

# More on Unemployment and Vacancy Fluctuations\*

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December 18, 2006

## Abstract

Shimer (2005a) argues that the textbook equilibrium search model of unemployment explains less than 10% of the volatility in U.S. vacancies and unemployment when fluctuations are driven by productivity shocks. His paper as well as other recent work inspired by it are reviewed and extended here. Although there seems to be excessive feedback from the job-finding rate to the wage built into the Nash bargaining mechanism assumed to determine wages in the model, we argue that he and others overemphasize the need for wage rigidity to explain the data on labor-market fluctuations. Indeed, a modified version of the model can explain the magnitude of the empirical relationship between the vacancy-unemployment ratio and labor productivity when wages are the outcome of a strategic bargaining game and when the elasticity of the matching function and the opportunity cost of a match are set at reasonable values. The modified model also explains almost two thirds of the volatility in the ratio relative to that of productivity when separation shocks are taken into account, as well as the strong negative correlation between vacancies and unemployment found in Shimer's data.

**Keywords:** Labor-market search, unemployment and vacancies volatility, job-finding rate, productivity shocks, wage rigidity.

**JEL Classifications:** E24, E32, J41, J63, J64

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\*Marcus Hagedorn, Robert Hall, John Kennan, Rasmus Lentz, Iourii Manovskii, Guido Menzio, Monika Merz, and Robert Shimer have all provided valuable comments and suggestions.

<sup>†</sup>Financial support from the National Science Foundation is acknowledged.

# 1 Introduction

In this paper, we review Shimer's (2005a) critique of the equilibrium search model of unemployment and the considerable subsequent debate that it has generated in the literature. (See Mortensen and Pissarides (1994,1999a,1999b), Pissarides (2000), and Rogerson, Shimer, and Wright (2005) for an extended development of the model and its implications.) Our purpose is to clarify and further the debate generated by Shimer's paper. We do so by incorporating the points made in the literature into a common framework.

Shimer argues that the volatility in unemployment is induced primarily by movements in the job-finding rate, the rate of transition from unemployment to employment, rather than the separation rate. He then demonstrates that the magnitude of the response of unemployment, vacancies, and the job-finding rate to labor-productivity shocks predicted by the model explains less than 10% of the observed volatility in U.S. data when productivity shocks are assumed to be the sole driving force, given reasonable specification assumptions and parameter values.<sup>1</sup> A principal reason for this lack of explanatory power, he argues, is that the wage, set as the outcome of a bilateral wage bargain, responds procyclically to offset almost all the effects of productivity shocks on job creation.

We argue that a flexible wage *per se* is not the principal problem with the model. Rather, Shimer's results are due to 1) a relatively low estimate of the elasticity of the matching function with respect to vacancies, 2) a large difference between labor productivity and the opportunity cost of a match implied by the assigned magnitudes of parameters, and 3) the excessively strong feedback from the job-finding rate to the wage. Even if the wage were rigid, its level must be such that the future flow of quasi-rent attributable to the creation of a new job is very small, if the model is to account for the volatility of the vacancy-unemployment ratio observed in the data.

As Hagedorn and Manovskii (2005) demonstrate, the model has no problem explaining vacancy and unemployment fluctuations if the parameters of the wage outcome function are set to match observed average profit rates and wage volatility. However, the opportunity cost of employment required to explain the observed volatility of the vacancy-unemployment ratio is unrealistically high. Nonetheless, Hall's (2006) calculations based on recent micro

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<sup>1</sup>Costain and Reiter (2005) make a similar point in a less well-known, but independently developed paper.

evidence on labor supply and employment-contingent consumption support a substantially larger value than that assumed by Shimer. We also argue that the opportunity cost of a match is not only influenced by the unemployment benefit paid and the value of forgone leisure, but also depends on the cost of hiring and training workers. If these costs are significant, then the worker's opportunity cost of employment required to match observed volatility is much lower than that derived by Hagedorn and Manovskii.

The standard search model of unemployment is designed to account for the fact that it takes time to match jobs and workers. As a consequence of this friction, match-specific rents exist when a worker meets a prospective employer. The designers of the original model assume that these rents are shared according to Nash's (1950) axioms with the value of searching for an alternative job serving as the threat point.

Hall (2005a) argues that any wage in the bargaining set, that is consistent with individual rationality for the employer-worker pair, should be regarded as a legitimate equilibrium candidate. He then proceeds to demonstrate by simulation that a rigid wage, one not conditioned on aggregate productivity, generally exists with the property that it is in the bargaining set. His argument, however, relies critically on the assumption that the only source of variability is a small aggregate shock.

Gertler and Trigari (2005) show that a version of Calvo's (1983) staggered price setting model provides an adequate explanation for observed volatility in labor-market aggregates when wages are revised only every three quarters on average. Their results demonstrate that the quantitative implications of the general framework for fluctuations in labor-market variables are quite sensitive to even small amounts of wage rigidity.

Hall and Milgrom (2005) argue that the outcome of a strategic bargaining game in which the disagreement payoff is delay rather than unemployed search, along the lines suggested by Binmore, Rubinstein and Wolinsky (1986), is a more realistic specification of a bargaining model. They also claim that the amended model substantially raises the implied degree of amplification because the alternative wage rule is less sensitive to productivity shocks. Although indeed the solution to this wage-bargaining game is less volatile, the job-creation response to productivity shocks is not that much larger if the value of delay is roughly equal to Shimer's value of the unemployment benefit. Alternatively, the Hall-Milgrom model explains the observed volatility of the job-finding rate only if the worker's benefit from delay plus the employer's cost, which is the opportunity cost of a match in their model,

is equal to the required value derived by Hagedorn and Manovskii (2005) for the standard model.

The fact that a third of the variation in the unemployment rate is explained by variation in the separation rate is potentially an important factor that is ignored in the simplest equilibrium unemployment model and in Shimer's analysis. We find that incorporating separation rate shocks improves the performance of the model in part because they are negatively correlated with productivity shocks. Indeed, given a value of the elasticity of the matching function with respect to vacancies at the midpoint of the range of estimates reported by Petrongolo and Pissarides (2001), Hall's (2006) estimate of the value of leisure, and the assumption that wages are determined as the outcome of a strategic bargaining game as Hall and Milgrom (2005) suggest, we find that the model can explain the response of the vacancy-unemployment ratio to variation in labor productivity as reflected in the OLS regression coefficient. Furthermore, the model also explains 66% of the observed volatility in the vacancy-unemployment ratio relative to that of labor productivity when separation shocks as well as productivity shocks are viewed as a driving force and matches the fact that the correlation coefficient between vacancies and unemployment is close to minus one.

## 2 The Textbook Model

For the purpose at hand, we restate Shimer's (2005) model and review his analysis in this section and the next. All workers and jobs are respectively identical and every agent has a life of indefinite length and is risk-neutral. For the sake of comparability, we use Shimer's notation when possible. Specifically, every job-worker match produces market output at flow rate  $p$ . Auto-correlated shocks to  $p$  occur from time to time. Hence, the current value of match productivity is an aggregate state variable. The possible dependency of any endogenous variable on the current value of productivity is represented by using  $p$  as a subscript. Following Shimer, we assume that the time sequence  $\{p_t\}$  is a Markov jump process characterized by arrival rate  $\lambda$  and a distribution of new values  $F$  that is generally dependent on the previous realization.

