Energy Efficiency Analysis of Small Cell Networks

Chang Li, Jun Zhang, and K. B. Letaief, *Fellow, IEEE*
Dept. of ECE, The Hong Kong University of Science and Technology
Email: {changli, eejzhang, eekhaled}@ust.hk

***Abstract*—Small cell networks have recently been proposed as an important evolution path for the next-generation cellular networks. While such approach has the potential of meeting the growing network throughput requirement, the energy efficiency of small cell networks is of great concern as the base station (BS) density will be significantly increased. The objective of this paper is to analyze the energy efficiency in small cell networks. To do so, we adopt a random spatial network model, where BSs and users are modeled as two independent spatial Poisson point processes (PPPs). We shall derive analytical results for the network energy efficiency, which show that the BS power consumption model plays a critical role. In particular, it will be shown that increasing the BS density can actually improve the energy efficiency if the BS power consumption that is not related to signal transmission is less than a certain threshold. By comparing the cases between single-antenna and multi-antenna BSs, we find that single-antenna BSs provide a higher energy efficiency if the circuit power is larger than a threshold. Simulation results will demonstrate that our conclusions which are based on the random network model also hold in a regular grid-based model.**

***Keywords*: Green communications, energy efficiency, cellular networks, Poisson point process.**

I. Introduction

With the increasing demand for high throughput wireless services, cellular networks are evolving from the conventional structure with high-power macro base stations (BSs) each covering a large geographic area to a small cell structure with low-power BSs such as micro, pico- and femto-BSs. With a higher BS density and short transmission distances, small cell networks can provide more uniform and higher data rates to mobile users [1]. Meanwhile, given the growing concerns over global warming, a more energy efficient communication network is desirable. It was pointed out that BSs contribute nearly 60% of the total energy consumption in cellular networks [2]. Hence, more efforts should be put to improve the BS energy efficiency. Compared with a macro BS, the power consumption for a micro, pico- or femto-BS is much lower. However, due to the high BS density in small cell networks, it is not clear whether it will be more energy efficient than the conventional network structure.

Previous research on energy-efficient communications, i.e., green communications, mainly focused on a point-to-point link [3]–[5], while inter-cell interference is largely ignored. Energy efficiency in a cellular setting was investigated in [6], where different cellular network architectures were compared, mainly through simulation. While evaluating the network performance through simulation can provide insights on some specific settings, the results may not be extended to other scenarios and the computational complexity is quite high.

The lack of analytical results for the network energy efficiency is due to the complexity of the cellular network topologies that mainly include irregular BS positions, pathloss, small-scale and large-scale fading. Conventionally, cellular network analysis is based on the hexagonal model, which becomes intractable as the network size grows. Another common model is the Wyner model, which is a simplified and tractable model but may lose essential characteristics of real and practical networks [7]. Recently, a random spatial model was proposed where BSs are modeled as a spatial Poisson point process (PPP) [8]. With the help of stochastic geometry, this model was shown to be tractable and can be used to analyze the outage probability and coverage in cellular networks. In the original model, it was assumed that each BS always has some users to serve, and all the analyses focused on a typical user. Essentially, this model only considers the spatial distribution of BSs, while the density and distribution of users are irrelevant. However, in a practical network, not all BSs are always active, i.e., there may be no user in some cells, and this will affect the interference and subsequently the performance of the network. Such effect becomes more prominent as the BS density increases. In more recent works [9], [10], the BSs and users were modeled as two independent PPPs. By doing so the user density effects can be analyzed. However, all of these works assume single-antenna BSs and more importantly the network energy efficiency has not been analyzed.

