Extended Delivery Time Analysis for Cognitive Packet Transmission with Application to Secondary Queuing Analysis

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Abstract

Cognitive radio transceiver can opportunistically access the underutilized spectrum resource of primary systems for new wireless services. With interleave implementation, the secondary transmission may be interrupted by the primary user's transmission. To facilitate the delay analysis of such secondary transmission for a fixed-size secondary packet, we study the resulting extended delivery time that includes both transmission time and waiting time. In particular, we derive the exact distribution function of extended delivery time of secondary transmission for both continuous sensing and periodic sensing cases. Selected numerical and simulation results are presented for illustrating the mathematical formulation. Finally, we consider a generalized M/G/1 queue set-up at the secondary user and formulate the closed-form expressions for the expected delay with Poisson traffic. The analytical results will greatly facilitate the design of the secondary system for particular target application.

Index Terms

Cognitive radio, spectrum access, single-channel sensing, traffic model, primary user, secondary user, M/G/1 Queue.

I. INTRODUCTION

Radio spectrum resource scarcity is one of the most serious problems nowadays faced by the wireless communications industry. Cognitive radio is a promising solution to this emerging problem by exploiting temporal/spatial spectrum opportunities over the existing licensed frequency bands [1]–[8]. Different techniques exist for opportunistic spectrum access (OSA). In underlay cognitive radio implementation, the primary and secondary users simultaneously access the same spectrum, with a constraint on the interference to primary transmission caused by the secondary user (SU). With interleaved cognitive implementation, the secondary transmission creates no interference to the primary user (PU). Specifically, the SU can access the channel only when it is not used by PU and must vacate the occupied channel when the PU appears. Spectrum handoff procedures are adapted for returning the channel to the PU and then re-accessing that channel or another channel later to complete the transmission. As such, the secondary transmission of a given amount of data may involve multiple spectrum handoffs, which results in extra transmission delay. The total time required for the SU to complete a given packet transmission will include the waiting periods before accessing the channel and become more than the actual time needed for transmission. In this paper, we investigate the statistical characteristics of the resulting extended delivery time (EDT) [9] and apply them to evaluate the delay performance of secondary transmission.

A. Previous Work

There has been a continuing interest in the delay and throughput analysis for secondary systems, especially for underlay implementation. [10] analyzes the delay performance of a point-to-multipoint secondary network, which concurrently shares the spectrum with a point-to-multipoint primary network in the underlay fashion, under Nakagami-m fading. The packet transmission time for secondary packets under PU interference is investigated in [11], where multiple secondary users are simultaneously using the channel. An optimum power and rate allocation scheme to maximize the effective capacity for spectrum sharing channels under average interference constraint is proposed in [12]. [13] examines the PDF and CDF of secondary packet transmission time in underlay cognitive system. [14] investigates the M/G/1 queue performance of the secondary packets under the PU outage constraint. [15] analyzes the interference caused by multiple SUs in a “mixed interleave/underlay” implementation, where each SU starts its transmission only when the PU is off, and continues and completes its transmission even after the PU turns on.

For interleave implementation strategy, [16] discusses the average service time for the SU in a single transmission slot and the average waiting time, i.e. the time the SU has to wait for the channel to become available, assuming general primary traffic model. A probability distribution for the service time available to the SU during a fixed period of time was derived in [17]. A model of priority virtual queue is proposed in [18] to evaluate the delay performance for secondary users. [19] studies the probability of successful data transmission in a cooperative wireless communication scenario with hard delay constraints. A queuing analysis for secondary users dynamically accessing spectrum in cognitive radio systems was carried out in [20]. [9] derives bounds on the throughput and delay performance of secondary users in cognitive scenario based on the concept of EDT. [21] calculates the expected EDT of a packet for a cognitive radio network with multiple channels and users.

When the secondary transmission is interrupted by PU activities, the secondary system can adopt either non-work-preserving strategy, where interrupted packets transmission must be repeated [9], or work-preserving strategy, where the secondary transmission can continue from the point where it was interrupted, without wasting the previous transmission [21]. These can be achieved with the application of rateless codes such as Fountain code [22], [23]. Work-preserving strategy also applies to the transmission scenario with small and individually coded sub-packets transmission.

B. Contribution

In this paper, we carry out a thorough statistical analysis on the EDT of secondary packet transmission with work-preserving strategy. In general, the transmission of a secondary packet involves an interleaved transmission sequence of transmission and waiting time slots, both of which can have random time duration. We first derive the exact closed-form expression for the distribution function of EDT assuming a fixed packet transmission time. Both, the ideal scenario of continuous sensing, in which the SU will continuously sense for the channel availability, and the practical scenario of periodic sensing, in which the SU will sense the channel periodically, are considered. We also generalize the analysis to the case where the transmission time depends on the instantaneous channel quality, and as such, is random. The exact statistics for the EDT for secondary packet transmission can be directly used to predict the delay performance of some low-traffic intensity secondary applications.

We then apply these statistical results on EDT to the secondary queuing analysis. The queuing analysis for secondary transmission is a challenging problem even for Poisson arrival traffic. The main difficulty results from the fact that packets will experience two different types of service time depending on whether the packets see an empty queue or not upon arrival. In this paper, we solve this problem by generalizing the conventional M/G/1 queuing model. Both average queuing delay and average queue length are calculated in closed form. Simulation results are included to validate the obtained analytical results.

The rest of this paper is organized as follows. In section II, we introduce the system model and the problem formulation. In section III, we analyze the EDT of a single packet for both continuous sensing case and periodic sensing case. We also consider the case of short packets with a variable transmission time. In section IV, we analyze the average queuing delay of the secondary system in a general M/G/1 queuing set-up. Finally, this paper is concluded in section V.

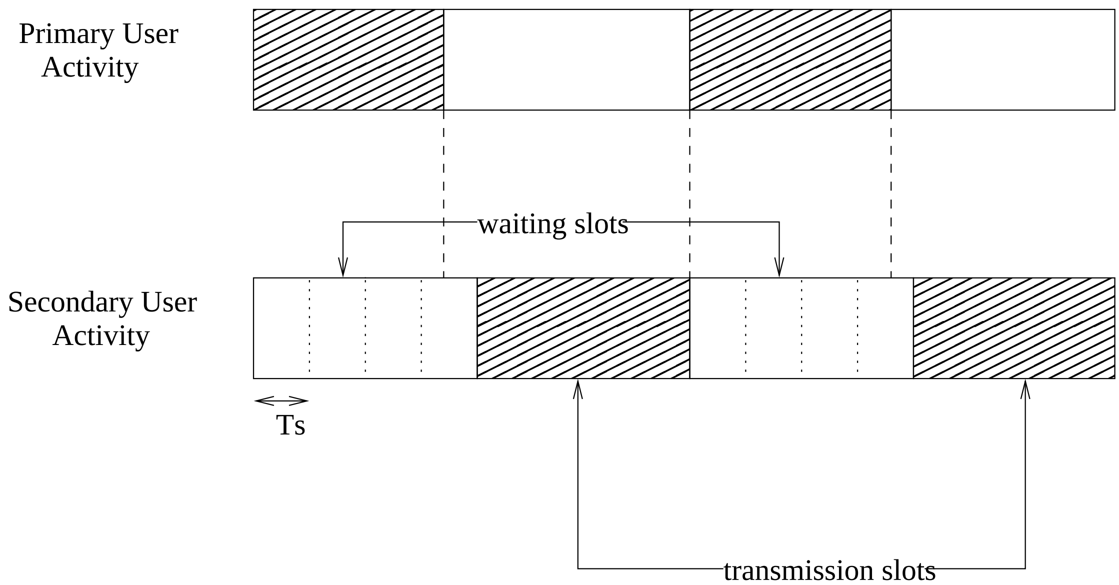

Fig. 1. Illustration of PU and SU activities and SU sensing for periodic sensing case.

II. System Model and Problem Formulation

We consider a cognitive transmission scenario where the SU opportunistically accesses a channel of the primary system for data transmission. The occupancy of that channel by the PU evolves independently according to a homogeneous continuous-time Markov chain with an average busy period of λ and an average idle period of μ. Thus, the duration of busy and idle periods are exponentially distributed. The SU opportunistically accesses the channel in an interleave fashion. Specifically, the SU can use the channel only after PU stops transmission. As soon as the PU restarts transmission, the SU instantaneously stops its transmission, and thus no interference is caused to the PU.