The opportunity cost of employment to the worker and the cost of posting a vacancy to the firm, measured in units of output, are non-state-contingent parameters, denoted by  $z$  and  $c$ , respectively. Since all matches are identical,

the flow of new matches is determined by a matching function, denoted as  $m(u, v)$ , where  $u$  and  $v$  represent the number of unemployed workers currently looking for a job and the number of currently open job vacancies, respectively. By assumption, the matching function is non-negative, increasing, concave, and homogeneous of degree one. As a consequence, the job-finding rate,  $f(\theta) \equiv m(u, v)/u = m(1, \theta)$ , is positive, increasing, and concave in “market tightness,” defined as the ratio of vacancies to unemployment,  $\theta \equiv v/u$ . Moreover, the rate at which vacancies are filled,  $m(u, v)/v = f(\theta)/\theta$ , is a positive, decreasing, and convex function of market tightness. Finally, matches are destroyed at the exogenous separation rate  $s$  and all agents discount future income flows at the common rate  $r$ . The matching function  $m(\cdot)$ , the productivity process  $(\lambda, F)$ , and the set of parameters  $\{z, c, s, r\}$  fully characterize the environment of interest.

We restrict our attention to the class of equilibria that depend only on the current value of  $p$ .<sup>2</sup> The levels of unemployment and vacancies are endogenous to the model. They are determined by the match surplus-sharing rule, free entry, and the law of motion for unemployment. To characterize these conditions, one needs to define the concept of match surplus.

Match surplus is the difference between the expected present value of the future incomes that the two parties to a match earn and the expected present value of income that they forgo by participating in the employment relationship. Because the state-contingent value of a vacancy is driven to zero by entry, match surplus in aggregate state  $p$  is  $V_p \equiv J_p + W_p - U_p$  where the value of a match to the employer,  $J_p$ , the value of a match to the worker,  $W_p$ , and the value of unemployment,  $U_p$ , are recursively defined by the continuous-time Bellman equations

$$rU_p = z + f(\theta_p)(W_p - U_p) + \lambda(E_p U_{p'} - U_p) \quad (1)$$

$$rW_p = w_p - s(W_p - U_p) + \lambda(E_p W_{p'} - W_p) \quad (2)$$

$$rJ_p = p - w_p - sJ_p + \lambda(E_p J_{p'} - J_p), \quad (3)$$

where  $E_p$  represents the expectation operator conditional on the current state  $p$ . In all cases, these equations imply that the return on the value of an agent’s state is equal to the income flow obtained plus the product of the change in value attributable to a state transition and the relevant transition rate

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<sup>2</sup>Because the number of unemployed workers is not information relevant when the matching function is homogenous of degree one and output is linear in employment (see Pissarides (2000)), dependence on this aggregate variable can be ignored.

summed over all possible transitions. In the case of an unemployed worker, the possible changes in state include a transition to employment as well as a transition to another aggregate productivity state. Similarly, changes in the value of employment and of a filled job occur when the match is destroyed and when the aggregate state changes. Notice that these equations are consistent with individual rationality only if  $W_p - U_p \geq 0$  and  $J_p \geq 0$  for all  $p$ . As Hall (2005a) emphasizes, any reasonable wage rule agreed to by an employer and a worker engaged in a match must satisfy these inequalities.

By summing Equations (2) and (3) and then subtracting the corresponding sides of (1), one obtains the following functional equation that the surplus value of a match must satisfy:

$$rV_p = p - z - f(\theta_p)(W_p - U_p) - sV_p + \lambda(E_p V_{p'} - V_p). \quad (4)$$

Given that each agent's threat point is assumed to be the value of not being matched, the generalized Nash solution to the bargaining problem that the worker and the employer face upon meeting maximizes the so-called Nash product, the geometric average of their respective shares of the match surplus,  $(W_p - U_p)^\beta J_p^{1-\beta}$ , where the parameter  $\beta$  reflects the worker's "bargaining power". By assumption, appropriate side payments are made to support the following solution to the problem in every state:

$$\frac{W_p - U_p}{\beta} = V_p = \frac{J_p}{1 - \beta}. \quad (5)$$

Finally, the free-entry condition requires that the expected cost of posting a vacancy is equal to the expected return. That is, given that the average time to fill a vacancy is  $\frac{\theta}{f(\theta)}$ ,

$$\frac{c\theta_p}{f(\theta_p)} = J_p. \quad (6)$$

An equilibrium solution to the model is a vector of functions  $(\theta_p, w_p, U_p, W_p, J_p, V_p)$ , all defined on the set of possible values of productivity, that satisfy Equations (1)-(6). To complete Shimer's analysis, we prove that a unique equilibrium exists and that all the functions increase with productivity given reasonable technical restrictions on the matching function.

**Proposition 1** *If  $\theta/f(\theta)$  is a strictly increasing and concave function of  $\theta$  such that  $\lim_{\theta \rightarrow 0} \{\theta/f(\theta)\} = 0$ , then a unique equilibrium exists within the class studied. Furthermore, the equilibrium functions  $(\theta_p, w_p, U_p, W_p, J_p, V_p)$  are all strictly increasing in  $p$  if  $p'$  is stochastically increasing in  $p$ .*

**Proof.** See the Appendix. ■

An explicit equilibrium wage rule that supports the Nash division of surplus can easily be derived by noting that Equations (1), (2), (3), and (5) imply

$$\begin{aligned} & (1 - \beta)(r + s + \lambda)(W_p - U_p) \\ = & (1 - \beta)(w_p - z - f(\theta_p)(W_p - U_p) + \lambda E_p(W_{p'} - U_{p'})) \\ = & \beta(r + s + \lambda)J_p = \beta(p - w_p + \lambda E_p J_{p'}). \end{aligned}$$

Since a new bargain is made after every aggregate shock, Equation (5) holds for all  $p'$ , so  $(1 - \beta)E_p(W_{p'} - U_{p'}) = \beta E_p J_{p'}$ . Together with the free-entry condition (6), this implies that the wage function takes the form

$$w_p = \beta p + (1 - \beta)(z + \beta f(\theta_p)V_p) = \beta(p + c\theta_p) + (1 - \beta)z. \quad (7)$$

The wage depends on the current value of aggregate productivity and increases with its realized value for two reasons – because current output is shared and because the value of search while unemployed is increasing in the job-finding rate, which in turn increases in market tightness.

Under the assumption that all workers desire employment and are either employed or unemployed, the unemployment rate adjusts according to the law of motion

$$\dot{u} = s(1 - u) - f(\theta_p)u,$$

where the size of the labor force is normalized at unity. Because productivity per worker is independent of employment and the matching function has constant return to scale, the unemployment rate is not an information-relevant state variable. Instead, unemployment simply converges toward the state-contingent target

$$u_p = \frac{s}{s + f(\theta_p)}. \quad (8)$$

Elsewhere, Shimer (2005b) demonstrates that the speed of adjustment, equal to the sum of the separation and job-finding rate, is large enough in practice that the negative relationship between vacancies,  $v_p = \theta_p u_p$ , and unemployment that Equation (8) implies can be interpreted as the empirical Beveridge curve, the downward-sloping relationship between vacancies and unemployment commonly observed.