In this paper, we will analyze network energy efficiency in a small cell network, where BSs and users are modeled as two independent homogeneous spatial PPPs. We first derive a closed-form expression for the successful transmission probability, which is the probability that a typical user can be successfully served by its associated BS. We will then analyze the network energy efficiency, with a focus on the effect of the BS density and the number of BS antennas. Our results will show that the BS power consumption model plays a critical role, and that in general we need to make a tradeoff between the network energy efficiency and throughput. While increasing the BS density always increases throughput, it can also increase energy efficiency if the ratio of non-transmission power to the total BS power is less than a threshold. Similarly, adding more antennas at each BS can also increase throughput, but if the circuit power of each radio frequency (RF) chain is larger than a threshold, deploying single-antenna BSs provides

This work is supported in part by the Hong Kong Research Grant Council under Grant No. 610212.

higher energy efficiency than multi-antenna BSs. Through simulations, we show that our conclusions based on the random network model also hold in a grid-based model, which demonstrates the generality of our approach.

II. System model

A. Network Model

We consider a cellular network where BSs and users are distributed spatially according to two independent homogeneous PPPs in $\mathbb{R}^2$, denoted as Φ_b and Φ_u respectively. Denote the BS density as λ_b and the user density as λ_u. Such a random spatial network model is well suited for small cell networks, where BS positions are becoming irregular. We consider the downlink transmission and assume that each user is served by the nearest BS, which results a Voronoi tesselation relative to Φ_b. Due to the independent locations of BSs and users, there may be some BSs that do not have any user to serve. These BSs are called *inactive* BSs and will not transmit any signals. A typical BS will have a certain probability to be *active*, i.e., have some users to serve, and we denote this BS active probability as p_{a}. Equivalently, p_{a} can be regarded as the ratio of the number of active BSs to the total number of BSs. It is shown in [9] that

$$p_{\mathrm{a}} = 1 - \left(1 + \frac{\lambda_u}{3.5\lambda_b}\right)^{-3.5}. \tag{1}$$

An active BS may have more than one user in its cell, and in this case the BS will randomly choose one user to serve at each time slot, i.e., intra-cell TDMA is adopted.

Different BSs may be of different types, such as macro BSs, pico- or femto-BSs. We assume each BS is equipped with M antennas, while each user has a single antenna. As the optimal usage of multiple antennas in small cell networks is unknown, we consider a simple transmission scheme, where each active BS transmits to its own user with maximal ratio transmission (MRT) beamforming. BS cooperation is not considered as we assume that the backhaul links between different BSs are of very limited capacity. We consider universal frequency reuse, so each user will receive information from its home BS, but also suffer interference from all the other active BSs. For a typical user, denoted as the 0th user, the discrete-time received signal is given by

$$y_0 = r_0^{-\frac{\alpha}{2}} \mathbf{h}_{00}^{\dagger} \mathbf{w}_0 \sqrt{P_{\mathrm{t}}} s_0 + \sum_{i \neq 0} R_{i0}^{-\frac{\alpha}{2}} \mathbf{h}_{i0}^{\dagger} \mathbf{w}_i \sqrt{P_{\mathrm{t}}} s_i + n_0, \tag{2}$$

where we consider both small-scale Rayleigh fading and large-scale pathloss between BSs and users. Thus $\mathbf{h}_{i0} \sim \mathcal{CN}(0, I)$, r_0 is the distance between the 0th BS to the 0th user, while R_{i0} is the distance from the ith BS to the 0th user. The pathloss exponent is α, the precoding vector is $\mathbf{w}_i = \frac{\mathbf{h}_{ii}}{\|\mathbf{h}_{ii}\|}$, P_{t} is the transmit power, and n_0 denotes the additive white Gaussian noise (AWGN) at the receiver. Then the receive signal-to-interference plus noise ratio (SINR) is given by

$$\mathrm{SINR} = \frac{P_{\mathrm{t}} g_{00} r_0^{-\alpha}}{\sum_{i \in \tilde{\Phi}_b \backslash 0} P_{\mathrm{t}} g_{i0} R_{i0}^{-\alpha} + \sigma_n^2}, \tag{3}$$

where $\tilde{\Phi}_b$ is the thinned PPP with density $\lambda_b p_{\mathrm{a}}$ representing those active BSs, and g_{i0} is the channel gain from the ith BS to the 0th user, i.e., $g_{00} = \|\mathbf{h}_{00}\|^2 \sim \mathrm{Gamma}(1, M)$ and $g_{i0} = \mathbf{h}_{i0}^{\dagger} \frac{\mathbf{h}_{ii}}{\|\mathbf{h}_{ii}\|} \sim \mathrm{Exp}(1)$ for $i \neq 0$ [11].