The SU monitors PU activity through spectrum sensing[1]. With continuous sensing, the SU continuously senses the channel for availability. Thus, the SU starts its transmission as soon as the channel becomes available. We also consider the case where the SU senses the channel periodically, with an interval of T_s. If the PU is sensed busy, the SU will wait for T_s time period

[1]We assume perfect spectrum sensing here. The consideration of imperfect sensing will be treated in future work.

and re-sense the channel. With periodic sensing, there is a small amount of time when the PU has stopped its transmission, but the SU has not yet acquired the channel, as illustrated in Fig. 1. During transmission, the SU continuously monitors PU activity. As soon as the PU restarts, the SU stops its transmission. The continuous period of time during which the PU is off and the SU is transmitting is referred to as a transmission slot. Similarly, the continuous period of time during which the PU is transmitting is referred to as a waiting slot. For periodic sensing case, the waiting slot also includes the time when the PU has stopped transmission, but the SU has not sensed the channel yet.

In this work, we analyze the packet delivery time of secondary system, which includes an interleaved sequence of the transmission time and the waiting time. The resulting EDT for a packet is mathematically given by $T_{ED} = T_w + T_{tr}$, where T_w is the total waiting time for the SU and T_{tr} is the packet transmission time. Note that both T_w and T_{tr} are, in general, random variables, with T_w depending on T_{tr}, PU behaviour and sensing strategies, and T_{tr} depending on packet size and secondary channel condition when available. In what follows, we first derive the exact distribution of the EDT T_{ED} for both continuous sensing and periodic sensing cases, which are then applied to the secondary queuing analysis in section IV.

III. Extended Delivery Time Analysis

In this section, we investigate the EDT of secondary system for a single packet arriving at a random point in time. We first consider a fast varying channel and/or a long packet, where the transmission time T_{tr} can be estimated as a constant, given by

$$T_{tr} \approx \frac{H}{W \int_0^\infty \log_2(1+\gamma) f_\gamma(\gamma) d\gamma}, \tag{1}$$

where H is the entropy of the packet, W is the available bandwidth and $f_\gamma(\gamma)$ is the PDF of the SNR of the fading channel. We then consider the case of short packets, where T_{tr} cannot be treated as a constant. For both continuous sensing and periodic sensing scenarios, we derive the

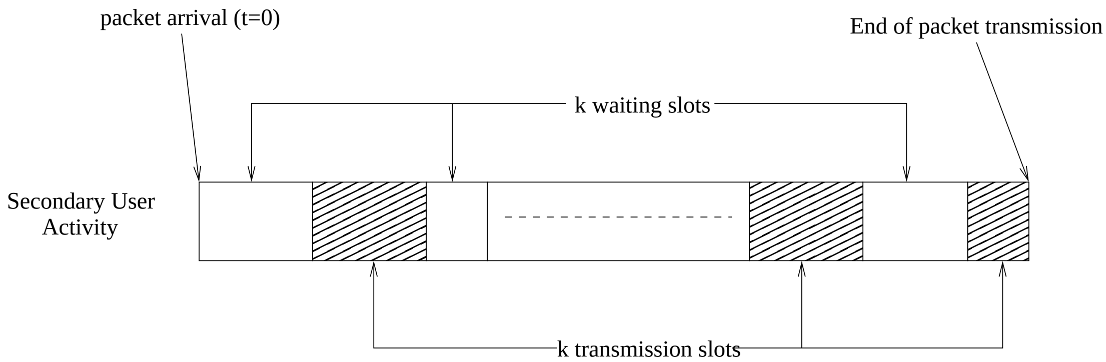

Fig. 2. Illustration of secondary transmission when the PU is on at $t = 0$.

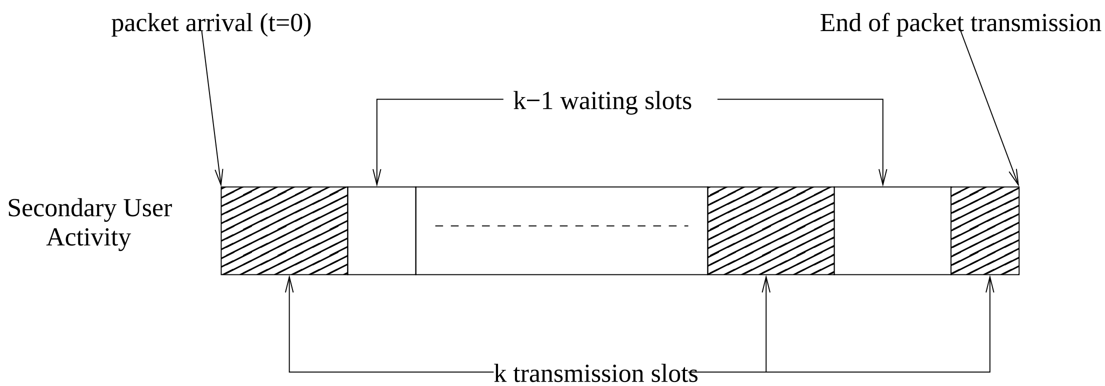

Fig. 3. Illustration of secondary transmission when the PU is off at $t = 0$.

exact distribution of T_{ED}. These analyses also characterize the delay of some low-rate secondary applications. For example, in wireless sensor networks for health care monitoring, forest fire detection, air pollution monitoring, disaster prevention, landslide detection etc., the transmitter needs to periodically transmit measurement data to the sink with a long duty cycle. The EDT essentially characterizes the delay of measurement data collection.

A. Continuous Sensing

The EDT for packet transmission by the SU consists of interleaved waiting slots and transmission slots. We first focus on the distribution of T_w. We assume, without loss of generality, that the packet arrives at $t = 0$. The distribution of T_w depends on whether the PU was on or off at that instance, as illustrated in Figs. 2 and 3. We denote the PDF of the waiting time of the SU for the case when PU is on at $t = 0$, and for the case when PU is off at $t = 0$, by $f_{T_w,p_{on}}(t)$ and $f_{T_w,p_{off}}(t)$, respectively. The PDF of the waiting time T_w for the SU is then given by

$$f_{T_w}(t) = \frac{\lambda}{\lambda+\mu} f_{T_w,p_{on}}(t) + \frac{\mu}{\lambda+\mu} f_{T_w,p_{off}}(t), \tag{2}$$

where $\frac{\lambda}{\lambda+\mu}$ and $\frac{\mu}{\lambda+\mu}$ are the stationery probabilities that the PU is on or off at $t = 0$, respectively. The two probability density functions $f_{T_w,p_{on}}(t)$ and $f_{T_w,p_{off}}(t)$ above are calculated independently as follows.

When the PU is on at $t = 0$, T_w includes k waiting slots if k transmission slots are needed for packet transmission. Let $\mathcal{P}_k$ represent the probability that the SU completes packet transmission in k transmission slots, and $f_{T_w,k}(t)$ represent the PDF of the total time duration of k SU waiting slots. Then the PDF of the total waiting time for the SU, for the case when PU is on at $t = 0$, is given by

$$f_{T_w,p_{on}}(t) = \sum_{k=1}^{\infty} \mathcal{P}_k \times f_{T_w,k}(t). \tag{3}$$

Note that $f_{T_w,k}(t)$ is the PDF of the sum of k independent and identically distributed exponential random variables with average λ. Therefore $f_{T_w,k}(t)$ is given by

$$f_{T_w,k}(t) = \frac{1}{\lambda^k} \frac{t^{k-1}}{(k-1)!} e^{\frac{-t}{\lambda}}. \tag{4}$$

$\mathcal{P}_k$ can be calculated as the probability that k SU transmission slots have a total time of more than T_{tr}, whereas $k-1$ transmission slots have a total time of less than T_{tr}. Since the total time for k transmission slots follows the Erlang distribution with PDF

$$f_{T_{tr},k}(t) = \frac{1}{\mu^k} \frac{t^{k-1}}{(k-1)!} e^{\frac{-t}{\mu}}, \tag{5}$$

we can show that

$$\mathcal{P}_k = \int_{T_{tr}}^{\infty} \frac{1}{\mu^k} \frac{t^{k-1}}{(k-1)!} e^{\frac{-t}{\mu}} dt - \int_{T_{tr}}^{\infty} \frac{1}{\mu^{k-1}} \frac{t^{k-2}}{(k-2)!} e^{\frac{-t}{\mu}} dt. \tag{6}$$

After using integration by parts on the first integral and cancelling the terms, $\mathcal{P}_k$ can be calculated as

$$\mathcal{P}_k = \frac{{T_{tr}}^{k-1} e^{\frac{-T}{\mu}}}{\mu^{k-1}(k-1)!}. \tag{7}$$

After substituting Eqs. (4) and (7) into Eq. (3), we get

$$f_{T_w,p_{on}}(t) = \sum_{k=1}^{\infty} \frac{{T_{tr}}^{k-1} e^{\frac{-T_{tr}}{\mu}}}{\mu^{k-1}(k-1)!} \times \frac{1}{\lambda^k} \frac{t^{k-1}}{(k-1)!} e^{\frac{-t}{\lambda}}. \tag{8}$$

Finally, applying the definition of Bessel function, we arrive at the following closed-form expression for $f_{T_w,p_{on}}(t)$

$$f_{T_w,p_{on}}(t) = \frac{1}{\lambda} e^{\frac{-T_{tr}}{\mu}} I_0 \left(2\sqrt{\frac{T_{tr} t}{\mu\lambda}} \right) e^{\frac{-t}{\lambda}}, \tag{9}$$

where $I_n(.)$ is the modified Bessel function of the first kind of order n.