### 3 Volatility Implied by the Textbook Model

Shimer's (2005a) principal claim is that the volatility of the job-finding rate and its determinant, the vacancy-unemployment ratio, is an order of magnitude larger in U.S. data than the value implied by the standard model for "reasonable" parameter values when fluctuations are induced by shocks to labor productivity. To demonstrate this point, we substitute appropriately from the free-entry condition in Equation (6) and use the Nash bargaining outcome in Equation (5), to conclude that the Bellman equation in Equation (4) implies

$$(r + s + \lambda) \frac{c\theta_p}{f(\theta_p)} + c\beta\theta_p = (1 - \beta)(p - z + \lambda E_p V_{p'}) . \quad (9)$$

The vacancy-unemployment ratio in state  $p$ , market tightness  $\theta_p$ , is determined by this equation. By taking logs and differentiating the result with respect to  $\ln p$ , one obtains the following expression for the elasticity of market tightness with respect to aggregate productivity:

$$\frac{\partial \ln \theta_p}{\partial \ln p} = \frac{r + s + \lambda + \beta f(\theta_p)}{(1 - \eta(\theta_p))(r + s + \lambda) + \beta f(\theta_p)} \times \frac{p \left(1 + \lambda \frac{\partial E_p V_{p'}}{\partial p}\right)}{p - z + \lambda E_p V_{p'}} ,$$

where  $\eta(\theta) \equiv \theta f'(\theta)/f(\theta) \equiv vm_v(u, v)/m(u, v)$  is the elasticity of the matching function with respect to vacancies.

At this point, Shimer documents through his subsequent simulations that the value of the elasticity when there are no aggregate shocks ( $\lambda = 0$ ) serves as an adequate approximation for computational purposes. That is, suppressing the dependence of the job-finding rate on  $\theta$ ,

$$\frac{\partial \ln \theta}{\partial \ln p} = \frac{r + s + \beta f}{(1 - \eta)(r + s) + \beta f} \times \frac{p}{p - z} \quad (10)$$

holds as an approximation. When evaluated at Shimer's choice of parameters, which are median labor productivity normalized to  $p = 1$ , quarterly rates  $r = 0.012$ ,  $s = 0.10$ , and  $f = 1.355$ , matching function elasticity  $\eta = 0.28$ , labor bargaining power  $\beta = 1 - \eta = 0.72$ , and opportunity cost of employment  $z = 0.4$ , the numerical value is

$$\frac{\partial \ln \theta}{\partial \ln p} = \frac{0.112 + 0.72 \times 1.355}{0.72 \times 0.112 + 0.72 \times 1.355} \times \frac{1}{1 - 0.4} = 1.72. \quad (11)$$

In contrast, Shimer finds that the volatility in the log of the vacancy-unemployment ratio relative to that of log productivity is over ten times as large in U.S. data. Namely,

$$\frac{\sigma_\theta}{\sigma_p} = \frac{0.382}{0.02} = 19.10 \quad (12)$$

given the data moments reported in Table 1 below (reproduced from Shimer (2005a)), where  $\sigma_x$  and  $\rho_{xy}$  represent the standard deviation of  $\ln x$  and the correlation between  $\ln x$  and  $\ln y$ , respectively, here and in the rest of the paper. In other words, shocks to labor productivity explain less than 10% of the variation in market tightness.

Table 1: Shimer’s Summary Statistics, Quarterly U.S. data, 1951-2003.

$x$		$u$	$v$	$v/u$	$f$	$s$	$p$
Standard Deviation		0.190	0.202	0.382	0.118	0.075	0.020
Autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
Correlation Matrix	$u$	1	-0.894	-0.971	-0.949	0.709	-0.408
	$v$	-	1	0.975	0.897	-0.684	0.364
	$v/u$	-	-	1	0.948	-0.715	0.396
	$f$	-	-	-	1	-0.574	0.396
	$s$	-	-	-	-	1	-0.524
	$p$	-	-	-	-	-	1

Source: Shimer (2005a), Table 1. All variables reported are log deviations from an HP trend with smoothing parameter  $10^5$ .

There are two points worth raising regarding the comparison made by Shimer. First, the empirical counterpart of the derivative computed in Equation (10) is the OLS regression coefficient

$$\rho_{\theta p} \sigma_\theta / \sigma_p = 0.396 \times 19.1 = 7.56. \quad (13)$$

When Shimer compares the elasticity of market tightness to the ratio of standard deviations,  $\sigma_\theta / \sigma_p$ , he is implicitly assuming that a productivity shock is the only cause of fluctuations in vacancies and unemployment. If true, the correlation between the two would be (approximately) unity and the two measures would be equivalent. The fact that the correlation is substantially below unity suggests the importance of other omitted driving forces such as the interest rate and the separation rate, a point we return to in Section 5. Given that there are other causes, one cannot expect a one factor model to

explain more than the observed relationship between it and the dependent variable as reflected in the regression coefficient. Even though the model with Shimer’s parameter values also fails this test, the goal post for modifications of the model is substantially lower when one allows for other sources of employment volatility.

Second, we know that the comparative static implications of the model for the effect of productivity on market tightness will approximate the magnitude of the dynamic stochastic model if the productivity process is sufficiently persistent. Since persistence in the process is inversely related to the arrival rate, the fact that Equation (10) serves as an adequate approximation in Shimer’s simulations would seem to be inconsistent with the rather large arrival rate of the  $p$  process actually estimated,  $\lambda = 4.0$  per quarter. However, the process will be also persistent if the changes that occur in  $p$  conditional on arrival are small enough even when  $\lambda$  is large. Indeed, Shimer’s estimate of the variance of the change in  $p$  is small which is why the simulation results reported by Shimer correspond well to the predictions of Equation (10) .

Formally, Shimer assumes that, given grid size  $n$ , the change in “net productivity,” defined as  $p - z$ , can be characterized as

$$\ln(p' - z) - \ln(p - z) = \pm\Delta \text{ w/ prob } \frac{1}{2} \left( 1 \mp \frac{\ln(p - z) - \ln(p^* - z)}{n\Delta} \right), \quad (14)$$

where  $p^*$  is the long-run mean of productivity. At the estimated standard deviation parameter,  $\sigma = \sqrt{\lambda}\Delta = 0.0165$ ,  $\Delta = 0.0165/2 = 0.0083$  is small. Hence, the following result justifies the use of the approximation.

**Proposition 2** *Equation (10) holds in the limit as either  $\lambda \rightarrow 0$  or  $\Delta \rightarrow 0$ .*

**Proof.** See the Appendix. ■

Beyond these methodological points, Shimer’s result is also naturally affected by the values of the parameters he uses. We turn to the discussion of these next.

### 3.1 The Elasticity of the Matching Function

The elasticity of the job-finding rate with respect to the vacancy-unemployment ratio is an important determinant of how strongly the model economy’s job-finding rate responds to changes in its driving forces. Shimer’s value of the elasticity,  $\eta = 0.28$ , is obtained by regressing the detrended log of his measure

of the job-finding rate, derived from CPS data, on the detrended log of the ratio of vacancies, as reflected in the Conference Board Help Wanted index, to detrended CPS unemployment. The resulting estimate is somewhat outside the “plausible range” of 0.3 to 0.5 reported by Petrongolo and Pissarides (2001) in their review of the literature on the matching function.