B. The BS Power Consumption Model

In a cellular network, most of the power consumption is due to BSs [2]. Therefore, we consider BS power consumption, denoted as P_{BS}, to evaluate network energy efficiency. In practice, transmit power P_{t} is only one part of the total BS power consumption. To take other power consumption into consideration, we adopt a linear BS power consumption model, given by

$$P_{\mathrm{BS}} = \frac{1}{\eta} P_{\mathrm{t}} + M P_{\mathrm{c}} + P_0, \tag{4}$$

which is widely utilized in the literature and standards organizations [12], [13]. Here η denotes the power amplifier efficiency, P_{c} accounts for the circuit power of the corresponding RF chain, and P_0 is determined by the non-transmission power consumption, including baseband processing, battery backup, cooling, etc.

C. Network Energy Efficiency

Before defining the network energy efficiency, we will first investigate the network throughput. We consider fixed-rate transmission, where outage happens if the receive SINR falls below a given threshold. The associated outage probability is denoted as $p_{\mathrm{out}} = \Pr(\mathrm{SINR} \le \hat{\gamma})$, with SINR given in (3) and $\hat{\gamma}$ as the threshold.

Then the network throughput is defined as the average number of successfully transmitted bits per sec·Hz·unit-area [10], [14], [15], and is written as

$$R_{\mathrm{area}} = \lambda_b p_{\mathrm{a}} (1 - p_{\mathrm{out}}) \log_2 (1 + \hat{\gamma}), \tag{5}$$

where $\lambda_b p_{\mathrm{a}}$ is the active BS density. This can be regarded as the area spectral efficiency [16].

By taking the BS power model into consideration, the average power consumption per unit area is the transmit power and circuit power consumption from active BSs and non-transmission power consumption from both active and inactive BSs, which is written as

$$P_{\mathrm{area}} = \lambda_b p_{\mathrm{a}} \left(\frac{1}{\eta} P_{\mathrm{t}} + M P_{\mathrm{c}}\right) + \lambda_b P_0. \tag{6}$$

Then, the network energy efficiency is defined as the ratio of network throughput to the power consumption per unit area [14], and is given by

$$\eta_{\mathrm{EE}} = \frac{R_{\mathrm{area}}}{P_{\mathrm{area}}} = \frac{p_{\mathrm{a}} p_{\mathrm{s}} \log_2 (1 + \hat{\gamma})}{p_{\mathrm{a}} \left(\frac{1}{\eta} P_{\mathrm{t}} + M P_{\mathrm{c}}\right) + P_0}, \tag{7}$$

where we denote $p_{\mathrm{s}} = 1 - p_{\mathrm{out}}$ as the successful transmission probability, and the unit of energy efficiency is bit/J/Hz.

In the following sections, we will first analyze the successful transmission probability and then investigate the effects of different system parameters on network energy efficiency in the small cell network.

III. Analysis of Successful Transmission Probability

The successful transmission probability is critical for energy efficiency, for which we will provide a closed-form expression in this section. This new result can be generally applied for performance analysis in random spatial network models, especially with multi-antenna BSs. Since the small cell network is interference-limited, we ignore additive noise in the following analysis. Later we will justify such approximation through simulation.

Based on the SINR expression in (3), which is now for signal to interference ratio (SIR) as we ignore the noise, the successful transmission probability for user 0 is given by

$$p_{\mathrm{s}} = \Pr\left(\mathrm{SIR} \geq \hat{\gamma}\right) = \Pr\left(\frac{P_{\mathrm{t}} g_{00} r_0^{-\alpha}}{\sum_{i \in \tilde{\Phi}_b \setminus 0} P_{\mathrm{t}} g_{i0} R_{i0}^{-\alpha}} \geq \hat{\gamma}\right). \quad (8)$$