Similarly, the PDF for T_w when PU is off at $t = 0$ can be obtained as

$$f_{T_w,p_{off}}(t) = \sum_{k=1}^{\infty} \mathcal{P}_k \times f_{T_w,k-1}(t) = e^{\frac{-T_{tr}}{\mu}} \delta(t) + \sum_{k=2}^{\infty} \frac{{T_{tr}}^{k-1} e^{\frac{-T_{tr}}{\mu}}}{\mu^{k-1}(k-1)!} \times \frac{1}{\lambda^{k-1}} \frac{t^{k-2}}{(k-2)!} e^{\frac{-t}{\lambda}}, \tag{10}$$

which simplifies to

$$f_{T_w,p_{off}}(t) = e^{\frac{-T_{tr}}{\mu}} \delta(t) + \sqrt{\frac{T_{tr}}{\mu\lambda t}} e^{\frac{-T_{tr}}{\mu}} I_1 \left(2\sqrt{\frac{T_{tr} t}{\mu\lambda}} \right) e^{\frac{-t}{\lambda}}, \tag{11}$$

where $\delta(t)$ is the delta function. Note that the term $e^{\frac{-T_{tr}}{\mu}} \delta(t)$ corresponds to the the case that the number of waiting slots is equal to 0.

After substituting Eqs. (9) and (11) into (2), and noting $T_{ED} = T_w + T_{tr}$, the PDF for the EDT T_{ED} for continuous sensing case is given by

$$\begin{aligned} f_{T_{ED}}(t) =& u(t-T_{tr}) \frac{1}{\lambda+\mu} e^{\frac{-t}{\lambda}} \left[I_0 \left(2\sqrt{\frac{T_{tr}(t-T_{tr})}{\mu\lambda}} \right) + \sqrt{\frac{T_{tr}\mu}{\lambda(t-T_{tr})}} I_1 \left(2\sqrt{\frac{T_{tr}(t-T_{tr})}{\mu\lambda}} \right) \right] \\ &+ \frac{\mu}{\lambda+\mu} e^{\frac{-T_{tr}}{\mu}} \delta(t-T_{tr}), \end{aligned} \tag{12}$$

where $u(.)$ is the step function.

Fig. 4 plots the analytical expression for the PDF of the EDT with continuous sensing, given in Eq. (12). The corresponding plot for the simulation results is also shown. The perfect match between analytical and simulation results verify our analytical approach.

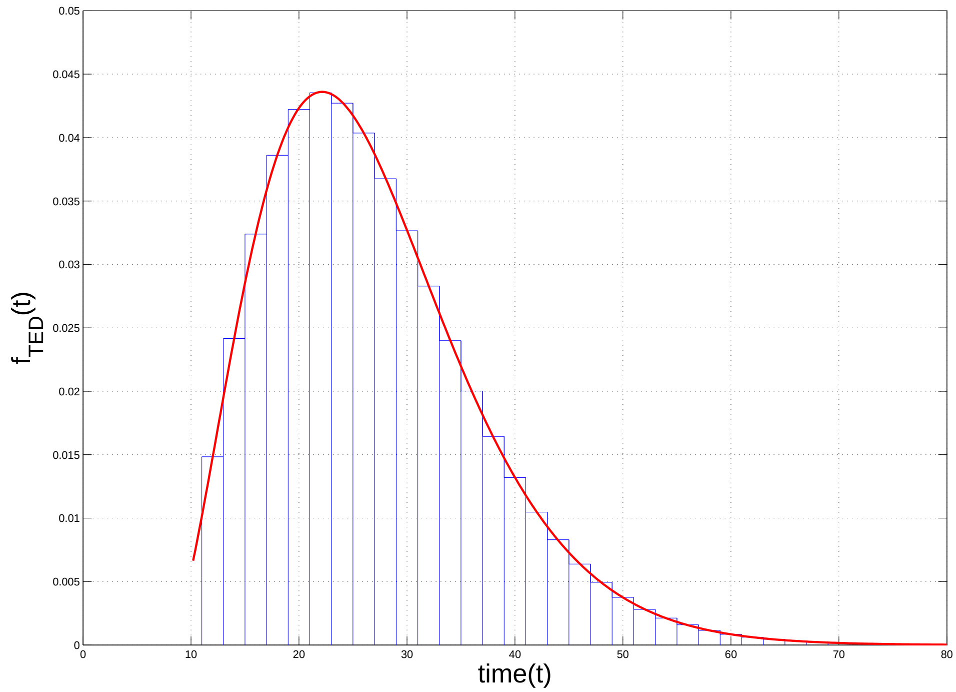

Fig. 4. Simulation verification for the analytical PDF of T_{ED} with continuous sensing ($T_{tr} = 10$, $\lambda = 3$, and $\mu = 2$).

B. Periodic Sensing

In the case of periodic sensing, the waiting time T_w will be a multiple of T_s, which is a known constant quantity. Therefore T_w will have a discrete distribution. Similar to continuous sensing case, we can write the probability that the waiting time T_w is nT_s by considering the PU is on or off at $t = 0$ separately, as

$$\Pr[T_w = nT_s] = \frac{\lambda}{\lambda+\mu}\Pr[T_w, p_{on} = nT_s] + \frac{\mu}{\lambda+\mu}\Pr[T_w, p_{off} = nT_s], \tag{13}$$

where $\Pr[T_w, p_{on} = nT_s]$ is the probability that the total waiting time for the SU is nT_s when PU is on at $t = 0$, and $\Pr[T_w, p_{off} = nT_s]$ the probability when PU is off at $t = 0$.

It can be shown based on the illustration in Fig. 2 that

$$\Pr[T_w, p_{on} = nT_s] = \sum_{k=1}^{\infty} \mathcal{P}_k \times \Pr[T_{w,k} = nT_s], \tag{14}$$

where $\mathcal{P}_k$ is the probability that the SU completes its transmission in k slots, given in Eq. (7), and $\Pr[T_{w,k} = nT_s]$ is the probability that the SU waiting time in k slots is nT_s, given by

$$\Pr[T_{w,k} = nT_s] = (1-\beta)^k (\beta)^{n-k} \binom{n-1}{k-1}, \tag{15}$$

where

$$\beta = \frac{\lambda}{\lambda+\mu} + \frac{\mu}{\lambda+\mu} e^{-(\frac{1}{\lambda}+\frac{1}{\mu})T_s}, \tag{16}$$

as shown in Appendix A. If we assume that T_s is very small, we can approximate β with $e^{\frac{-T_s}{\lambda}}$. After substituting Eqs. (7) and (15) into Eq. (14), and some manipulation, we can calculate $\Pr[T_w, p_{on} = nT_s]$ as

$$\Pr[T_w, p_{on} = nT_s] = (1-\beta)\beta^{n-1} e^{\frac{-T_{tr}}{\mu}} \times \sum_{0}^{n-1} \left[\frac{T_{tr}(1-\beta)}{\mu\beta}\right]^k \frac{1}{k!} \binom{n-1}{k}, \tag{17}$$

which simplifies to

$$\Pr[T_w, p_{on} = nT_s] = (1-\beta)\beta^{n-1} e^{\frac{-T_{tr}}{\mu}} \times {}_1F_1\left(1-n; 1; \frac{-T_{tr}(1-\beta)}{\mu\beta}\right), \tag{18}$$

where ${}_1F_1(.,.,.)$ is the generalized Hyper-geometric function. Similarly, $\Pr[T_w, p_{off} = nT_s]$ in Eq. (13) can be calculated as

$$\Pr[T_w, p_{off} = nT_s] = \sum_{k=1}^{\infty} \mathcal{P}_k \times \Pr[T_{w,k-1} = nT_s]. \tag{19}$$