There are alternative ways to estimate the elasticity of the matching function using Shimer’s data. In particular, Shimer’s data on vacancies and unemployment clearly imply that  $\ln v + \ln u$  is almost constant, and hence the Beveridge curve is close to a rectangular hyperbola. Specifically, the data moments in Table 1 imply that the OLS regression of (log) vacancies on (log) unemployment yield the coefficient estimate  $\rho_{vu}\sigma_v/\sigma_u = -0.894 \times 0.202/0.190 = -0.950$ . Given the rapid adjustment of unemployment to the state-contingent target value at rate  $s + f = 0.485$  per month, Equation (8) accurately represents the Beveridge curve relating vacancies and unemployment. Since this equation and a Cobb-Douglas specification of the matching function,  $m(v, u) = \mu v^\eta u^{1-\eta}$ , can be written as

$$\ln \mu + \eta \ln v + (1 - \eta) \ln u = \ln s + \ln(1 - u),$$

the regression coefficient implied by Equation (8) is

$$\frac{\partial \ln v}{\partial \ln u} = -\frac{1}{\eta} \left( \frac{u}{1 - u} + 1 - \eta \right).$$

Using Shimer’s long-run average of the unemployment rate,  $u = 0.0567$ , and the estimated regression coefficient  $\frac{\partial \ln v}{\partial \ln u} = -0.950$  gives an estimate of the elasticity of the matching function of  $\eta = 0.544$ , somewhat above the upper bound on the “plausible range” of Petrongolo and Pissarides (2001). Since the approximate elasticity of market tightness in Equation (10) increases in  $\eta$ , this estimate suggests that Shimer’s measure of the volatility of the market tightness may be biased downward.

Notice, however, that even if one uses our alternative estimate of  $\eta$  and his parameter values, the elasticity of market tightness with respect to productivity implied by Equation (10) is

$$\begin{aligned} \frac{\partial \ln \theta}{\partial \ln p} &= \frac{r + s + \beta f}{(1 - \eta)(r + s) + \beta f} \times \frac{p}{p - z} \\ &= \frac{0.112 + 0.456 \times 1.355}{0.456 \times 0.112 + 0.456 \times 1.355} \times \frac{1}{1 - 0.4} = 1.82, \end{aligned}$$

which is still only a quarter of the value of the regression coefficient,  $\rho_{\theta p} \sigma_{\theta} / \sigma_p = 7.56$ , implied by Shimer’s data. In other words, uncertainty and disagreement regarding the magnitude of the elasticity of the matching function alone does not overturn Shimer’s conclusion given his choices of the other parameter values. Nevertheless, in our view, an elasticity value in between the two implied by Shimer’s data,  $\eta = 0.4$ , which also in the middle of the Petrongolo-Pissarides range, is a more appropriate value to use in the calculations that follow.

### 3.2 The Opportunity Cost of Employment

Shimer (2005a) sets  $z = 0.4$  as a “generous estimate” of the unemployment insurance replacement ratio. Hagedorn and Manovskii (2005) argue that Shimer’s choice of the opportunity cost of employment is too low because it does not allow for the “value of leisure” or “home production” forgone when employed above and beyond the unemployment insurance benefit. Moreover, they calibrate both the opportunity cost of employment and the bargaining share parameter to match the cyclical response of wages observed in post WWII BLS data and an inferred cost of vacancy posting as well as standard labor-force statistics. Their results imply  $z = 0.955$  and  $\beta = 0.052$ . Using these numbers and setting the other values equal to those of Shimer’s calibration, the elasticity of market tightness with respect to productivity is

$$\frac{\partial \ln \theta}{\partial \ln p} = \frac{0.112 + 0.052 \times 1.355}{0.72 \times 0.112 + 0.052 \times 1.355} \times \frac{1}{1 - 0.955} = 26.83.$$

In other words, with these parameter, the model over-predicts the relative magnitudes of the variability of the job-finding rate and labor productivity found in the data even when Shimer’s estimate of  $\eta$  is used in the calculation and shocks to productivity are the only source of labor-market fluctuations.

Frankly, the Hagedorn-Manovskii value for  $z$  seems implausibly large. For example, the flow surplus enjoyed by an employed worker in the model for these parameter values is miniscule. Indeed, the wage at the modal value of productivity,  $p = 1$ , is

$$w = \beta(p + c\theta) + (1 - \beta)z = 0.052 \times (1 + 0.382) + (1 - 0.052) \times 0.955 = 0.977,$$

given the parameters they calibrate ( $c = 0.212$  and  $\theta = 1.8$ ), and the surplus flow when employed is

$$\frac{w - z}{z} = \frac{0.977 - 0.955}{0.955} = 0.023.$$

But, do workers work for a 2.3% surplus?

Hagedorn and Manovskii do demonstrate that their implied value of  $z$  is much smaller once taxes are taken into account. Given a consumption tax  $\tilde{\tau}$  and a wage tax  $\tau_w$ , they show that the ex-post profit earned and wage paid are

$$\begin{aligned}\pi &= (1 - \tilde{\tau})p - w \\ w &= \beta((1 - \tilde{\tau})p + c\theta) + (1 - \beta)\frac{z}{1 - \tau_w}.\end{aligned}$$

Given the value  $\tilde{\tau} = 0.051$  and  $\tau_w = 0.291$  drawn from the literature, the implied solution for the opportunity true cost of employment expressed as a ratio of before tax output is  $z = (1 - 0.291) \times 0.955 = 0.677$ . Nonetheless, the after tax surplus enjoyed by an employed worker,

$$\begin{aligned}\frac{(1 - \tau_w)w - z}{z} &= \frac{\beta[(1 - \tau_w)((1 - \tilde{\tau})p + c\theta) - z]}{z} \\ &= \frac{0.052[(1 - 0.291)(1 - 0.051 + 0.382) - 0.677]}{0.677} = 0.020,\end{aligned}$$

is still trivial given their estimate of the worker's share of surplus.

In their paper, Hagedorn and Manovskii respond to this criticism by arguing that a gross value of the opportunity cost parameter near  $p$  is reasonable for the marginal worker. While this argument is essentially correct, it is irrelevant in the context of this model, because job creation depends on the average value of  $z$ , not its value for the marginal worker. To see this point more clearly, notice that, given heterogeneity in  $z$ , the value of unemployment for the marginal worker is equal to the value of non-participation. Thus the value of  $z$  for the marginal participant, denoted by  $\underline{z}$ , solves  $\underline{z} = r\underline{U}$  where  $\underline{U}$  represents the marginal worker's value of unemployed search. Because

$$\begin{aligned}rU(z) &= z + f(\theta)(W(z) - U(z)) \\ rW(z) &= w(z) - s(W(z) - U(z))\end{aligned}$$

hold as steady-state approximations,

$$0 = r\underline{U} - \underline{z} = f(\theta)(\underline{W} - \underline{U}),$$

so  $\underline{U} = \underline{W}$  and  $\underline{z} = r\underline{U} = r\underline{W} = \underline{w}$ . This in turn implies that the surplus from the relationship is  $\underline{V} = \frac{\underline{W} - \underline{U}}{1 - \beta} = 0$ , which in turn necessitates that  $p = \underline{w} = \underline{z}$ .

So not only is the marginal worker's value of leisure close to  $p$ , it is exactly equal to it.<sup>3</sup>

However, the incentive to create a job depends on the average value of  $z$ , not its marginal value. Indeed, as the value of a match with a worker of type  $z$  is

$$J(z) = \frac{p - w(z)}{r + s},$$

but the worker's type is not known when the employer posts the job, the free-entry condition is

$$\frac{cf(\theta)}{\theta} = E\{J(z)|z \leq p\} = \frac{p - \{Ew(z)|z \leq p\}}{r + s} = \frac{(1 - \beta)(p - \{Ez|z \leq p\}) - \beta c\theta}{r + s},$$

where the last equality can be derived from the wage equation as before. In sum, it is the average opportunity cost of unemployed participants that matters in the determination of market tightness.

