

# Load Balancing and Pricing for Spectrum Access Control in Cognitive Radio Networks

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**Abstract**—In dynamic spectrum access (DSA) control, the prevalent approach to provide economics incentives for operators is pricing, whereas load balancing gives congestion-avoidance incentives to secondary users (SUs). Despite complexities of i) the couplings between pricing, load balancing and SUs' spectrum access decision, and ii) the heterogeneity of primary users' traffic and SUs types, we propose to solve the joint load balancing and pricing problem to maximize operator' revenue in a monopoly market. In this market, we first show there exists a unique SUs' equilibrium arrival rate to the monopolist's channels, and then we show that the joint problem can be solved efficiently by exploiting its convex structure. We next propose a low-complexity algorithm that enable the operator to maximize its revenue.

**Index Terms**—Pricing, Load balancing, Nash Equilibrium, Dynamic Spectrum Access, Cognitive Radio.

## I. INTRODUCTION

To efficiently utilize the scarce wireless spectrum, dynamic spectrum access (DSA) has been introduced, which is commonly controlled by static licensing. There are various DSA approaches, including two popular paradigms shared-use and exclusive-use, have been proposed to give secondary users (SUs) the flexibility to access the underutilized legacy spectrum, that is used sporadically by primary users (PUs) [1], [2]. The shared-use allows SUs to opportunistically access the (secondary) operators' "interruptible spectrum" without negative effects on PUs' activities, and the exclusive-use allows operators to lease parts of a temporarily unused spectrum (i.e. no PUs operations) for SUs' service provisioning.

Within these two paradigms, *pricing* is one of the effective market-based methods to distribute spectrum from operators to SUs since it not only provides economics incentives for operators, but also reduces the amount of overhead operations [3]–[8]. However, once the operator sets its price, various SU applications (classes) with distinguished physical conditions (types) will have different spectrum access decisions. As a result, when the heterogeneity of SUs' classes and types is considered, designing an efficient pricing mechanism to achieve the optimal revenue for operators is one of the market-based challenges.

While pricing provides the economics incentive for operators, *load balancing*, which distributes the SUs' traffic loads

to the right channels, gives a congestion-avoidance incentive to SUs [9]–[11]. Nevertheless, SUs' congestion is influenced by service times of the operator's channels, which are affected by PUs traffic patterns variation. Therefore, when the heterogeneity of PU traffic is considered, designing a low-complexity load balancing mechanism is another great challenge.

In this research, by taking into account the heterogeneity of PUs traffic as well as SUs types and conditions, we study a load balancing and pricing problem jointly for spectrum access control in multi-channel cognitive radio networks (CRNs). At the operator level, the operator will try to maximize its revenue by deciding the corresponding prices and load balancing information. Based on these information, at the SU level, an arriving SU with a specific application and physical condition will decide whether or not to join the operator to maximize its expected utility. Because how SUs make joining decisions depends on both price and load balancing information of the operator, and how the operator sets its prices and load balancing depends on SUs' joining policy, there are certain couplings among pricing, load balancing, and SUs' joining policy.

Particularly, we consider a monopoly market with one operator employing shared-use DSA. Given the information of operator's load balancing and prices, we first characterize the SUs' joining policy and show that there exists a unique SUs' equilibrium arrival rate to the operator's channels. By putting this equilibrium constraint into the operator's revenue maximization problem, we then show that this problem can be solved efficiently by a sequential optimization method, which exposes its convex structure. We also propose a low-complexity algorithm for the monopolist's revenue maximization.

To overcome many previous simplified assumptions by taking the heterogeneity of SUs' classes and types into account, we study the jointly optimal pricing and load balancing spectrum access control with low-complexity mechanisms which, to the best of our knowledge, has not been addressed in the literature before.

## II. RELATED WORKS

In the literature, two popular DSA research directions are load balancing and pricing. On one hand, many researchers have their interest in load balancing spectrum control in

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multi-channel CRNs [9]–[11]. The authors in [11] proposed a dynamic learning scheme to determine a load balancing strategy that can lead to a Nash equilibrium, but it does not guarantee the result is a global optimal point. The work in [9] tried to minimize the system time by providing a solution for optimal channel selection which, however, relies on a numerical optimization that uses a high-complexity exhausted search algorithm. The recent work in [10] proposes a low-complexity algorithm to provide optimal load balancing based on theory of convex optimization; however, its channels are limited to only exponential distributions.

