MAXIMALLY FAST RECURSIVE ALGORITHMS

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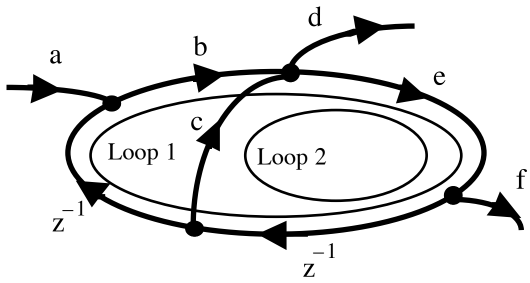
1. INTRODUCTION

Every recursive algorithm has a maximum sample frequency determined by the computational loops in the algorithm. The algorithm may, however, be modified in order to reach the true maximal sample frequency. In this paper, we introduce a technique using the distributive and associative properties of the operations to modify the algorithms.

2. MAXIMALLY FAST RECURSIVE ALGORITHMS

The minimal sample period for a recursive algorithm that is described by a fully specified signal-flow graph is:

$$T_{min} = \max_i \left\{ \frac{T_{opi}}{N_i} \right\}$$



where

T_{opi} is the total delay due to the arithmetic operations, etc. in the directed loop i.

N_i is the number of delay elements in the directed loop i.

The loops that yield T_{min} are called critical loops.

3. INCREASING THE MAXIMUM SAMPLE RATE

The minimal sample period can be reduced by the use of the distributive and associative laws to alter the expressions involved in the critical loop. The aim of this alteration is to move some of the operation outside the loop. However, the modifications may not change the numerical properties of the original algorithm. It is often possible to reduce the critical loop to a single addition and multiplication. This alteration does, however, change the surrounding operations and this may introduce new critical loops.

Distributive: x*(y+z) = x*y + x*z

Associative: (x + y) + z = x + (y + z)

It is also possible to apply some algorithm transformation to get faster algorithms [2], but the numerical properties are changed. This is unfortunately not acceptable in many applications.

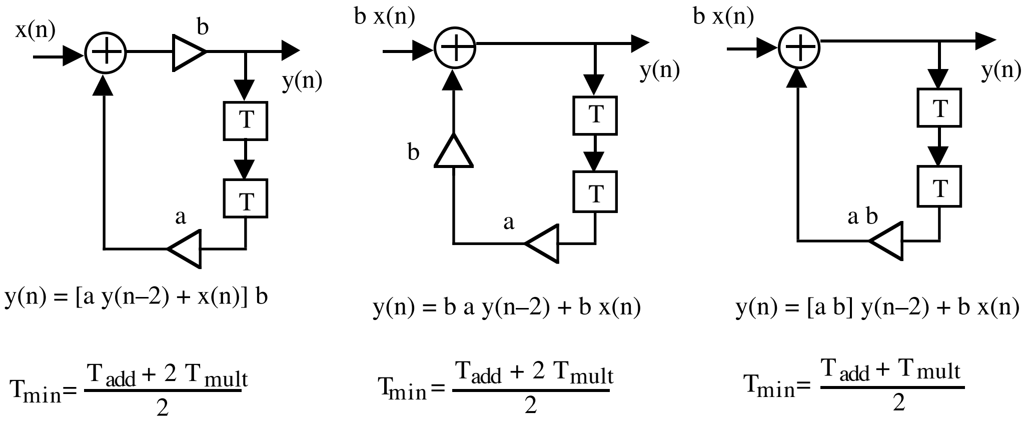
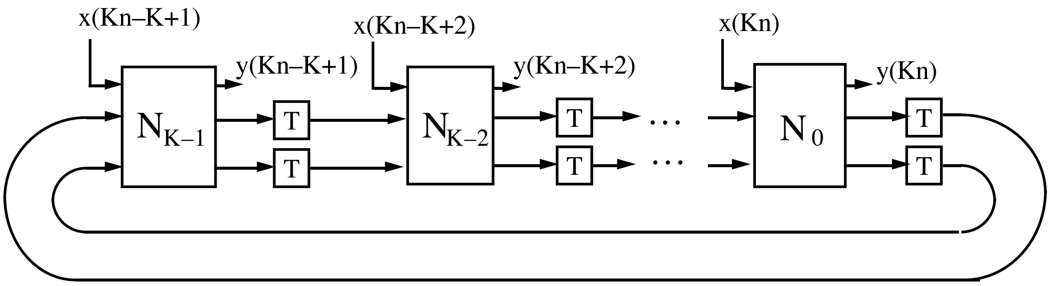


Illustration of the use of distributive and associative properties.

4. SCHEDULING FORMULATION FOR MAXIMAL SAMPLE FREQUENCY

The recursive property of the algorithm must be exploited in order to reach the maximal sample frequency. The operation schedule must be done over multiple sample periods. This is accomplished by expanding the signal-flow graph over multiple sample periods. This general periodic scheduling formulation allows one operation to be mapped onto multiple resources that are working concurrently. This scheduling function is necessary in order to reach maximally fast implementations if the critical loop contains more than one delay element but also in most cases when the loop has only one delay element.



5. BIT-SERIAL ARITHMETIC

The first bit of the result of a bit serial calculation using least significant bit first will be available before the calculation is complete. The next operation can therefore start immediately using this bit. The same operation in the next sample period may thereby have its input data available before the previous operation is completed. This property can be exploited by the periodic scheduling. Multiple processing elements is automatically induced by the schedule thereby allowing the immediate usage of a calculated result.

REFERENCES

[1] Renfors M. and Neuvo Y.: The Maximal Sampling Rate of Digital Filters Under Hardware Speed Constraints, IEEE Trans. on Circuits and Systems, CAS-28, No. 3, pp. 196-202, March 1981.

[2] Wanhammar L.: *System Design: DSP Integrated Circuits*, Prentice-Hall, 1994. (in preparation).