Hall (2006) also argues that Shimer's opportunity cost of employment is too low. Under the assumption that households can insure against income risk, which he views as a reasonable benchmark, the flow payoff a worker enjoys while unemployed expressed in units of output is given by the expression

$$z = b + c_1 - c_0 + \frac{u(c_0, 0) - u(c_1, 1)}{\xi},$$

where  $b$  represents the government provided unemployment benefit,  $\xi$  is the marginal utility of income, and  $u(c, n)$  denotes the instantaneous utility as a function of the employment-contingent level of consumption,  $c_i \in R_+$ , and labor services provided,  $i \in \{0, 1\}$ . If the utility function was additively separable, the working household would fully insure consumption ( $c_0 = c_1$ ) and the last term on the right side would simply be the value of forgone leisure. However, recent research documents the fact that consumption falls about 15% in response to earning loss attributable to unemployment. As Hall points out, this evidence is consistent with complementarity between consumption and labor supply.

Using utility parameter values based on the empirical literature on household consumption demand and labor supply, Hall's estimate of the flow value of leisure forgone is 0.43. As the earning replacement ratio is less than 50%

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<sup>3</sup>Adding search costs could drive a wedge between the value of leisure for the marginal worker and match output  $p$ .

in the U.S. as a consequence of the 26 week limitation on the annual benefit period in most states and because not all qualified workers actually claim benefits, a reasonable value of  $b$  relative to marginal product  $p$  is between 0.2 and 0.4. For a value of  $b = 0.3$ ,  $z = 0.73$ . Given this number and a more reasonable value of the elasticity of the matching function and corresponding worker's share of surplus,  $\eta = 1 - \beta = 0.4$ , one obtains

$$\begin{aligned} \frac{\partial \ln \theta}{\partial \ln p} &= \frac{r + s + \beta f}{(1 - \eta)(r + s) + \beta f} \times \frac{p}{p - z} \\ &= \frac{0.112 + 0.6 \times 1.355}{0.6 \times (0.112 + 1.355)} \times \frac{1}{1 - 0.73} = 3.89. \end{aligned}$$

This quantitative response is more than twice as large as that implied by Shimer's parameter values and about 51% ( $3.89/7.56 = 0.514$ ) of the slope of the OLS relationship between market tightness and productivity implied by his data.

### 3.3 Turnover Costs

Shimer's analysis also abstracts from turnover costs. As has been pointed out by several authors (see Braun (2005), Nagypál (2005), Silva and Toledo (2005), and Yashiv (2005)), the presence of fixed turnover costs makes a firm's net payoff (after paying the fixed cost) more responsive to productivity variation. In other words, allowing for hiring and firing costs increases the opportunity cost of the match without resorting to very high values of opportunity cost of employment for the worker.

For the sake of illustration, suppose that the cost of hiring and training a new worker is  $H$  and the termination cost is  $T$ . In this case, the deterministic steady-state value of filling a job solves

$$rJ = p - w - s(J + T).$$

Since employer's surplus is  $J - H$  in this case, the Nash sharing rule<sup>4</sup> gives

$$\beta(J - H) = \beta \left( \frac{p - w - sT - (r + s)H}{r + s} \right)$$

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<sup>4</sup>Here, for the sake of simplicity, we assume that wages are set prior to the firm incurring the hiring cost. One could alternatively assume that wages are (re)negotiated after the relationship has been established. In that case, too, the elasticity of market tightness increases with  $T + H$ , but the exposition is more cumbersome.

$$= (1 - \beta)(W - U) = (1 - \beta) \left( \frac{w - z - f(\theta)\beta S}{r + s} \right),$$

where

$$S = W - U + J - H.$$

Combining with the free-entry condition to eliminate  $S$  implies

$$w = \beta p + (1 - \beta)z + \beta c\theta - \beta (sT + (r + s)H).$$

Since the free entry condition is

$$\frac{c\theta}{f(\theta)} = J - H = \frac{(1 - \beta)(p - z - sT - (r + s)H) - \beta c\theta}{r + s},$$

the amortized turnover cost flow  $sT + (r + s)H$  is simply added to the value of leisure. As the elasticity of market tightness is given by

$$\frac{\partial \ln \theta}{\partial \ln p} = \frac{r + s + \beta f}{(1 - \eta)(r + s) + \beta f} \times \frac{p}{p - z - sT - (r + s)H},$$

the model explains the relative volatility of the job-finding rate if the amortized costs of hiring and firing are high enough.

For this mechanism to work, however, it is important that these turnover costs are indeed fixed as the underlying driving force varies and depend neither on labor productivity nor on the level of wages in the economy. Whether this is indeed the case is an empirical question that deserves further attention.

## 4 Alternative Wage-Setting Mechanisms

Several authors have argued that the model can match the empirically observed volatility of labor-market variables if the wage setting mechanism generates a less procyclical wage than implied by the standard model. However, a rigid wage is not enough, its level must also be sufficiently high. Only when the wage is large and not too procyclical does the per-period profit that motivates job creation,  $p - w$ , respond strongly to changes in  $p$ . To understand the importance of this point, note that when  $\beta = 0$  the wage is rigid and equal to  $z$  from Equation (7). Hence, given the values assigned by Shimer for the other parameters, the elasticity of market tightness with respect to productivity would still only be

$$\frac{\partial \ln \theta}{\partial \ln p} = \frac{1}{1 - \eta} \times \frac{p}{p - z} = \frac{1}{0.72} \times \frac{1}{0.6} = 2.31.$$

## 4.1 Rational Wage Rigidity

Of course, if the wage level is high enough, then a rigid wage can easily account for the observed volatility. For example, suppose that the wage  $w$  is rigid as in Hall (2005a). In this case, the free-entry condition

$$\frac{c\theta}{f(\theta)} = J = \frac{p - w}{r + s}$$

holds as a steady-state approximation. Setting the rigid wage equal to the Nash solution at the median productivity,  $w = \beta(p + c\theta) + (1 - \beta)z = 0.983$ , as in Hall’s analysis, Shimer’s parameter values imply

$$\frac{\partial \ln \theta}{\partial \ln p} = \frac{1}{1 - \eta} \times \frac{p}{p - w} = \frac{1}{1 - 0.28} \times \frac{1}{1 - 0.983} = 81.70.$$

This number is  $81.70/7.56 = 10.81$  times that needed to explain the observed response of market tightness to productivity as measured by the regression coefficient in the data.

The rigid wage assumption is difficult to swallow. Since aggregate shocks are common knowledge, why wouldn’t negotiated wages reflect the fact that the worker’s outside search option is procyclical as the Nash bargaining solution implies? Hall (2005a) argues, as many others have done in the past, that the solution to a bilateral monopoly problem is simply indeterminate. According to Hall, any solution in the bargaining set available to a worker and employer when they meet should be regarded as a legitimate equilibrium. Furthermore, under these circumstances, it is reasonable to suppose that the wage set in previous bargains with other workers will serve as either a “norm” or a “focal point” for the outcome of any current bargain in every state for which this solution is jointly rational, that is when both  $W_p - U_p \geq 0$  and  $J_p \geq 0$  hold.<sup>5</sup> He then proceeds to show that the shocks to aggregate productivity required to explain the volatility of unemployment are so small that this condition is always satisfied in simulations.

