system can be reduced by $2^{N_P}/2^{(N_P-j)}$ times, by using the optimization results for an $n_1 \times n_1$ system. Note that the inversion of a channel matrix $\mathbf{H}$, which is needed for the V-BLAST linear receiver, requires high computational complexity, especially when a matrix is large. From the statements above, the optimization strategy for a system with a large number of antennas can be derived as follows: Before optimizing the space-time coding of a specific layered source for a system with a large number of antennas, we perform the optimization for the same source in a test system with a small number of antennas, and exploit the results as described above.

V. Numerical Evaluation

The outage probabilities of V-BLAST and OSTBC are numerically evaluated for various numbers of antennas with R = 4 (bits/s/Hz) in i.i.d. Rayleigh fading channels. The results are shown in Fig. 2. It is seen that as the number of antennas increases, the crossover point of the outage probabilities behave in a manner as predicted by the analysis in Sec. III (see Fig. 1).

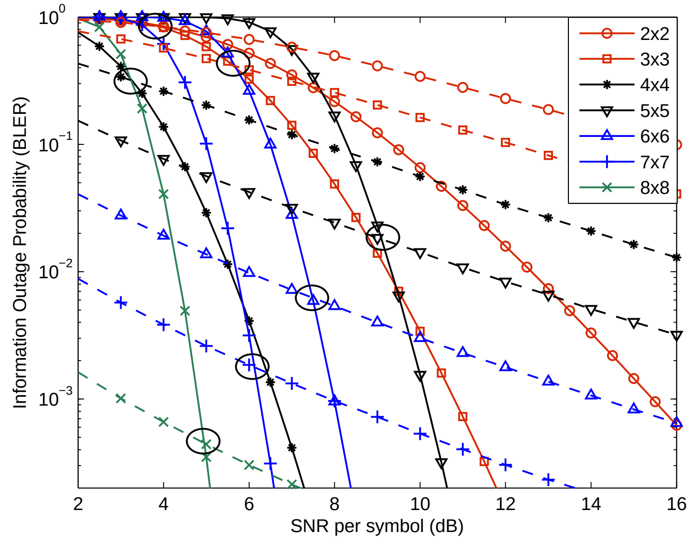

Fig. 2. The outage probabilities in i.i.d. Rayleigh fading channels. Solid curves denote the outage probabilities of OSTBC, and dashed curves denote those of V-BLAST. The crossover points are marked with circles.

VI. Conclusions

The behavior of the crossover point of the outage probability curves for V-BLAST with an MMSE receiver and OSTBC was analyzed in terms of the number of antennas. We showed that the results can be used to simplify computations involved with the optimal space-time coding of a sequence of progressive packets in a system with a large number of antennas.

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