The case with single-antenna BSs has been derived by [8], [9], but the result cannot be easily extended to the multi-antenna case. Eq. (8) can be written as

$$p_{\mathrm{s}} = \Pr\left(g_{00} \geq \hat{\gamma} r_0^{\alpha} \sum g_{i0} R_{i0}^{-\alpha}\right). \quad (9)$$

As g_{00} follows the gamma distribution, then we have

$$p_{\mathrm{s}} = \mathrm{E}_{r_0}\left[\mathrm{E}_I\left[\sum_{n=0}^{M-1} \frac{r_0^{\alpha n}}{n!} I^n e^{-r_0^{\alpha} I}\right]\right], \quad (10)$$

where $I = \hat{\gamma} \sum g_{i0} R_{i0}^{-\alpha}$. Denote $s = r_0^{\alpha}$, then $\mathrm{E}_I\left[e^{-sI}\right]$ can be regarded as the Laplace transform of I, denoted as $\mathcal{L}_I(s)$. Following the property of the Laplace transform, we have $\mathrm{E}_I\left[I^n e^{-sI}\right] = (-1)^n \frac{d^n}{ds^n}\mathcal{L}(s)$, which subsequently gives

$$p_{\mathrm{s}} = \mathrm{E}_{r_0}\left[\sum_{n=0}^{M-1} \frac{r_0^{\alpha n}}{n!} (-1)^n \frac{d^n}{ds^n} \mathcal{L}_I(s)\right]. \quad (11)$$

The major difficulty of expanding (11) is to simplify the nth derivative of $\mathcal{L}_I(s)$, which is similar to the case in [11]. However, only approximation results by Taylor expansion were provided in [11]. In [15], a similar problem was treated and closed-form expressions were derived, but their method cannot be directly used for our problem, as the network model is different. While an ad hoc network model was considered in [15], where the interfering nodes can be arbitrarily close, in our cellular model the interfering BSs will be farther away than the home BS. In addition, our solution will be presented in a clean expression and it can provide more insights.

Our approach is different from [11], [15], and the complex expression of (11) will be transformed to a lower triangular Toeplitz matrix form, which possesses nice analytical properties for further performance evaluation. The new closed-form expression of the successful transmission probability is given in the following Theorem.

Theorem 1: The successful transmission probability with M antennas at each BS is given by

$$p_{\mathrm{s}} = \frac{1}{1 + k_0 p_{\mathrm{a}}} \sum_{n=0}^{M-1} c^n \left\|\mathbf{Q}_M^n\right\|_1, \quad (12)$$

where $\|\cdot\|_1$ is the L_1 induced matrix norm (i.e., $\|\mathbf{A}\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^{m} |a_{ij}|$ for $\mathbf{A} \in \mathbb{R}^{m \times n}$), $c = \frac{p_{\mathrm{a}}}{1 + k_0 p_{\mathrm{a}}}$,

$$\mathbf{Q}_M = \begin{bmatrix} 0 & & & & \\ k_1 & 0 & & & \\ k_2 & k_1 & 0 & & \\ \vdots & & & \ddots & \\ k_{M-1} & k_{M-2} & \cdots & k_1 & 0 \end{bmatrix},$$

and $k_i = \frac{\frac{2}{\alpha}\hat{\gamma}^i}{i - \frac{2}{\alpha}} {}_2F_1\left(i+1, i-\frac{2}{\alpha}; i+1-\frac{2}{\alpha}; -\hat{\gamma}\right)$ for $i \geq 1$, $k_0 = \frac{\frac{2}{\alpha}\hat{\gamma}}{1 - \frac{2}{\alpha}} {}_2F_1\left(1, 1-\frac{2}{\alpha}; 2-\frac{2}{\alpha}; -\hat{\gamma}\right)$, where ${}_2F_1(\cdot)$ is the Gauss hypergeometric function [17].