There are two problems with Hall’s argument. First, to maintain that the rigid wage is jointly rational, only small shocks can affect the employment relationship of workers and firms in the economy. This assumption

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<sup>5</sup>Wage norms are not the only way to rationalize the existence of a rigid wage. Kennan (2005), Menzio (2005), and Moen and Rosen (2005) all develop contracting models with asymmetric information that deliver a rigid wage as an equilibrium outcome.

is greatly at odds with the extent of gross flows in the labor market that reflect the importance of idiosyncratic variability. Second, the limited empirical evidence available does not support the claim that wages of workers in new employment relationships are rigid over the business cycle, in fact, these wages have been found to be more cyclically sensitive than wages of workers in continuing relationships.<sup>6</sup>

## 4.2 Infrequent Wage Bargaining

Since the work of Taylor (1980) and Calvo (1983), the idea that nominal prices and wages are revised infrequently has played an important role in macroeconomics. Gertler and Trigari (2005) have shown that this mechanism also has the power to generate substantial labor-market volatility in a matching model. Although we abstract from their assumption that wage negotiations are at the firm level and are staggered, the basic point can be made within our simple framework. That is, suppose that all workers in all matches receive the same wage which is revised from time to time to reflect new information about aggregate productivity.

By assumption, a new wage bargain is made with all workers, old and new.<sup>7</sup> In the Calvo version of the model, wage renegotiation takes place with Poisson frequency  $\alpha$ . Hence, the value of a new match depends on the currently prevailing wage as well as future anticipated wages. Indeed, at the moment a new productivity shock  $p$  is realized, the Bellman equation can be expressed as

$$rJ_p(w) = p - w - sJ_p(w) + \alpha(J_p - J_p(w)).$$

where  $w$  is the wage inherited from the last renegotiation. Hence,  $J_p(w)$  is the value of the match when worker and employer meet and  $J_p$  is the value after the wage is renegotiated in the future. That is

$$rJ_p = p - w_p - sJ_p = \frac{p - w_p}{r + s},$$

where  $w_p$  represents the bargained wage given that common match productivity is  $p$ . Equivalently,  $J_p = J_p(w_p)$  is the value of the match after the wage has been adjusted to reflect the value of  $p$ .

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<sup>6</sup>See Vroman (1977), Bilal (1985), and Barlevy (2001).

<sup>7</sup>For the Gertler and Trigari story to work, it is imperative that the wages of new employees are negotiated only at the time that all wages of worker in the firm are revised and not when the new worker is hired.

When a wage is renegotiated, it reflects prevailing demand conditions as in the original model. Specifically, equation (7)

$$w_p = \beta(p + c\theta_p) + (1 - \beta)z$$

holds by virtue of the Nash sharing rule and the free-entry condition at the renegotiation date as in the original model. Therefore,

$$J_p = \frac{p - w_p}{r + s} = \frac{(1 - \beta)(p - z) - \beta c\theta_p}{r + s}$$

as before, but,

$$J_p(w) = \frac{p - w + \alpha J_p}{r + s + \alpha} = \frac{p - w}{r + s + \alpha} + \frac{\alpha}{r + s + \alpha} \frac{(1 - \beta)(p - z) - \beta c\theta_p}{r + s}.$$

Note that this formulation nests both the standard model in which wages are continually revised ( $\alpha = \infty$ ) and the rigid wage model ( $\alpha = 0$ ).<sup>8</sup>

The free-entry condition given the current wage  $w$  is

$$\frac{c\theta}{f(\theta)} = J_p(w) = \frac{r + s}{r + s + \alpha} \frac{p - w}{r + s} + \frac{\alpha}{r + s + \alpha} \frac{(1 - \beta)(p - z) - \beta c\theta_p}{r + s}.$$

where  $c\theta_p/f(\theta_p) = J_p(w_p) = J_p$  determines market tightness once the wage is renegotiated. Suppose, as Gertler and Trigari do, that wages are renegotiated once every three quarters on average,  $\alpha = 0.33$ . When evaluated at a wage equal to that prevailing in the median productivity state  $w = \beta(p + c) + (1 - \beta)z = 0.983$  and  $\frac{\partial \ln \theta_p}{\partial \ln p} = 1.72$  from Equation (11) and  $\eta = 0.28$ . Hence,

$$\begin{aligned} \frac{\partial \ln \theta}{\partial \ln p} &= \frac{p}{1 - \eta} \left( \frac{r + s + \alpha \left(1 - \beta - \beta c \frac{\theta_p}{p} \frac{\partial \ln \theta_p}{\partial \ln p}\right)}{(r + s)(p - w) + \alpha [(1 - \beta)(p - z) - \beta c\theta_p]} \right) \\ &= \frac{1}{0.72} \left( \frac{0.112 + 0.33 \times (0.28 - 0.72 \times 0.209 \times 1.72)}{0.112 \times (1 - 0.983) + 0.33 \times (0.28 \times (1 - 0.4) - 0.72 \times 0.209)} \right) \\ &= 21.61. \end{aligned}$$

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<sup>8</sup>For simplicity, we are allowing agents to take account of a future change in the wage to bring it in line with current productivity but continue to suppose that they ignore the fact that the productivity will change in the future. Again, these assumptions provide quantitative implications that are reasonable approximations if the productivity process is sufficiently persistent.

Clearly, the initial impact of the productivity shock on market tightness, that is computed in the equation above, is substantially larger than its volatility relative to that of productivity observed in the data. However, note that in the future when the wage is actually renegotiated, the wage will jump up from  $w$  to its equilibrium value  $w_p$  in response to a positive productivity shock which will then induce a drop in  $\theta$  to  $\theta_p$ . That is, the implied response in the vacancy-unemployment ratio to a productivity shock is large on impact but then dissipates as wages adjust to the shock with a lag. A precise analysis of the average response requires the lag in adjustment to be explicitly taken into account.

### 4.3 Strategic Wage Bargaining

The Nash bargaining wage-setting mechanism of the textbook model is often justified by noting that the wage outcome it implies is equivalent to the outcome of an alternating-offers strategic bargaining game. Hall and Milgrom (2005) note that for this equivalence to hold, one needs to assume not only that the outside option of the worker (the payoff she gets in case negotiations break down) but also the disagreement payoff of the worker (the payoff she gets when agreement is delayed) is equal to the value of search. They take issue with the second assumption and argue that unemployed search is not the relevant payoff from delay during negotiations.<sup>9</sup> When the disagreement payoff is fixed rather than tied to the value of search, the wage agreed to is more rigid than that implied by the standard sharing rule.

For the sake of illustration, suppose that the worker receives payoff  $z$  and the employer incurs no cost while bargaining continues and that they renegotiate the division of the match product  $p$  whenever it changes. In this case, the outcome of a symmetric alternating-offers game is

$$w = z + 0.5(p - z). \quad (15)$$

As workers do not share turnover costs under the assumed wage mechanism, the free entry condition is

$$\frac{c\theta}{f(\theta)} = J - H = \frac{p - w - sT}{r + s} - H \quad (16)$$

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<sup>9</sup>Although Binmore, Rubinstein and Wolinsky (1986) provide conditions under which the strategic bargaining outcome corresponds to the Nash sharing rule, they also make the Milgrom and Hall point. Namely, if the probability that negotiations will breakdown is zero, then the cost and benefits of delay determine the relevant default options.

$$= \frac{0.5(p - z) - sT - (r + s)H}{r + s},$$

which implies that the response in market tightness is given by

$$\frac{d \ln \theta}{d \ln p} = \frac{1}{1 - \eta} \times \frac{0.5p}{0.5(p - z) - sT - (r + s)H}.$$

Given an elasticity of the matching function set at the midpoint of the Petrongolo and Pissarides (2001) plausible range,  $\eta = 0.4$ , and a value of leisure equal to Hall's (2006) estimate,  $z = 0.73$ , the model exactly matches the OLS response of tightness to productivity,  $\sigma_\theta \rho_{\theta p} / \sigma_p = 7.56$ , when the turnover cost flow is equal to 2.48% of match output ( $sT + (r + s)H = 0.0248$ ). Even if there are no firing costs, the cost of training a worker required to support this number is less than 22.1% of quarterly match output ( $H = 0.0248 / 0.112 = 0.221$ ). In sum, the model with strategic bargaining does explain the observed partial relationship between labor market tightness and labor productivity given reasonable alternative estimates of the elasticity of the matching function, the household's value of leisure, and the cost of turnover.