On the other hand, pricing methods which address the DSA economic aspect, have received tremendous attention recently [3]–[5]. However, very few papers apply the strategic queuing system for the pricing aspect, which originated from [12], as compared with our work. In this field, [6] proposed a pricing scheme to maximize a monopolist operator's revenue, and [7] accounted for a socially-maximizing pricing mechanism; however, both consider homogeneous SUs with the same class and type, which is an over-simplified model. The recent work of [8] investigated not only revenue but also socially-optimal pricing schemes; however, its assumptions are limited to a single-channel case and the same type for all SUs.

### III. SYSTEM MODEL

In this section, a network that consists of one operator with multiple shared-use channels is considered. The network is assumed to receive a sequence of arriving SUs jobs and each SU will make a spectrum access decision for its job. The model in this section can be quantitatively described as follows.

1) *Shared-use Monopolist Operator*: The operator is assumed to have a set of channels, which are denoted by  $\mathcal{L} = \{1, \dots, L\}$  and licensed to legacy PUs. Traffic patterns of PUs are modeled as an ON-OFF renewal process, and it's alternating between ON (busy) and OFF (idle) periods. The sojourn times of the ON and OFF periods on each channel  $l \in \mathcal{L}$  are modeled by i.i.d. random variables (r.v.)  $Y_l$  and  $Z_l$ , respectively. These ON and OFF periods are assumed to be independent with SUs' arrival process and service time. The SU services can regard this ON-OFF process as a channel model for them. This model lets us obtain the idle time period in which the SUs can utilize the channel without causing harmful interference to the PUs.

The shared-use operator gains revenue by allowing SUs to share the PUs' channels opportunistically. Before each period of all stationary distributions, the operator broadcasts a load balancing vector  $\vec{s} = \{s_l\}_{l \in \mathcal{L}}$  and an admission price vector  $\vec{p} = \{p_l\}_{l \in \mathcal{L}}$  to all potential SUs. While  $\vec{s}$  with  $\sum_{l=1}^L s_l = 1$  probabilistically guides the SUs in selecting channels,  $\vec{p}$  helps SUs decide whether or not to join the network.

2) *SUs*: A set of classes of SUs denoted by  $\mathcal{K} = \{1, \dots, K\}$  is assumed in the network where class- $k$  SUs arrive at the network following a Poisson process with a potential rate  $\Lambda_k$ ,  $\forall k \in \mathcal{K}$ . Each class- $k$  SU has a distinct job (e.g. a packet, session, or connection) upon arrival and its job is associated with a specific delay-sensitive application characterized by a value  $\theta_k$ . For instance, multimedia applications

with stringent delay requirements will have high values of  $\theta_k$ . Without loss of generality, we assume  $0 < \theta_1 < \dots < \theta_K$ .

The requested time to finish a SU's job is represented by a r.v.  $X$ , and it is assumed to be independent of the arrival process.

Furthermore, we also assume that each SU belonging to a type  $\alpha$  has a monetary evaluation value  $\alpha r$  about the operator's service, where  $\alpha$  is a random variable characterizing how *heterogeneous* SUs evaluates the operator's quality (e.g. SUs with different physical conditions such as location, temperature, etc.) and  $r$  is the operator's intrinsic quality (e.g. the coverage area). We assume that  $\alpha$  follows a uniform distribution on  $[0, 1]$ , which is often used in the literature [13], [14], with their cumulative distribution function  $F(\cdot)$ . It has been shown in [14] that this assumption only affects the magnitude of the finding results but does not alter the engineering insights.