Proof: The proof is omitted due to space limitation. ■

There are some basic properties for a lower triangular Toeplitz matrix, which can be applied to the further performance analysis based on Eq. (12). Some of useful properties are listed as follows:

- As $\mathbf{Q}_M$ is a lower triangular Toeplitz matrix, $(c\mathbf{Q}_M)^n$ is also a lower triangular Toeplitz matrix for $n \in \mathbb{N}^+$.
- $\sum_{n=0}^{M-1} c^n \left\|\mathbf{Q}_M^n\right\|_1 = \left\|\sum_{n=0}^{M-1} (c\mathbf{Q}_M)^n\right\|_1$.
- For different values of M, denoted as M_1 and M_2, $\mathbf{Q}_{M_1}^n(t,1) = \mathbf{Q}_{M_2}^n(t,1)$ for $\forall n \in \mathbb{N}$.

These basic properties can be directly derived from [18]. Moreover, it can be shown from (12) that c is a monotone decreasing function with the BS density, and $\left\|\mathbf{Q}_M^n\right\|_1$ is unrelated to either the BS density or the user density, which will make the analysis with respect to the BS density tractable.

Based on the closed-form expression (12), we can easily prove that p_{s} is an increasing function with both the BS density (λ_b) and the number of BS antennas (M). Therefore, we can increase network throughput via deploying more BSs or increasing the number of BS antennas, but how this will affect network energy efficiency is not clear yet. In the following sections, we will investigate this aspect.

IV. Energy Efficiency Analysis

Based on the analytical result of the successful transmission probability, in this section, we will analyze energy efficiency of the small cell network.

As the network throughput in the interference-limited regime is independent of the absolute value of transmit power P_{t}, the network energy efficiency increases with the decrease of the total power consumption at each BS, i.e., P_{BS}. Table I shows power consumptions for typical types of BSs, which clearly shows the advantage of deploying small BSs such as micro or pico-BSs for higher network energy efficiency. On the other hand, the effects of the BS density and multiple BS antennas on energy efficiency require more effort to reveal, and we will show that different components of P_{BS} play critical roles.

A. The Effect of the BS Density

In Section III, we have shown that the throughput increases with the BS density (λ_b). However, the effect of λ_b on energy efficiency is unclear, as increasing the BS density will also

TABLE I
POWER CONSUMPTION FOR DIFFERENT TYPES OF BSS [13]

	η	P_t (W)	P_c (W)	P_0 (W)
Macro BS	0.14	80	244	225
Micro BS	0.32	6.3	35	34
Pico-BS	0.23	0.25	6.1	2.6

increase the total power consumption in the network. In the following we will show that the effect of λ_b on energy efficiency depends critically on the BS non-transmission power consumption P_0.

Since p_a is a monotone decreasing function with λ_b, the derivative of energy efficiency with respect to λ_b has the opposite sign with the derivative of energy efficiency with respect to p_a. Due to such relationship, to analyze the effect of λ_b, we can consider p_a instead. From (7) and (12), we have

$$\frac{\partial\eta_\mathrm{EE}}{\partial p_\mathrm{a}}=\frac{\sum_{n=0}^{M-1}c^{n+1}\|\mathbf{Q}_M^n\|_1\left[(P_\mathrm{BS}-P_0)\left(n-k_0p_\mathrm{a}\right)+\frac{(n+1)P_0}{p_\mathrm{a}}\right]}{\left[p_\mathrm{a}\left(\frac{1}{\eta}P_\mathrm{t}+MP_\mathrm{c}\right)+P_0\right]^2\left(1+k_0p_\mathrm{a}\right)}. \tag{13}$$

The denominator in (13) is always greater than 0 when $P_0\neq 0$. Define the numerator as $f\left(p_\mathrm{a}\right)$, and then we have

$$\frac{df(p_\mathrm{a})}{dp_\mathrm{a}}=\sum_{n=0}^{M-1}c^{n+1}\|\mathbf{Q}_M^n\|_1\left\{-k_0\left(P_\mathrm{BS}-P_0\right)-\frac{(n+1)\,P_0}{p_\mathrm{a}^2}\right\}\leq 0, \tag{14}$$

for $p_\mathrm{a}\in(0,1)$. Combining with the fact $\lim_{p_\mathrm{a}\to 0}f\left(p_\mathrm{a}\right)>0$, there can only be two cases for the monotonicity of energy efficiency with respect to p_a: 1) Energy efficiency is a quasi-concave function with p_a if $f\left(1\right)<0$; 2) Energy efficiency is an increasing function with p_a if $f\left(1\right)\geq 0$.