## 5 Separation Rate and Other Shocks

Clearly, one important source of fluctuations missing from the above analysis is variation in the rate of discounting of future income flows,  $r$ . Qualitatively, the question is whether variations in the discount rate amplify or dampen the response of labor-market variables, which in turn depends on whether the discount rate is pro- or counter-cyclical. When Hall (2005b) embeds his version of the matching model in a DSGE framework, he finds that increases in the discount rate induced by an increase in productivity dampen labor-market amplification. If the source of variation in the match output,  $p$ , are monetary policy shocks, however, as in Braun (2005), the discount rate moves countercyclically, and labor-market responses are amplified by variation in the discount rate. Quantitatively, Yashiv (2005), using observed values of the discount rate in his version of the matching model, finds that variations in the discount rate have a small role at best in explaining the volatility of the job-finding rate. This is because the pure discount rate  $r$  makes up a small fraction of the total rate of "discounting" that firms apply to match profits,  $r + s$ , both in terms of levels and in terms of volatility.

Given that the separation rate makes up the bulk of firms' total discount rate,  $r + s$ , it is natural to consider volatility in the separation rate as a potential driving force. As is clear from Table 1, changes in the separation rate,  $s$ , are present in the data and are negatively correlated with labor productivity. These changes, which are implied by the Mortensen and Pissarides (1994) model with endogenous separation, serve as an important source of volatility in unemployment. Moreover, because countercyclical movements in the separation rate imply that the rate at which firms discount match profits falls in a boom and rises in a bust, these shocks tend to also amplify the effects of productivity shocks on vacancy creation and the job-finding rate if the feedback from the job-finding rate to the wage is weak.

## 6 Volatility of Market Tightness

Consider the simplified version of the Hall-Milgrom strategic wage bargain discussed above, where wages do not directly depend on market tightness, and assume that the model is subject to two shocks – productivity and separation rate shocks. Given the free-entry condition as stated in Equation (16),

$$\Delta \ln \theta = \frac{a\Delta \ln p - b\Delta \ln s}{1 - \eta} \quad (17)$$

holds as a linear approximation, where  $\Delta \ln x$  is the difference between  $\ln x$  and its mean in the data and

$$\begin{aligned} a &= \frac{\partial \ln(J - H)}{\partial \ln p} = \frac{0.5p}{0.5(p - z) - sT - (r + s)H} \\ b &= -\frac{\partial \ln(J - H)}{\partial \ln s} = \frac{s}{r + s} + \frac{s(T + H)}{0.5(p - z) - sT - (r + s)H} \end{aligned}$$

are the indicated partial derivatives. Since Equation (17) implies

$$E(\Delta \ln \theta)^2 = \sigma_\theta^2 = \left( \frac{1}{1 - \eta} \right)^2 \left( a^2 \sigma_p^2 - 2ab\rho_{ps}\sigma_s\sigma_p + b^2\sigma_s^2 \right),$$

it follows that

$$\frac{\sigma_\theta}{\sigma_p} = \frac{a}{1 - \eta} \left( 1 - \frac{2b}{a} \frac{\rho_{ps}\sigma_s}{\sigma_p} + \left( \frac{b\sigma_s}{a\sigma_p} \right)^2 \right)^{1/2}. \quad (18)$$

Given our preferred parameter values,  $r + s = 0.112$ ,  $\eta = 0.4$ ,  $z = 0.73$ ,  $T = 0$ , and  $H = 0.221$ , one obtains

$$\begin{aligned} a &= \frac{0.5}{0.5 \times (1 - .73) - 0.112 \times 0.221} = 4.54. \\ b &= \frac{0.1}{0.112} + \frac{0.1 \times 0.221}{0.5 \times (1 - .73) - 0.112 \times 0.221} = 1.09. \end{aligned} \tag{19}$$

These values together with equation (18) imply that

$$\frac{\sigma_\theta}{\sigma_p} = \frac{4.54}{1 - 0.4} \left( 1 + \frac{2 \times 1.09 \times 0.524 \times 0.075}{4.54 \times 0.02} + \left( \frac{1.09 \times 0.075}{4.54 \times 0.02} \right)^2 \right)^{1/2} = 12.56$$

while the estimate from the data is 19.10. In other words, we find that productivity and separation shocks in the amended version of the model explain 66% of the variation in the vacancy-unemployment ratio relative to that in productivity ( $12.56/19.01 = 0.66$ ). Although the model fails to explain the entire ratio, this value implies a response that is six times larger than that obtained using the standard model with Shimer's parameter values.

## 6.1 The Beveridge Curve

Shimer (2005a) argues that separation rate shocks induce positive co-moments in vacancies and unemployment which is an implication inconsistent with the commonly observed negative association between these two variables captured by the Beveridge curve. However, his argument ignores the fact that market tightness will fall in response to the shock and that this effect is likely to dominate as long as it is not offset by a sufficiently large drop in the wage. Of course, the wage response is large if the threat of unemployment is credible in the bargaining game. To see this point, note that the wage derived in Equation (7) implies that

$$\begin{aligned} \left. \frac{\partial \ln(p - w)}{\partial \ln \theta} \right|_{p \text{ fixed}} &= - \frac{\beta c \theta}{(1 - \beta)(p - z) - \beta c \theta} \\ &= - \frac{0.72 \times 0.209}{0.28 \times 0.6 - 0.72 \times 0.209} = -8.59 \end{aligned}$$

at Shimer's parameter values. In other words, any change in the vacancy-unemployment ratio that is not induced by a shock to  $p$  has a huge impact

on the employer flow profit which offsets the equilibrium effect of the change. This effect is not present when wages are the outcome of strategic bargaining game in which delay rather than unemployment is the no-agreement default. In this section, we show that this variant of the model implies a strong negative correlation between unemployment and vacancies even when separation shocks contribute to variation in unemployment.