After substituting Eqs. (7) and (15) into Eq. (19), and some manipulation, we get

$$\Pr[T_w, p_{off} = nT_s] = \left[\frac{T_{tr}(1-\beta)\beta^{n-1}}{\mu}\right] e^{\frac{-T_{tr}}{\mu}} \times \sum_{k=0}^{n-1} \left[\frac{T_{tr}(1-\beta)}{\mu\beta}\right]^k \frac{1}{(k+1)!} \binom{n-1}{k} + e^{-\frac{T_{tr}}{\mu}} \delta[n], \tag{20}$$

which eventually simplifies to

$$\begin{aligned} \Pr[T_w, p_{off} = nT_s] =& \left[\frac{T_{tr}(1-\beta)\beta^{n-1}}{\mu}\right] e^{\frac{-T_{tr}}{\mu}} \\ & \times {}_1F_1\left(1-n; 2; \frac{-T_{tr}(1-\beta)}{\mu\beta}\right) u[n-1] + e^{-\frac{T_{tr}}{\mu}} \delta[n]. \end{aligned} \tag{21}$$

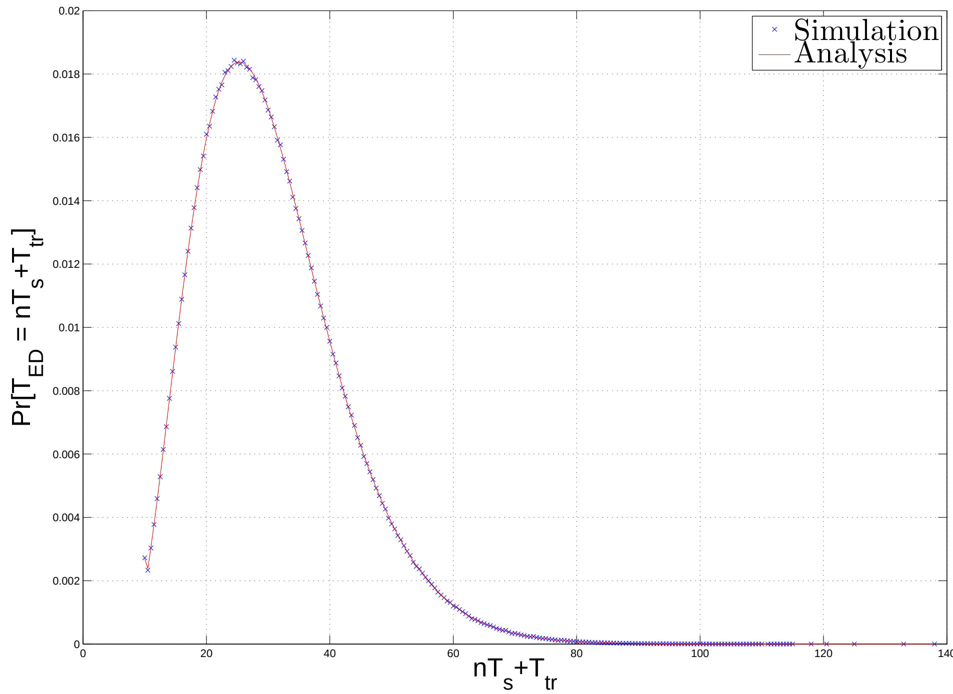

Fig. 5. Simulation verification of the analytical PMF of T_{ED} with periodic sensing ($T_{tr} = 10$, $\lambda = 3$, $\mu = 2$, and $T_s = 0.5$).

After substituting Eqs. (18) and (21) into Eq. (13), the probability mass function (PMF) of the EDT for periodic sensing case is given by

$$\Pr[T_{ED} = nT_s + T_{tr}] = \frac{\lambda}{\lambda+\mu}(1-\beta)\beta^{n-1}e^{\frac{-T_{tr}}{\mu}} \times {}_1F_1\left(1-n;1;\frac{-T_{tr}(1-\beta)}{\mu\beta}\right)u[n]$$
$$+\frac{\mu}{\lambda+\mu}\left[\left(\frac{T_{tr}(1-\beta)\beta^{n-1}}{\mu}\right)e^{\frac{-T_{tr}}{\mu}}{}_1F_1\left(1-n;2;\frac{-T_{tr}(1-\beta)}{\mu\beta}\right)u[n-1]+e^{-\frac{T_{tr}}{\mu}}\delta[n]\right]. \quad (22)$$

Fig. 5 plots the PMF of delivery time T_{ED} for periodic sensing case, and the corresponding simulation result. The plots show that the analytical results conform to the simulation results. Fig. 6 shows the PMF envelope of the packet delivery time with periodic sensing for various values of sensing period T_s. As can be seen, the performance of periodic sensing improves with reduction in the sensing interval T_s. As T_s approaches 0, the performance of periodic sensing comes close to that of continuous sensing, as expected.

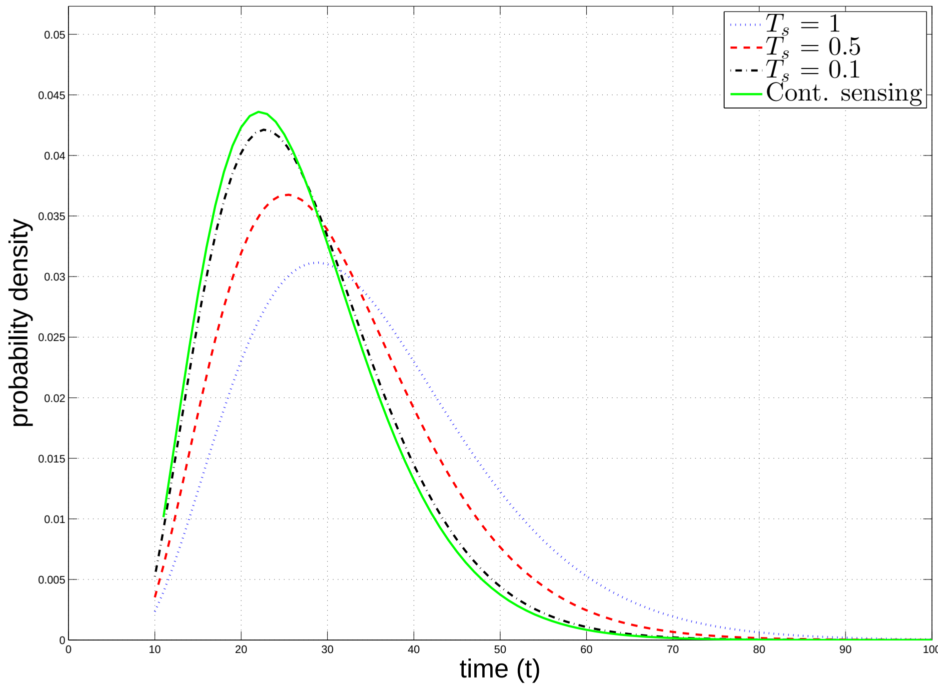

Fig. 6. Distribution of the EDT with continuous and periodic sensing ($T_{tr} = 10$, $\lambda = 3$, and $\mu = 2$).

C. *Short Packets*

In the earlier subsections, the packet transmission time T_{tr} was considered to be a constant depending on the average SNR, which applies to long packets and/or fast fading scenario. Now we consider the transmission of short packets, where T_{tr} depends on the instantaneous SNR of the secondary channel.

1) Constant SNR during packet transmission: For a short enough packet and/or slow fading scenario, the received SNR of the secondary channel γ can be assumed to be constant for the complete duration of the packet transmission. The transmission time T_{tr} will be a function of the received γ, defined as

$$T_{tr} = \frac{H}{W \log_2(1+\gamma)}, \tag{23}$$

where H is the entropy of the data packet in bits. Assuming a Rayleigh fading model for the secondary channel with an SNR PDF given by

$$f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{\frac{\gamma}{\bar{\gamma}}}, \tag{24}$$

the PDF of the transmission time T_{tr} can be derived as [24]

$$f_{T_{tr}}(T) = \frac{H}{\bar{\gamma} T^2} e^{\left[\frac{1}{\bar{\gamma}} + \frac{H}{T} - \frac{e^{\frac{H}{T}}}{\bar{\gamma}}\right]}, \tag{25}$$

where $\bar{\gamma}$ is the average link SNR. The exact distribution of the EDT of a single packet for the continuous sensing case can then be calculated as

$$f^{v,c}_{T_{ED}}(t) = \int_0^t f_{T_{ED}}(t|T_{tr}) \cdot f_{T_{tr}}(T_{tr}) dT_{tr}, \tag{26}$$

where $f_{T_{ED}}(.|T_{tr})$ is the conditional PDF of the EDT of the SU for a given T_{tr}, as given in Eq. (12). For the discrete sensing case, the exact distribution of EDT will be given by

$$f^{v,p}_{T_{ED}}(t) = \sum_{n=0}^{\lfloor \frac{t}{T_s} \rfloor} \Pr[T_{ED} = nT_s + T_{tr} | T_{tr} = t - nT_s] \times f_{T_{tr}}(t - nT_s), \tag{27}$$

where $\Pr[T_{ED} = nT_s + T_{tr} | T_{tr} = t - nT_s]$ is the probability that the EDT of a packet for given data transmission time T_{tr} is $nT_s + T_{tr}$, as defined in Eq. (22). More specifically, each summation term in the above equation refers to the probability that the SU waiting time is nT_s and the physical packet transmission time is $t - nT_s$.