Since the same licensed channel  $l \in \mathcal{L}$  may be shared by many SUs, congestion can happen. An arriving SU at this channel will be informed about its job's delay in a queue containing many other SU jobs that also want to use that licensed channel. Hence, the operator is assumed that it should maintain a parallel queuing system of  $L$  M/G/1 queues whose service time of each queue  $l$ , denoted by a random variable  $\chi_l$ , has a general distribution dictated by distributions of  $X$ ,  $Y_l$  and  $Z_l$ . We denote  $\bar{T}_l(\lambda_l)$  the mean steady-state queuing delay induced by an effective arrival rate  $\lambda_l$ . Respectively we also denote the first and second moments of channel  $l$ 's service time by  $\bar{\chi}_l$  and  $\bar{\chi}_l^2$ , and the mean queuing delay is defined as follows according to the Pollaczek-Khinchin formula [15]

$$\bar{T}_l(\lambda_l) = \begin{cases} \frac{\lambda_l \bar{\chi}_l^2}{2(1-\lambda_l \bar{\chi}_l)} + \bar{\chi}_l, & \text{if } \lambda_l < 1/\bar{\chi}_l; \\ \infty, & \text{otherwise,} \end{cases} \quad (1)$$

where  $\bar{\chi}_l$  and  $\bar{\chi}_l^2$  are provided in [16]. Without loss of generality, we also assume  $0 < \bar{\chi}_1 < \dots < \bar{\chi}_L$ .

Let  $\lambda_k$  denotes the effective arrival rate of class- $k$  SUs into the system, then the effective arrival rate into channel  $l$  is  $\lambda_l(s_l) = s_l \sum_{k=1}^K \lambda_k$ . For given  $\vec{s}$  and  $\vec{p}$  from the operator, the utility of a class- $k$  type- $\alpha$  SU that joins channel  $l$  is

$$U_{k,l}(\alpha, s_l, p_l) = \alpha r - \theta_k \bar{T}_l(\lambda_l(s_l)) - p_l, \quad \forall k, l. \quad (2)$$

This utility function strikes a balance between a reward  $\alpha r$  and a total cost  $\theta_k \bar{T}_l(\lambda_l(s_l)) + p_l$ , where  $\theta_k$  can be interpreted as a waiting cost per unit time. We see that this utility is generalized to capture the heterogeneity of both user types (i.e.  $\alpha$ ) and classes (i.e.  $\theta_k$ ). In the literature, most utility functions either have equal value  $r$  [8], or  $\theta_k$  [17] for all users. A variant form is  $\alpha q_l - p_l$  [13], [18], where the QoS function  $q_l$  presents an inverse effect of our congestion function  $d_l$ .

### IV. MONOPOLY MARKET

In this section, firstly, we present the SUs' decision-making equilibrium. Based on these information, we then study how the operator maximizes its revenue.

### A. SUs' Decision Policy and Equilibrium

We assume that the SUs are rational decision-makers. Thus, they only join the network when their utilities are positive. Therefore, we have:

**Definition 1.** A class- $k$  type- $\alpha$  SU in channel  $l$  with its utility  $U_{k,l}(\alpha, s_l, p_l)$  will follow a joining decision policy such that

- it joins the channel  $l$  if  $U_{k,l}(\alpha, s_l, p_l) > 0$ , which requires  $\alpha > \alpha_{k,l}(\lambda_l(s_l)) = \frac{\theta_k \bar{T}_l(\lambda_l(s_l)) + p_l}{r}$ ;
- it balks, otherwise.

Then, the effective arrival rate of class- $k$  SU into channel  $l$ , defined by  $\lambda_{k,l} \triangleq s_l \Lambda_k$ , is as follows

$$\begin{aligned} \lambda_{k,l} &= s_l \Lambda_k \Pr [U_{k,l}(\alpha, s_l, p_l) > 0] = s_l \Lambda_k \Pr [\alpha > \alpha_{k,l}] \\ &= s_l \Lambda_k \left( 1 - F\left(\alpha_{k,l} \left( \sum_{k=1}^K \lambda_{k,l} \right)\right) \right). \end{aligned} \quad (3)$$

**Proposition 1.** For given  $\vec{p}$  and  $\vec{s}$  of a shared-use monopolist operator, if  $p_l \geq r - \theta_k \bar{\lambda}_l$ , then  $\lambda_{k,l}^{\text{eq}} = 0$ . Otherwise, there exists a unique equilibrium  $\lambda_{k,l}^{\text{eq}}$  of the class- $k$  SU into channel  $l$  such that  $\lambda_{k,l}^{\text{eq}} = s_l \Lambda_k \left( 1 - \alpha_{k,l} \left( \sum_{k=1}^K \lambda_{k,l}^{\text{eq}} \right) \right) > 0$ .