Accordingly, there are two cases for the effect of λ_b on energy efficiency, as shown in Fig. 1. The condition of the second case, i.e., the energy efficiency decreases with λ_b, is given by $f\left(1\right)\geq 0$, which can be written equivalently as

$$\frac{P_0}{P_\mathrm{BS}}\geq\frac{\sum_{n=0}^{M-1}\left(k_0-n\right)\left(\frac{1}{1+k_0}\right)^{n+1}\|\mathbf{Q}_M^n\|_1}{\left(1+k_0\right)\sum_{n=0}^{M-1}\left(\frac{1}{1+k_0}\right)^{n+1}\|\mathbf{Q}_M^n\|_1}\triangleq\gamma_{P_0}. \tag{15}$$

Therefore, the non-transmission power consumption P_0 is the key parameter to the effect of the BS density. Specifically, when the ratio of non-transmission power to the total BS power is large, i.e., $\frac{P_0}{P_\mathrm{BS}}\geq\gamma_{P_0}$, increasing the BS density will always decrease energy efficiency, although it can improve throughput. This implies that in this case, we should deploy the least number of BSs in the area to satisfy the throughput requirement. On the other hand, when $\frac{P_0}{P_\mathrm{BS}}<\gamma_{P_0}$, there is a non-zero BS density that maximizes energy efficiency. We can obtain the optimal density numerically from (13), which is instructive when designing and operating a cellular network.

B. The Effect of the Number of BS Antennas

We have seen that deploying multi-antenna BSs can increase throughput, but it will also consume more circuit power.

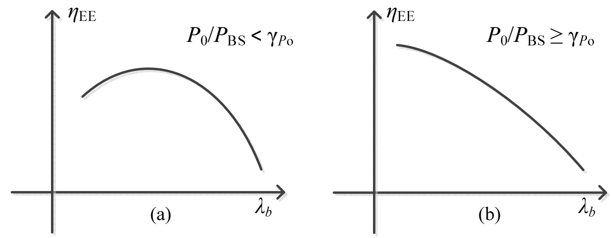

Fig. 1. The diagram showing the possible relationship between energy efficiency and the BS density: Fig. 1(a) means increasing the BS density can increase energy efficiency to a certain extent, while Fig. 1(b) means increasing the BS density will always decrease energy efficiency.

Denote $\eta_\mathrm{EE}\left(M\right)$ as the energy efficiency with M antennas at each BS, then it can be proved that

$$\eta_\mathrm{EE}\left(M+1\right)+\eta_\mathrm{EE}\left(M-1\right)-2\eta_\mathrm{EE}\left(M\right)\leq 0, \tag{16}$$

for $M>1$, which implies that it is not possible that the two inequalities $\eta_\mathrm{EE}\left(M-1\right)\geq\eta_\mathrm{EE}\left(M\right)$ and $\eta_\mathrm{EE}\left(M\right)\leq\eta_\mathrm{EE}\left(M+1\right)$ hold simultaneously. Therefore, it will never happen that the energy efficiency will first decrease and then increase as we keep increasing M. Moreover, we have

$$\lim_{M\to\infty}\eta_\mathrm{EE}\left(M\right)=0,\text{ and }\eta_\mathrm{EE}\left(1\right)>0.$$

Considering all these facts, there can only be two different cases for the effect of M on energy efficiency: 1) Energy efficiency decreases with M, so deploying a single antenna at each BS is more energy efficient than using multiple antennas; 2) Deploying multi-antenna BSs can achieve higher energy efficiency than single-antenna BSs and there is an optimal value of M.