Fig. 7 shows the numerically computed PDFs of the EDT for short packets for continuous sensing case and periodic sensing cases with $T_s = 0.1$, $T_s = 0.5$, and $T_s = 1$. As the periodic sensing interval approaches 0, the corresponding PDF curve comes closer to the PDF curve for continuous sensing.

2) One shot transmission: For a very short packet, where the packet transmission will complete in only one secondary transmission slot, the packet needs to wait for at most one slot. Using the PDF given Eq. (4) with $k = 1$, the PDF of the EDT for such packets with continuous

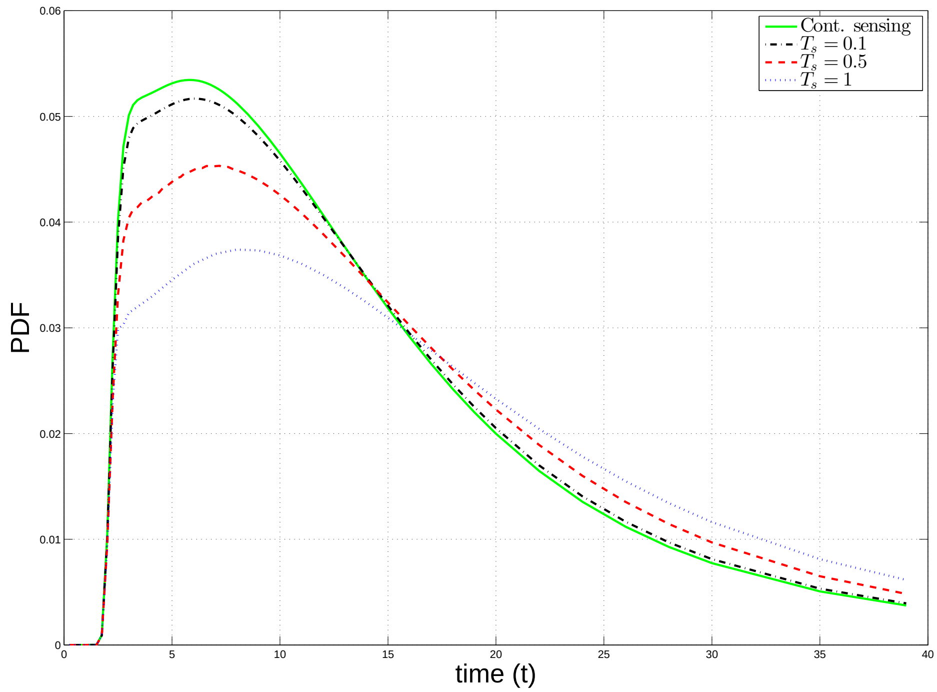

Fig. 7. PDF of EDT for short packets ($H = 100$, $W = 10$, $\bar{\gamma} = 8$ dB $\lambda = 3$, and $\mu = 2$).

sensing can be calculated as

$$f_{T_{ED}}^{o,c}(t) = \frac{\lambda}{\lambda+\mu}\int_0^t \frac{1}{\lambda} e^{\frac{-(t-T_{tr})}{\lambda}} f_{T_{tr}}(T_{tr}) dT_{tr} + \frac{\mu}{\lambda+\mu} f_{T_{tr}}(t). \tag{28}$$

Similarly, for periodic sensing case, using $\Pr[T_{w,k} = nT_s]$ from Eq. (15) with $k = 1$, the probability distribution function of the EDT for such packets is given by

$$f_{T_{ED}}^{o,p}(t) = \frac{\mu}{\lambda+\mu} f_{T_{tr}}(t) + \frac{\lambda}{\lambda+\mu} \sum_{n=1}^{\lfloor \frac{t}{T_s} \rfloor} (1-\beta)(\beta)^{n-1}.f_{T_{tr}}(t-n.T_s). \tag{29}$$

Fig. 8 displays the numerically computed PDFs of the EDT for very short packets for continuous sensing and periodic sensing cases. The first peak in all the curves correspond to the case that the incoming packet finds the PU to be off, and hence gets transmitted immediately. The oscillations seen in the curves for $T_s = 0.5$ and $T_s = 1$ can be attributed to sharp-peaked

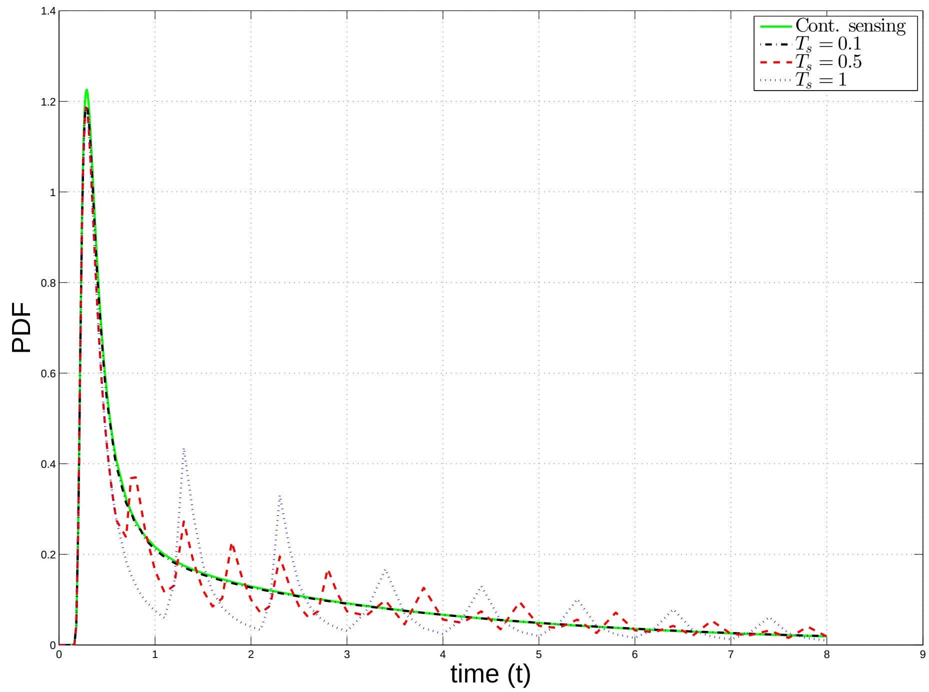

Fig. 8. PDF of EDT for one shot transmission ($H = 10$, $W = 10$, $\bar{\gamma} = 8$ dB $\lambda = 3$, and $\mu = 2$).

nature of the PDF of the transmission time T_{tr}, where each peak in the above curve corresponds to a different value of n in Eq. (29).

IV. Application to Secondary Queuing Analysis

In this section, we consider the transmission delay for the secondary system in a queuing set-up. In particular, the secondary traffic intensity is high and, as such, a first-in-first-out queue is introduced to hold packets until being transmitted. We assume that equal-sized packet arrival follows a Poisson process with intensity $\frac{1}{\psi}$, i.e. the average time duration between packet arrivals is ψ. For the sake of simplicity, the packets are assumed to be of the same long length, such that their transmission time T_{tr} is a fixed constant in the following analysis. As such, the secondary packet transmission can be modelled as a general M/G/1 queue, where the service time is closely related to the EDT we studied in the previous section.

Note also from the EDT analysis, the waiting time of a packet depends on whether the PU is on or off when the packet is available for transmission. As such, different secondary packets will experience two types of service time characteristics. Specifically, some packets might see that there are one or more packets waiting in the queue or being transmitted upon arrival. Such packets will have to wait in the queue until transmission completion of previous packets. Once all the previous packets are transmitted, the new arriving packet will find the PU to be off. We term such packets as type 1 packets. On the other hand, some packets will arrive when the queue is empty, and will immediately become available for transmission. Such packets might find the PU to be on or off. We will call this type of packets, type 2 packets. To facilitate subsequent queuing analysis, we now calculate the first and second moments of the service time for these two types of packets.