*Proof:* **Case (a):** if  $p_l \geq r - \theta_k \bar{\lambda}_l$ , it is equivalent to  $\alpha_{k,l}(0) \geq 1$ . Since  $\alpha_{k,l} \left( \sum_{k=1}^K \lambda_{k,l} \right) \geq \alpha_{k,l}(0)$ ,  $\forall \lambda_{k,l}$ , we have  $\alpha_{k,l} \left( \sum_{k=1}^K \lambda_{k,l} \right) \geq 1$ , with which there exists a unique  $\lambda_{k,l}^{\text{eq}} = 0$  satisfying (3)  $\forall k, l$ .

**Case (b):** if  $p_l < r - \theta_k \bar{\lambda}_l$ , it is equivalent to  $\alpha_{k,l}(0) < 1$ . Defining

$$g(\lambda_{k,l}) = \lambda_{k,l} - s_l \Lambda_k \left( 1 - F\left(\alpha_{k,l} \left( \sum_{k=1}^K \lambda_{k,l} \right)\right) \right),$$

when  $\alpha_{k,l}(0) < 1$ , we have

$$\begin{aligned} g(0) &= 0 - s_l \Lambda_k (1 - F(\alpha_{k,l}(0))), \\ &= -s_l \Lambda_k (1 - \alpha_{k,l}(0)) < 0. \end{aligned} \quad (4)$$

Since  $\alpha_{k,l} \left( \sum_{k=1}^K \lambda_{k,l} \right)$  is increasing and unbounded, we can always choose a value  $\lambda'_{k,l} > 0$ ,  $\forall k, l$  such that  $\alpha_{k,l} \left( \sum_{k=1}^K \lambda'_{k,l} \right) > 1$ . Therefore, we have

$$\begin{aligned} g(\lambda'_{k,l}) &= \lambda'_{k,l} - s_l \Lambda_k \left( 1 - F\left(\alpha_{k,l} \left( \sum_{k=1}^K \lambda'_{k,l} \right)\right) \right), \\ &= \lambda'_{k,l} - s_l \Lambda_k (1 - 1) > 0. \end{aligned} \quad (5)$$

Since  $F(\cdot)$  and  $\alpha_{k,l}(\cdot)$  are continuous and increasing, it is clear that  $g(\cdot)$  is continuous and increasing. Hence, there exists a unique equilibrium  $\lambda_{k,l}^{\text{eq}} \in (0, \lambda'_{k,l})$  satisfying  $\lambda_{k,l}^{\text{eq}} = s_l \Lambda_k \left( 1 - F\left(\alpha_{k,l} \left( \sum_{k=1}^K \lambda_{k,l}^{\text{eq}} \right)\right) \right)$ ,  $\forall k, l$ . Furthermore, with this positive  $\lambda_{k,l}^{\text{eq}}$ , we must have  $\alpha_{k,l} \left( \sum_{k=1}^K \lambda_{k,l}^{\text{eq}} \right) < 1$ ; otherwise  $\lambda_{k,l}^{\text{eq}} = 0$ ,  $\forall k, l$  according to case (a), which is a contradiction. ■

### B. Operator's Revenue Maximization

In this subsection, we give the fomulation of the revenue maximization problem and present a sequential optimization method, based on which we can achieve the optimal solution and algorithm.

1) *Problem Formulation:* At this stage, the operator temporarily assumes that there exists a unique SUs' equilibrium  $\lambda_{k,l}^{\text{eq}} > 0$ ,  $\forall k, l$ . From this knowledge, the operator's objective is to maximize its revenue, which can be formulated as the following optimization problem

$$\begin{aligned} &\text{maximize}_{\vec{s}, \vec{p}} \quad \sum_{l=1}^L p_l \lambda_l^{\text{eq}} \\ &\text{subject to} \quad \lambda_{k,l}^{\text{eq}} = s_l \Lambda_k \left( 1 - \alpha_{k,l} \left( \sum_{k=1}^K \lambda_{k,l}^{\text{eq}} \right) \right), \forall k, l, \\ &\quad \sum_{l=1}^L s_l = 1, \\ &\quad 0 \leq s_l \leq 1, p_l \geq 0, \quad \forall l. \end{aligned} \quad (6)$$

The first constraint is the SUs' equilibrium knowledge from Proposition 1. The second constraint is the load balancing constraint. This problem is a non-convex optimization problem due to the couplings between variables.