The necessary and sufficient condition for the first case is $\eta_\mathrm{EE}\left(1\right)\geq\eta_\mathrm{EE}\left(2\right)$, which is equivalent to

$$P_\mathrm{c}\geq\frac{k_1\left(p_\mathrm{a}\frac{1}{\eta}P_\mathrm{t}+P_0\right)}{1+\left(k_0-k_1\right)p_\mathrm{a}}. \tag{17}$$

The right hand side of (17) is a monotone function with respect to p_a, which means if the condition

$$P_\mathrm{c}\geq\max\left(k_1P_0,\frac{k_1\left(\frac{1}{\eta}P_\mathrm{t}+P_0\right)}{1+k_0-k_1}\right)\triangleq\gamma_{P_\mathrm{c}} \tag{18}$$

is satisfied, for any BS and user densities, deploying single-antenna BSs provides higher energy efficiency than multi-antenna BSs.

V. NUMERICAL RESULTS

In this section, we will demonstrate our results through numerical and simulation based analysis. We will also test whether the conclusions we draw from the random network model still hold in a regular grid-based network model.

Assume that the pathloss exponent $\alpha=4$, the SINR threshold $\hat{\gamma}$ is 1, and the user density is $\lambda_u=10^{-3}$ per square meter. Micro BS is considered, with the power consumption model shown in Table I. By substituting these values, we can

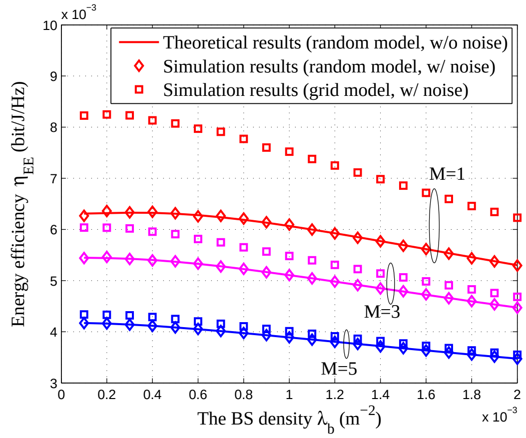

Fig. 2. Energy efficiency vs. BS density for different numbers of BS antennas, with $\alpha = 4$, $\hat{\gamma} = 1$, $\lambda_u = 10^{-3}\mathrm{m}^{-2}$, and noise power is considered in simulation where $\sigma_n^2 = -97.5$dBm.

first find that the condition (18) is satisfied, which means it is more energy efficient to deploy single-antenna BSs. On the other hand, by examining (15) with the above data, we see that when $M > 2$, energy efficiency is a decreasing function with the BS density. Hence, we need to make a tradeoff between energy efficiency and the network throughput. These design guidelines are confirmed by the simulation results in Fig. 2.

To confirm our conclusions based on the random network model, we also simulate a grid-based model with the same BS density, where each cell is modeled as a hexagon. Furthermore, we consider additive noise to test the interference-limited assumption. From Fig. 2, we have the following observations. First, the influence of the additive noise is negligible, and the theoretical results fit the simulation results. Second, the performance of the hexagonal-cell network provides an upper bound compared to the random network model, which was also shown in [8], but both network models have the same trend. Finally, when $M = 1$, there is an optimal BS density that maximizes the network energy efficiency,[1] which is about 0.3×10^{-3} per square meter, but when $M = 3$ or 5, the energy efficiency decreases with the BS density, which fits our analysis. Therefore, we can conclude that deploying single-antenna BSs provides higher energy efficiency than using multi-antenna BSs and there is an optimal BS density, upon which we can increase both the network throughput and energy efficiency by increasing the BS density.

VI. Conclusions

In this paper, we analyzed the network energy efficiency of a small cell network where BSs and users are modeled as two independent spatial point processes. Our analysis revealed the effects of the BS density and multiple BS antennas on energy efficiency. It was found that while increasing the BS density or increasing the number of BS antennas can always increase network throughput, their effects on energy efficiency depend on the non-transmission power and the circuit power at the BS, respectively. In particular, our results have shown that in most cases we will need to make a tradeoff between the network throughput and energy efficiency. However, there are special cases where both throughput and energy efficiency can be increased. Specifically, it was found that if the non-transmission power (or the circuit power) is smaller than a given threshold $\gamma_{P_0} P_{\mathrm{BS}}$ (or γ_{P_c}), we can increase both the throughput and energy efficiency by increasing the BS density (or the number of antennas) to a certain value. These insightful results can be used as guidelines for the network design.