A. *Moments of packet service time*

1) Type 1 packets: The average service time of type 1 packets is equal to the EDT of packets that find PU off at the start of their transmission. Specifically, the first moment of the service time of type 1 packets with continuous sensing can be calculated as

$$E_1[t] = \int_{T_{tr}}^{\infty} t f_{T_w, p_{off}}(t - T_{tr}) dt \triangleq E^c_{off}[t], \tag{30}$$

where $E^c_{off}[t]$ denotes the average EDT of a packet that finds PU off at the start of their transmission for continuous sensing. Substituting Eq. (8) into Eq. (30) and carrying out integration while following the steps in Appendix B, we can obtain the closed-form expression of $E_1[t]$ as

$$E_1[t] = E^c_{off}[t] = T_{tr} + \lambda \left(\frac{T_{tr}}{\mu} \right). \tag{31}$$

Similarly, the second moment of service time for type 1 packet with continuous sensing can be calculated as

$$E_1[t^2] = E^c_{off}[t^2] = \lambda^2 \left[\left(\frac{T_{tr}}{\mu} \right)^2 + 2\frac{T_{tr}}{\mu} \right] + 2\frac{\lambda {T_{tr}}^2}{\mu} + {T_{tr}}^2. \tag{32}$$

With periodic sensing, the first and second moment of service time of type 1 packets can be calculated using $\Pr[T_w, p_{off} = nT_s]$ as

$$E_1[t] = T_s \sum_{n=1}^{\infty} nPr[T_w, p_{off} = nT_s] + T_{tr} \triangleq E^p_{off}[t], \tag{33}$$

and

$$E_1[t^2] = {T_s}^2 \sum_{n=1}^{\infty} n^2 Pr[T_w, p_{off} = nT_s] + 2T_s T_{tr} \sum_{n=1}^{\infty} nPr[T_w, p_{off} = nT_s] + {T_{tr}}^2 \triangleq E^p_{off}[t^2], \tag{34}$$

respectively, where $E^p_{off}[t]$ and $E^p_{off}[t^2]$ denote the average EDT of a packet that find PU off at the start of their transmission for periodic sensing. Following similar steps in Appendix C, we can obtain the following closed-form expressions of $E_1[t]$ and $E_1[t^2]$ for periodic sensing case

$$E_1[t] = E^p_{off}[t] = T_{tr}\left(1 + \frac{T_s}{\mu(1-\beta)}\right) \tag{35}$$

and

$$E_1[t^2] = E^p_{off}[t^2] = \frac{{T_s}^2}{(1-\beta)^2}\left[\left(\frac{T_{tr}}{\mu}\right)^2 + 2\frac{T_{tr}}{\mu}\right] - \frac{{T_s}^2}{1-\beta}\left(\frac{T_{tr}}{\mu}\right) + \frac{T_s}{1-\beta}\frac{2{T_{tr}}^2}{\mu} + {T_{tr}}^2. \tag{36}$$

2) Type 2 packets: Type 2 packets may find PU on or off at the start of their service upon arrival. Therefore, the service time of type 2 packets is the weighted average of the EDTs of packets that find PU on at the start of their transmission, and those that find PU off. Mathematically speaking, $E_2[t]$ and $E_2[t^2]$ can be calculated as

$$E_2[t] = P_{on,2} \cdot E_{on}[t] + (1 - P_{on,2}) \cdot E_{off}[t], \tag{37}$$

and

$$E_2[t^2] = P_{on,2} \cdot E_{on}[t^2] + (1 - P_{on,2}) \cdot E_{off}[t^2], \tag{38}$$

where $P_{on,2}$ denotes the probability that a type 2 packet finds PU on upon arrival, $E_{on}[t]$ and $E_{on}[t^2]$ are the first and second moments of the EDT of a packet that finds PU on at $t = 0$, respectively, and $E_{off}[t]$ and $E_{off}[t^2]$ are the moments for PU off case. In particular, $E_{on}[t]$

and $E_{on}[t^2]$ have been calculated for continuous sensing case in Appendix B, and for periodic sensing case in Appendix C.

The following argument will lead to the derivation of an expression for $P_{on,2}$. Whenever the transmission of the last packet in the queue is completed, due to the memoryless property of exponential distribution, the time that takes for the next packet to arrive will follow an exponential distribution with average ψ. At the start of that time interval, it is known that the PU is off. The probability that the PU is on, $P_{p_{on}}(t)$, conditioned on the time elapsed since the completion of last packet transmission, t, is given by [25]

$$P_{p_{on}}(t) = \Pr[\text{PU on at } t_0 + t \mid \text{PU off at } t_0] = \frac{\lambda}{\lambda + \mu}\left[1 - e^{-(\frac{1}{\lambda}+\frac{1}{\mu})t}\right]. \tag{39}$$

Removing the conditioning on t, the probability for the PU being on when a type 2 packet arrives, $P_{on,2}$, is obtained as

$$P_{on,2} = E[P_{p_{on}}(t)] = \int_0^\infty P_{p_{on}}(t) \cdot \frac{1}{\psi} e^{-\frac{t}{\psi}} dt. \tag{40}$$

It can be shown that the above simplifies to

$$P_{on,2} = \frac{\lambda\psi}{\lambda\psi + \lambda\mu + \mu\psi}. \tag{41}$$

Substituting the moments $E_{on}[t]$, $E_{on}[t^2]$, $E_{off}[t]$, and $E_{off}[t^2]$ for continuous and periodic sensing cases, and $P_{on,2}$ into Eqs. (37) and (38), we can obtain the moments of type 2 packet service time. As an example, the first moment of the service time for type 2 packets with continuous sensing is given, after substituting Eqs. (31), (41), and (64) into Eq. (37), by

$$E_2[t] = \frac{\lambda^2\psi}{\lambda\psi + \lambda\mu + \mu\psi} + \left(1 + \frac{\lambda}{\mu}\right) T_{tr}. \tag{42}$$

The other moments can be similarly obtained.

B. Queuing Analysis

In this subsection, we derive the expression for the expected delay for a packet in the queue. For clarity, we focus on continuous sensing in the following. The expression for periodic sensing can be similarly obtained. The average total delay is given by

$$E[D] = E[t] + E[Q], \tag{43}$$

where $E[t]$ is the average service time of an arbitrary packet, and $E[Q]$ is the average wait time in the queue. $E[t]$ is a weighted average of $E_1[t]$ and $E_2[t]$, as defined in Eqs. (31) and (37), respectively, given by

$$E[t] = (1 - p_0) \cdot E_1[t] + p_0 \cdot E_2[t], \tag{44}$$

where p_0 is the probability of the queue being empty at any given time instance and $1 - p_0$ is the utilization factor of the queue, which is, in turn, related to $E[t]$ as

$$1 - p_0 = \frac{E[t]}{\psi}. \tag{45}$$

Simultaneously solving Eqs. (44) and (45), we can obtain $E[t]$ and p_0 as

$$E[t] = \frac{\psi E_2[t]}{\psi + E_2[t] - E_1[t]}, \tag{46}$$

and

$$p_0 = \frac{\psi - E_1[t]}{\psi + E_2[t] - E_1[t]}, \tag{47}$$

respectively.

The average delay in the queue, $E[D]$, can be calculated using the mean value technique [26] as

$$E[Q] = E[N_Q] \cdot E_1[t] + (1 - p_0) \cdot E[R], \tag{48}$$

where $E[N_Q]$ is the average number of packets waiting in the queue, not including the current packet in service, $E_1[t]$ is the average service time of a packet in the queue (type 1 packet), and $E[R]$ is the mean residual time of the packet currently being served. Specifically, the first addition term in corresponds to the average total service time of the packets currently waiting in the queue, if any, and the second term to the waiting time for the currently served packet, if any. Given that a packet is being served at a given instance, the probabilities that the packet is a type 1 packet or type 2 packet, are equal to $\frac{(1-p_0)E_1[t]}{(1-p_0)E_1[t]+p_0E_2[t]}$ and $\frac{p_0E_2[t]}{(1-p_0)E_1[t]+p_0E_2[t]}$, respectively. Therefore, mean residual service time, $E[R]$ can be calculated as

$$E[R] = \frac{(1-p_0)E_1[t]}{(1-p_0)E_1[t]+p_0E_2[t]} \cdot E[R_1] + \frac{p_0E_2[t]}{(1-p_0)E_1[t]+p_0E_2[t]} \cdot E[R_2], \tag{49}$$

where $E[R_1]$ and $E[R_2]$ are the mean residual times for type 1 and type 2 packets, respectively, defined by [27]

$$E[R_1] = \frac{E_1[t^2]}{2E_1[t]}, \tag{50}$$

and

$$E[R_2] = \frac{E_2[t^2]}{2E_2[t]}. \tag{51}$$

Recalling the Little's law stating that

$$E[N_Q] = \frac{E[Q]}{\psi}, \tag{52}$$

$E[Q]$ can be obtained after much simplification as

$$E[Q] = \frac{E[t^2]}{2(\psi - E_1[t])}, \tag{53}$$

where $E[t^2]$ is the second moment of the average service time of all packets, defined by

$$E[t^2] = \frac{(\psi - E_1[t]) \cdot E_2[t^2] + E_2[t] \cdot E_1[t^2]}{\psi + E_2[t] - E_1[t]}. \tag{54}$$