From the first constraint of (6), the equilibrium arrival rate into channel  $l$  can be obtained as  $\lambda_l^{\text{eq}} = \sum_{k=1}^K \lambda_{k,l}^{\text{eq}} = s_l \left( \Lambda - \frac{\Omega \bar{T}_l(\lambda_l^{\text{eq}}) + \Lambda p_l}{r} \right)$ ; therefore, we have

$$p_l(s_l, \lambda_l^{\text{eq}}) = r - \frac{r \lambda_l^{\text{eq}}}{\Lambda s_l} - \frac{\Omega}{\Lambda} \bar{T}_l(\lambda_l^{\text{eq}}), \quad (7)$$

where  $\Lambda \triangleq \sum_{k=1}^K \Lambda_k$  and  $\Omega \triangleq \sum_{k=1}^K \Lambda_k \theta_k$ . Eliminating the first constraint of problem (6) by substituting (7) into the objective function, we obtain an equivalent optimization problem as follows

$$\begin{aligned} &\text{maximize}_{\vec{s}, \vec{\lambda}^{\text{eq}} = \{\lambda_l^{\text{eq}}\}_{l \in \mathcal{L}}} \quad \sum_{l=1}^L r \lambda_l^{\text{eq}} - \frac{r (\lambda_l^{\text{eq}})^2}{\Lambda s_l} - \frac{\Omega}{\Lambda} \lambda_l^{\text{eq}} \bar{T}_l(\lambda_l^{\text{eq}}) \\ &\text{subject to} \quad \sum_{l=1}^L s_l = 1 \text{ and } 0 \leq s_l \leq 1, \forall l. \end{aligned} \quad (8)$$

Problem (8) reveals a structure that can be solved efficiently by using a sequential optimization technique as follows.

2) *Sequential Optimization:* By fixing  $\vec{\lambda}^{\text{eq}}$ , problem (8) is equivalent to

$$\begin{aligned} &\text{maximize}_{\vec{s}} \quad \sum_{l=1}^L -\frac{r (\lambda_l^{\text{eq}})^2}{\Lambda s_l} \\ &\text{subject to} \quad \sum_{l=1}^L s_l = 1 \text{ and } 0 \leq s_l \leq 1, \forall l. \end{aligned} \quad (9)$$

The partial Lagrangian of (9) is  $L(\vec{s}, \mu) = \sum_{l=1}^L -\frac{r (\lambda_l^{\text{eq}})^2}{\Lambda s_l} - \mu \left( \sum_{l=1}^L s_l - 1 \right)$ .

Since (9) is a convex problem, then the first-order necessary and sufficient condition  $\frac{\partial L(\vec{s}, \mu)}{\partial s_l} = 0$  yields  $s_l(\mu) = \lambda_l^{\text{eq}} \sqrt{\frac{r}{\Lambda \mu}}$ ,  $\forall l$ . Then, the optimal solution of (9) is  $s_l^* = s_l(\mu^*)$ ,  $\forall l$ , where  $\mu^*$  is the solution of  $\sum_{l=1}^L s_l(\mu) = 1$ ; therefore, we have

$$s_l^* = \frac{\lambda_l^{\text{eq}}}{\sum_{l=1}^L \lambda_l^{\text{eq}}}, \quad \forall l. \quad (10)$$