[1]Here we assume that the throughput requirement can be met, while further investigation on energy efficiency and throughput tradeoff will be carried out in our future work.

References

[1] J. Hoydis, M. Kobayashi, and M. Debbah, "Green small-cell networks," *IEEE Veh. Technol. Mag.*, vol. 6, no. 1, pp. 37–43, Mar. 2011.

[2] Z. Hasan, H. Boostanimehr, and V. K. Bhargava, "Green cellular networks: A survey, some research issues and challenges," *IEEE Commun. Surveys Tutorials*, vol. 13, no. 4, pp. 524–540, 2011.

[3] C. Li, S. Song, J. Zhang, and K. Letaief, "Maximizing energy efficiency in wireless networks with a minimum average throughput requirement," in *Proc. of IEEE Wireless Commun. and Networking Conf. (WCNC)*, Apr. 2012, pp. 1130–1134.

[4] C. Isheden, Z. Chong, E. Jorswieck, and G. Fettweis, "Framework for link-level energy efficiency optimization with informed transmitter," *IEEE Trans. Wireless Commun.*, vol. 11, no. 8, pp. 2946–2957, Aug. 2012.

[5] C. Xiong, G. Li, S. Zhang, Y. Chen, and S. Xu, "Energy- and spectral-efficiency tradeoff in downlink OFDMA networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 11, pp. 3874–3886, Nov. 2011.

[6] E. Kurniawan and A. Goldsmith, "Optimizing cellular network architectures to minimize energy consumption," in *Proc. of IEEE Int. Conf. on Commun. (ICC)*, Ottawa, Canada, Jun. 2012.

[7] J. Xu, J. Zhang, and J. Andrews, "On the accuracy of the Wyner model in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 9, pp. 3098–3109, Sept. 2011.

[8] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 3122–3134, Nov. 2011.

[9] S. Lee and K. Huang, "Coverage and economy of cellular networks with many base stations," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 1038–1040, Jul. 2012.

[10] D. Cao, S. Zhou, and Z. Niu, "Optimal base station density for energy-efficient heterogeneous cellular networks," in *Proc. of IEEE Int. Conf. on Commun. (ICC)*, Ottawa, Canada, Jun. 2012.

[11] A. M. Hunter, J. G. Andrews, and S. Weber, "Transmission capacity of ad hoc networks with spatial diversity," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5058–5071, Dec. 2008.

[12] Y. Chen, S. Zhang, and S. Xu, "Impact of non-ideal efficiency on bits per joule performance of base station transmissions," in *Proc. of IEEE Veh. Technol. Conf. (VTC)*, Budapest, Hungary, May 2011, pp. 1–5.

[13] G. Auer et al., "D2.3: energy efficiency analysis of the reference systems, areas of improvements and target breakdown," *INFSO-ICT-247733 Energy Aware Radio and NeTwork TecHnologies*, Nov. 2010.

[14] T. Q. S. Quek, W. C. Cheung, and M. Kountouris, "Energy efficiency analysis of two-tier heterogeneous networks," in *Proc. of Wireless Conf. 2011 - Sustainable Wireless Technologies*, Apr. 2011, pp. 1–5.

[15] Y. Wu, R. H. Y. Louie, M. R. McKay, and I. B. Collings, "Generalized framework for the analysis of linear MIMO transmission schemes in decentralized wireless ad hoc networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 8, pp. 2815–2827, Aug. 2012.

[16] M.-S. Alouini and A. Goldsmith, "Area spectral efficiency of cellular mobile radio systems," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1047–1066, Jul. 1999.

[17] I. Gradshteĭn, I. Ryzhik, A. Jeffrey, and D. Zwillinger, *Table of integrals, series, and products*. Academic press, 2007.

[18] D. Commenges and M. Monsion, "Fast inversion of triangular Toeplitz matrices," *IEEE Trans. Autom. Control*, vol. 29, no. 3, pp. 250–251, Mar. 1984.