The average number of packets waiting in the queue, not including the packet currently in transmission, is given by

$$E[N_Q] = \frac{E[t^2]}{2\psi(\psi - E_1[t])}. \tag{55}$$

Finally, the average total delay for secondary packets can be simply expressed as

$$E[D] = \frac{\psi E_2[t]}{\psi + E_2[t] - E_1[t]} + \frac{E[t^2]}{2(\psi - E_1[t])}. \tag{56}$$

Fig. 9 shows the variation of average total delay against the arrival rate of data packet with continuous sensing. The graph is based on the assumption that the average delay between packet arrival is greater than the average service time of the SU, as otherwise the queue will become

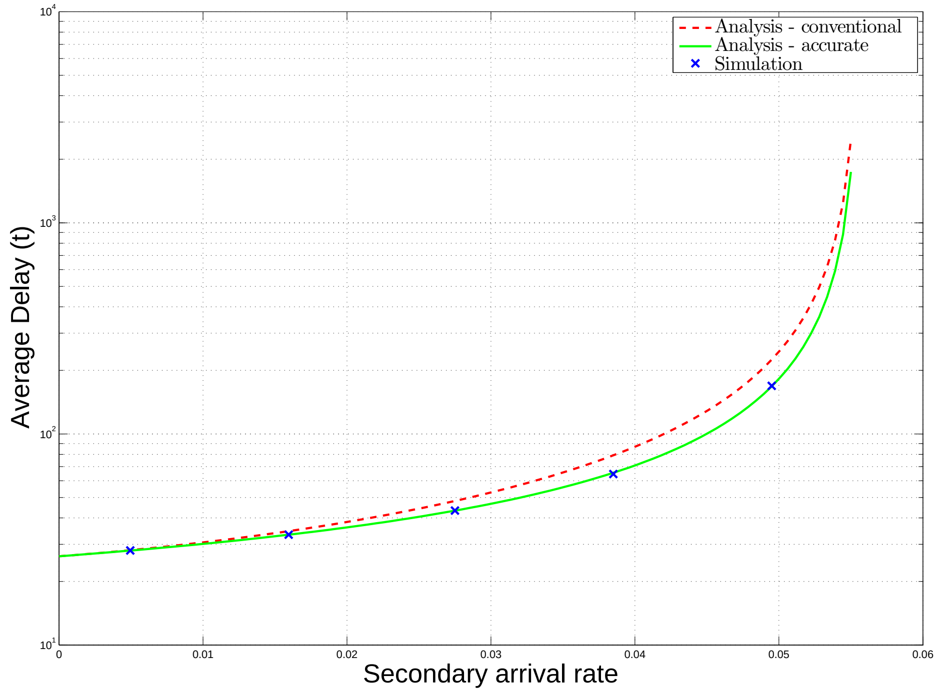

Fig. 9. Simulation verification for the analytical average queuing delay with continuous sensing ($T_{tr} = 3$, $\lambda = 10$, and $\mu = 2$)

unstable. For comparison purposes, we have included the average queuing delay obtained by modeling the secondary queue by conventional M/G/1 queue with service time moments simply given by Eqs. (44) and (54), $E[D] = E[t] + \frac{E[t^2]}{2(\psi - E[t])}$. The simulation results clearly show our analytical approach is more accurate.

Fig. 10 shows the variation of average delay including the queuing delay against the arrival rate of data packet. As the periodic sensing interval becomes small, the periodic sensing curves converge to the continuous sensing curve.

V. Conclusion

This paper studied the extended delivery time of a data packet appearing at the secondary user in an interleave cognitive setup. Exact analytical results for the probability distribution of the

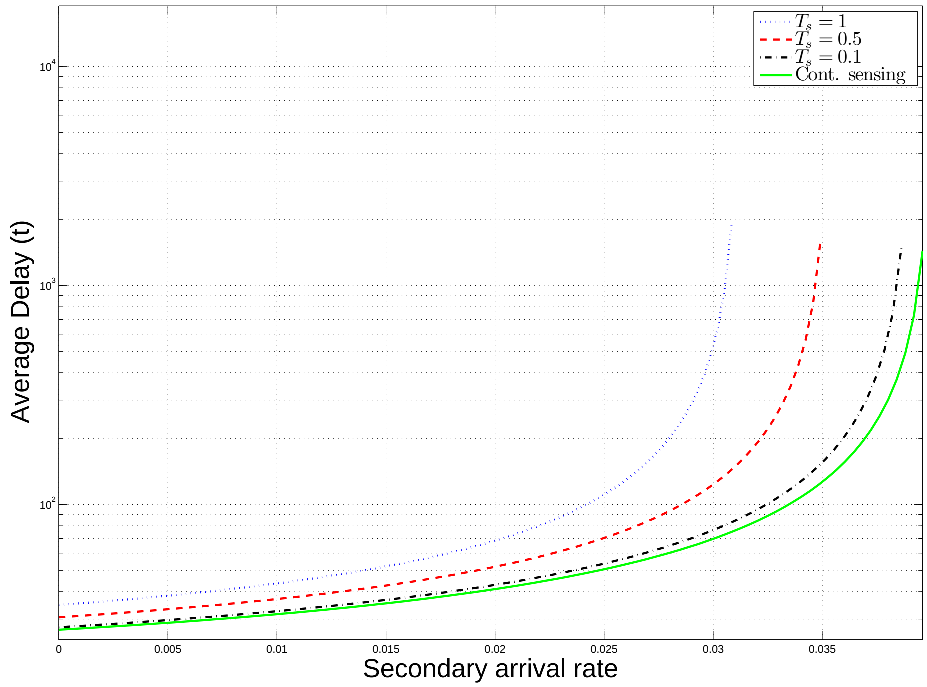

Fig. 10. Average queuing delay ($T_{tr} = 10$, $\lambda = 3$, and $\mu = 2$).

EDT for a fixed-size data packet were obtained for both continuous sensing and periodic sensing. These results were then applied to analyze the expected delay of a packet at SU in a queuing setup. Simulation results were presented to verify the analytical results. These analytical results will facilitate the design and optimization of secondary systems for diverse target applications. Ongoing effort is being carried out to extend the analysis to multiple primary channels scenario with the consideration of imperfect sensing.

APPENDIX A

In this appendix, we derive the expression for the probability that the total SU waiting time over k waiting slots is equal to nT_s for the periodic sensing case, $\Pr[T_{w,k} = nT_s]$. $\Pr[T_{w,k} = nT_s]$

can be equivalently calculated as the probability that it takes n sensing instances to find exactly k times that the PU is off. Applying the result of negative binomial distribution, we can calculate $\Pr[T_{w,k} = nT_s]$ as

$$\Pr[T_{w,k} = nT_s] = (1-\beta)^k (\beta)^{n-k} \binom{n-1}{k-1}, \tag{57}$$

where β is the probability that primary user is on at the sensing instant, which, due to the memoryless property of exponential distribution, is a constant. Since the SU only senses every T_s time period, there is chance that the PU turns off and then back on between two sensing instants. To account for such possibilities, we can model PU activity with a continuous-time Markov process, where the transition rates are given by $\frac{1}{\lambda}$ and $\frac{1}{\mu}$. Given that the PU was on at a given time instant, the probability that the PU is still on after time T_s is given by [25]

$$\beta = \frac{\lambda}{\lambda+\mu} + \frac{\mu}{\lambda+\mu} e^{-(\frac{1}{\lambda}+\frac{1}{\mu})T_s}. \tag{58}$$

The above definition is valid for any value of $T_s > 0$. If we assume that T_s is small, and the chance of the PU turning back on again before the SU senses a free channel is negligible, we can approximately use $\beta = e^{\frac{-T_s}{\lambda}}$.

APPENDIX B

In this appendix, we present the calculation for the first and second moments of the EDT of packets that find PU on at the start of their service for the continuous sensing case, $E^c_{on}[t]$ and $E^c_{on}[t^2]$. Similar steps can be used to compute $E^c_{off}[t]$ and $E^c_{off}[t^2]$, as defined in section IV-A.