To assess quantitatively the correlation between vacancies and unemployment implied by the two-shock model, notice that the steady state condition,  $u = s/(s + f)$  and equation (17), imply that the following relationships hold as linear approximations:

$$\begin{aligned}
\Delta \ln u &= \frac{f}{s+f} (\Delta \ln s - \eta \Delta \ln \theta) = & (20) \\
&= \frac{f}{s+f} \left( 1 + \frac{\eta b}{1-\eta} \right) \Delta \ln s - \frac{f}{s+f} \frac{\eta}{1-\eta} a \Delta \ln p \\
&= 1.61 \Delta \ln s - 2.82 \Delta \ln p. \\
\Delta \ln v &= \Delta \ln \theta + \Delta \ln u & (21) \\
&= \frac{4.54}{1-0.4} \Delta \ln p - \frac{1.09}{1-0.4} \Delta \ln s + 1.61 \Delta \ln s - 2.82 \Delta \ln p \\
&= 4.74 \Delta \ln p - 0.21 \Delta \ln s.
\end{aligned}$$

Note that vacancies and unemployment respond in opposite directions to both productivity and separation shocks. The implied volatility of unemployment and vacancies are

$$\begin{aligned}
\sigma_u &= \left( E \Delta \ln u^2 \right)^{\frac{1}{2}} = \left( 2.59 \sigma_s^2 - 9.07 \rho_{ps} \sigma_s \sigma_p + 7.93 \sigma_p^2 \right)^{\frac{1}{2}} = 0.158 \\
\sigma_v &= \left( E \Delta \ln v^2 \right)^{\frac{1}{2}} = \left( 22.50 \sigma_p^2 - 2.01 \rho_{ps} \sigma_p \sigma_s + 0.045 \sigma_s^2 \right)^{\frac{1}{2}} = 0.104
\end{aligned}$$

while the correlation is

$$\begin{aligned}
\sigma_{vu} &= \frac{E(\Delta \ln v \Delta \ln u)}{\sigma_v \sigma_u} = \frac{8.22 \rho_{ps} \sigma_p \sigma_s - 13.37 \sigma_p^2 - 0.34 \sigma_s^2}{\sigma_v \sigma_u} \\
&= \frac{-0.014}{0.104 \times 0.158} = -0.836.
\end{aligned}$$

In short, our variant of the model and parameter values implies a correlation between vacancies and unemployment that is very close to that observed in the data ( $-0.894$ ).

## 7 Conclusions

Shimer (2005a) argues that the Mortensen-Pissarides equilibrium search model of unemployment with shocks to productivity explains less than 10% of the volatility in the job-finding rate. Some of the recent papers inspired by his critique are reviewed and commented on here and compared within a unified framework that highlights the importance of parameter choices. Overall, we find that the literature has overemphasized the need to introduce wage rigidity into the model. Indeed, we show that a modified version of the model can explain nearly two thirds of the volatility in the ratio of vacancies to unemployment as well as the quantitative properties of the Beveridge curve inferred from U.S. data when the feedback from market tightness to wages is consistent with a strategic model of wage bargaining, turnover costs are taken into account, and the elasticity of the matching function and the flow value of leisure are set at more reasonable values.

While we attempted to give a broad overview of the issues raised in the literature following Shimer's work, our review is by no means complete. One important consideration we did not address is the fact that Shimer (2005a) and subsequent authors abstract from job-to-job flows. Considering these flows is important for two reasons. First, in the presence of job-to-job flows, the rate that firms use to discount match profits is determined by the total separation rate (which includes the quit rate) and not just the separation rate into unemployment. Second, employed workers represent well over half of those hired in any period and their fraction among new hires is strongly procyclical (Nagypál (2006)). Thus the payoff from meeting an employed worker has an important influence on the incentives to create vacancies. If employers profit more from meeting employed rather than unemployed workers, as is the case in Nagypál's (2005) model, then the increase in the fraction of employed workers in the application flow on the upswing both amplifies and propagates the effects of a positive productivity shock on job creation. Nagypál (2005) shows that this effect is not only qualitatively, but also quantitatively important in accounting for the data.

## 8 Appendix

### 8.1 Proof to Proposition 1

By substitution from Equations (5) and (6), Equation (4) implies that an equilibrium surplus value function is a fixed point of the map

$$(TV)_p = \Gamma^{-1} \left( \frac{p - z + \lambda E_p V_{p'}}{r + s + \lambda} \right)$$

from the set of real valued functions of  $p$  to itself where  $\Gamma(V)$  is the real valued function defined by

$$\Gamma(V) \equiv V + \frac{\beta c \theta(V)}{(1 - \beta)(r + s + \lambda)}$$

and  $\theta(V)$  is the function implicitly defined by the free-entry condition

$$\frac{c\theta}{f(\theta)} = (1 - \beta)V.$$

Because  $\theta(V)$  is continuous, increasing, and convex and  $\theta(0) = 0$  under the hypothesis,  $\Gamma(V)$  has these same properties.

To prove uniqueness, we show that the mapping  $T$  satisfies Blackwell's sufficient conditions for a contraction. Since  $\Gamma^{-1}(\cdot)$  is increasing and  $E_p(V_{p'} + k) \geq E_p(V_{p'})$  for all  $k \geq 0$ ,  $T$  is increasing. Hence,

$$\begin{aligned} (T(V + k))_p &= \Gamma^{-1} \left( \frac{p - z + \lambda E_p(V_{p'} + k)}{r + s + \lambda} \right) = \Gamma^{-1} \left( \frac{p - z + \lambda E_p V_{p'} + \lambda k}{r + s + \lambda} \right) \\ &\leq \Gamma^{-1} \left( \frac{p - z + \lambda E_p V_{p'}}{r + s + \lambda} \right) + \frac{1}{\Gamma'(V)} \left( \frac{\lambda k}{r + s + \lambda} \right) \\ &\leq (TV)_p + \beta k \end{aligned}$$

for any positive constant  $k$  and  $\beta = \lambda / (r + s + \lambda) < 1$  where the first inequality follows from the concavity of  $\Gamma^{-1}(\cdot)$  (recall that  $\Gamma(\cdot)$  is convex) and the second is implied by the fact that  $d\Gamma^{-1}(x)/dx = 1/\Gamma'(y) \leq 1$ .

If  $p'$  is stochastically increasing in  $p$ , then  $T$  maps the set of continuous and increasing function of  $p$  into itself. Hence, the fact that  $T$  is a contraction implies that its fixed point is increasing in  $p$ . All the other equilibrium outcomes can be expressed as increasing functions of  $p$  and  $V_p$ . Finally, the assertion that  $V_p$  is strictly increasing is implied by the fact that  $T$  transforms any increasing functions into the set of strictly increasing functions.

## 8.2 Proof to Proposition 2

The claim is an immediate implication of Equation (9) in the case of  $\lambda \rightarrow 0$ . As the specification in Equation (14) implies,

$$p' - z = (p - z)e^\Delta,$$

it follows that

$$\lim_{\Delta \rightarrow 0} E_p \phi_{p'} = \phi_p$$

for any real-valued integrable function  $\phi$  of  $p$ . The free-entry condition (4) and Equation (6) imply that the Bellman equation can be written as

$$V_p = \frac{p - z - \frac{\beta c \theta_p}{1 - \beta} + \lambda E_p V_{p'}}{r + s + \lambda}.$$

It follows that,

$$\begin{aligned} \lim_{\Delta \rightarrow 0} V_p &= \frac{p - z - \frac{\beta c}{1 - \beta} \lim_{\Delta \rightarrow 0} E_p \theta_{p'} + \lambda \lim_{\Delta \rightarrow 0} E_p V_{p'}}{r + s + \lambda} \\ &= \frac{p - z - \frac{\beta c}{1 - \beta} \theta_p + \lambda \lim_{\Delta \rightarrow 0} V_p}{r + s + \lambda} = \frac{p - z - \frac{\beta c}{1 - \beta} \theta_p}{r + s}. \end{aligned}$$

Hence, the free-entry condition can be approximated by

$$\frac{c \theta_p}{f(\theta_p)} = (1 - \beta) V_p = \frac{(1 - \beta)(p - z) - \beta c \theta_p}{r + s}.$$

By differentiating this expression with respect to  $\ln p$ , one obtains Equation (10).

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