The first moment of the EDT for the case when PU is on at $t = 0$, $E^c_{on}[t]$, is defined as

$$E^c_{on}[t] = \int_{T_{tr}}^{\infty} t f_{T_w,p_{on}}(t - T_{tr}) dt. \tag{59}$$

With a change of variable $s = t - T_{tr}$ we get

$$E^c_{on}[t] = \int_{0}^{\infty} (s + T_{tr}) f_{T_w,p_{on}}(s) ds. \tag{60}$$

After substituting Eq. (8) and some manipulation, we arrive at

$$E^c_{on}[t] = T_{tr} + e^{\frac{-T_{tr}}{\mu}} \lambda \sum_{k=1}^{\infty} \frac{{T_{tr}}^{k-1}}{\mu^{k-1}(k-1)!(k-1)!} \times \int_0^{\infty} \frac{s^k e^{\frac{-s}{\lambda}}}{\lambda^{k+1}} ds. \tag{61}$$

Applying the definition of the standard Gamma function, the above expression becomes

$$E^c_{on}[t] = T_{tr} + e^{\frac{-T_{tr}}{\mu}} \lambda \sum_{k=1}^{\infty} \frac{{T_{tr}}^{k-1} k}{\mu^{k-1}(k-1)!}. \tag{62}$$

With a change in summation variable, and after some manipulation, we get

$$E^c_{on}[t] = T_{tr} + e^{\frac{-T_{tr}}{\mu}} \lambda \left[\sum_{k=1}^{\infty} \frac{{T_{tr}}^{k}}{\mu^{k}(k-1)!} + \sum_{k=0}^{\infty} \frac{{T_{tr}}^{k}}{\mu^{k}, k!} \right], \tag{63}$$

which finally simplifies to

$$E^c_{on}[t] = T_{tr} + e^{\frac{-T_{tr}}{\mu}} \lambda \left(\frac{T_{tr}}{\mu} \right) e^{\frac{T_{tr}}{\mu}} + e^{\frac{-T_{tr}}{\mu}} \lambda e^{\frac{T_{tr}}{\mu}} = T_{tr} + \lambda \left(1 + \frac{T_{tr}}{\mu} \right). \tag{64}$$

The second moment, $E^c_{on}[t^2]$ is defined as

$$E^c_{on}[t^2] = \int_{T_{tr}}^{\infty} t^2 f_{T_w,p_{on}}(t - T_{tr}) dt. \tag{65}$$

With a change of variable $s = t - T_{tr}$, we get

$$E^c_{on}[t^2] = {T_{tr}}^2 + 2T_{tr}(E^c_{on}[t] - T_{tr}) + \int_0^{\infty} s^2 f_{T_w,p_{on}}(s) ds. \tag{66}$$

Following the similar derivation steps for $E^c_{on}[t]$, we get

$$\begin{aligned} E_{on}[t^2] =& {T_{tr}}^2 + 2T_{tr}(E^c_{on}[t] - T_{tr}) + e^{\frac{-T_{tr}}{\mu}} \lambda^2 \sum_{k=0}^{\infty} \left(\frac{T_{tr}}{\mu} \right)^{k+2} \frac{1}{k!} \\ &+ 4e^{\frac{-T_{tr}}{\mu}} \lambda^2 \sum_{k=0}^{\infty} \left(\frac{T_{tr}}{\mu} \right)^{k+1} \frac{1}{k!} + 2e^{\frac{-T_{tr}}{\mu}} \lambda^2 \sum_{k=0}^{\infty} \left(\frac{T_{tr}}{\mu} \right)^{k} \frac{1}{k!}. \end{aligned} \tag{67}$$

Replacing the summation by the natural exponent, we arrive at the following closed-form expression

$$E_{on}[t^2] = \lambda^2 \left[\left(\frac{T_{tr}}{\mu} \right)^2 + 4\frac{T_{tr}}{\mu} + 2 \right] + 2\lambda T_{tr} \left[1 + \frac{T_{tr}}{\mu} \right] + {T_{tr}}^2. \tag{68}$$

APPENDIX C

In this appendix, we derive expressions for the first and second moments of the EDT of packets that find PU on at the start of their service for the periodic sensing case, $E^p_{on}[t]$ and $E^p_{on}[t^2]$. Similar steps can be used to compute $E^p_{off}[t]$ and $E^p_{off}[t^2]$, as defined in section IV-A.

When PU is on at $t=0$, the first moment of the EDT, $E^p_{on}[t]$, is defined as

$$E^p_{on}[t] = E[nT_s + T_{tr}] = T_s E^p_{on}[n] + T_{tr}, \tag{69}$$

where

$$E^p_{on}[n] = \sum_{n=1}^{\infty} nPr[T_w, p_{on} = nT_s]. \tag{70}$$

After substituting Eq. (17) and some manipulation, we obtain

$$E^p_{on}[n] = e^{\frac{-T_{tr}}{\mu}} \frac{1}{(1-\beta)} \sum_{k=0}^{\infty} \left[\left(\frac{T_{tr}}{\mu}\right)^k \frac{k+1}{k!} \times \sum_{n=k+1}^{\infty} (1-\beta)^{k+2} \beta^{n-k-1} \frac{n!}{(n-1-k)!(k+1)!} \right]. \tag{71}$$

Noting that the second summation is equal to 1, the above expression can be written as

$$E^p_{on}[n] = e^{\frac{-T_{tr}}{\mu}} \frac{1}{(1-\beta)} \sum_{k=0}^{\infty} \left[\left(\frac{T_{tr}}{\mu}\right)^k \frac{k+1}{k!} \right], \tag{72}$$

which simplifies to

$$E^p_{on}[n] = \frac{1}{1-\beta}\left(1 + \frac{T_{tr}}{\mu}\right). \tag{73}$$

Thus, $E^p_{on}[t]$ can be finally expressed as

$$E^p_{on}[t] = T_{tr} + \frac{T_s}{1-\beta}\left(1 + \frac{T_{tr}}{\mu}\right) \tag{74}$$

The second moment, $E^p_{on}[t^2]$, is computed as

$$E_{on}[t^2] = E[(nT_s + T_{tr})^2] = {T_s}^2 E^p_{on}[n^2] + 2T_s T_{tr} E^p_{on}[n] + {T_{tr}}^2, \tag{75}$$

where $E^p_{on}[n]$ is given in Eq. (73), and

$$E^p_{on}[n^2] = \sum_{n=1}^{\infty} \left[n^2 (1-\beta)\beta^{n-1} e^{\frac{-T_{tr}}{\mu}} \times \sum_{k=0}^{n-1} \left(\frac{T_{tr}(1-\beta)}{\mu\beta}\right)^k \frac{1}{k!} \binom{n-1}{k} \right]. \tag{76}$$

Changing the sequence of the two summations and applying $n^2(n-1)! = (n+1)! - n!$, we obtain

$$E^p_{on}[n^2] = \sum_{k=0}^{\infty}\Bigg[(1-\beta)\left(\frac{T_{tr}(1-\beta)}{\mu\beta}\right)^k e^{\frac{-T_{tr}}{\mu}}\frac{1}{k!} \times\left(\sum_{n=k+1}^{\infty}\beta^{n-1}\frac{(n+1)!}{(n-1-k)!k!} - \sum_{n=k+1}^{\infty}\beta^{n-1}\frac{n!}{(n-1-k)!k!}\right)\Bigg]. \quad (77)$$

Using the similar manipulations for the calculation of $E^p_{on}[n]$, we obtain

$$E^p_{on}[n^2] = \sum_{k=0}^{\infty}\left[\left(\frac{T_{tr}}{\mu}\right)^k e^{\frac{-T_{tr}}{\mu}} \times \left(\frac{(k+1)(k+2)}{(1-\beta)^2 k!} - \frac{(k+1)}{(1-\beta)k!}\right)\right]. \quad (78)$$

With further manipulation on the factorial terms, we arrive at

$$E^p_{on}[n^2] = \frac{e^{\frac{-T_{tr}}{\mu}}}{(1-\beta)^2}\left[\sum_{k=2}^{\infty}\frac{1}{(k-2)!}\left(\frac{T_{tr}}{\mu}\right)^k + 4\sum_{k=1}^{\infty}\frac{1}{(k-1)!}\left(\frac{T_{tr}}{\mu}\right)^k + 2\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{T_{tr}}{\mu}\right)^k\right] - \frac{e^{\frac{-T_{tr}}{\mu}}}{(1-\beta)}\left[\sum_{k=1}^{\infty}\frac{1}{(k-1)!}\left(\frac{T_{tr}}{\mu}\right)^k + \sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{T_{tr}}{\mu}\right)^k\right], \quad (79)$$

which finally simplifies to

$$E^p_{on}[n^2] = \frac{1}{(1-\beta)^2}\left[\left(\frac{T_{tr}}{\mu}\right)^2 + 4\frac{T_{tr}}{\mu} + 2\right] - \frac{1}{(1-\beta)}\left[\frac{T_{tr}}{\mu} + 1\right]. \quad (80)$$

Thus the second moment, $E_{on}[t^2]$, can be calculated in the following closed-form expression

$$E^p_{on}[t^2] = \frac{{T_s}^2}{(1-\beta)^2}\left[\left(\frac{T_{tr}}{\mu}\right)^2 + 4\frac{T_{tr}}{\mu} + 2\right] + \frac{T_s}{1-\beta}(2T_{tr} - T_s)\cdot\left[1 + \frac{T_{tr}}{\mu}\right] + {T_{tr}}^2. \quad (81)$$

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