Substituting (10) back into (8) and introducing an auxiliary variable  $\lambda_{\text{tot}}^{\text{eq}} = \sum_{l=1}^L \lambda_l^{\text{eq}}$ , we have an equivalent problem of (8) as follows

$$\begin{aligned} &\text{maximize}_{\lambda_{\text{tot}}^{\text{eq}}, \vec{\lambda}^{\text{eq}} \geq 0} \quad r \lambda_{\text{tot}}^{\text{eq}} - \frac{r}{\Lambda} (\lambda_{\text{tot}}^{\text{eq}})^2 - \frac{\Omega}{\Lambda} \sum_{l=1}^L \lambda_l^{\text{eq}} \bar{T}_l(\lambda_l^{\text{eq}}) \\ &\text{subject to} \quad \sum_{l=1}^L \lambda_l^{\text{eq}} = \lambda_{\text{tot}}^{\text{eq}}. \end{aligned} \quad (11)$$

We see that  $-\frac{\Omega}{\Lambda}\lambda_l^{\text{eq}}\bar{T}_l(\lambda_l^{\text{eq}})$  is a strictly concave function because it's second derivative is

$$-\frac{\Omega\bar{\chi}_l^2}{2\Lambda}\left(\frac{2\bar{\chi}_l^2(\lambda_l^{\text{eq}})^2}{(1-\bar{\chi}_l\lambda_l^{\text{eq}})^3} + \frac{4\bar{\chi}_l\lambda_l^{\text{eq}}}{(1-\bar{\chi}_l\lambda_l^{\text{eq}})^2} + \frac{2}{1-\bar{\chi}_l\lambda_l^{\text{eq}}}\right) < 0 \quad (12)$$

when  $\lambda_l^{\text{eq}} < 1/\bar{\chi}_l$ ,  $\forall l$ . Therefore, the objective function of (11), which is the sum of a concave function  $r\lambda_{\text{tot}}^{\text{eq}} - \frac{r}{\Lambda}(\lambda_{\text{tot}}^{\text{eq}})^2$  with a sum of  $L$  concave functions  $-\frac{\Omega}{\Lambda}\lambda_l^{\text{eq}}\bar{T}_l(\lambda_l^{\text{eq}})$ ,  $\forall l$ , is a concave function [19]. Furthermore, the equality constraint of problem (11) is an affine function, which means that problem (11) is a convex optimization problem.

The partial Lagrangian of this problem is  $L(\lambda_{\text{tot}}^{\text{eq}}, \bar{\lambda}^{\text{eq}}, \mu) = L_{\text{tot}}(\lambda_{\text{tot}}^{\text{eq}}, \mu) + \sum_{l=1}^L L_l(\lambda_l^{\text{eq}}, \mu)$ , where  $L_{\text{tot}}(\lambda_{\text{tot}}^{\text{eq}}, \mu) = r\lambda_{\text{tot}}^{\text{eq}} - \frac{r}{\Lambda}(\lambda_{\text{tot}}^{\text{eq}})^2 + \mu\lambda_{\text{tot}}^{\text{eq}}$  and  $L_l(\lambda_l^{\text{eq}}, \mu) = -\frac{\Omega}{\Lambda}\lambda_l^{\text{eq}}\bar{T}_l(\lambda_l^{\text{eq}}) - \mu\lambda_l^{\text{eq}}$ ,  $\forall l$ . Using the first-order condition, we obtain

$$\lambda_{\text{tot}}^{\text{eq}}(\mu) = \frac{\Lambda(r + \mu)}{2r}, \quad (13)$$

$$\lambda_l^{\text{eq}}(\mu) = [\Phi_l(\mu)]^+, \quad \forall l, \quad (14)$$

where  $[\cdot]^+ \triangleq \max\{0, \cdot\}$  and  $\Phi_l(\mu) \triangleq \frac{2(\mu\Lambda + \Omega\bar{\chi}_l)}{-\zeta_l(\mu) - \sqrt{\Omega\bar{\chi}_l^2\zeta_l(\mu)}}$  with  $\zeta_l(\mu) = \Omega(\bar{\chi}_l^2 - 2\bar{\chi}_l^2) - 2\Lambda\bar{\chi}_l\mu$ . We have the following property.

**Lemma 1.**  $\Phi_l(\mu)$  is continuous, strictly decreasing, positive on  $(-\infty, -\frac{\Omega}{\Lambda}\bar{\chi}_l)$ , and non-positive on  $[-\frac{\Omega}{\Lambda}\bar{\chi}_l, -\frac{\Omega}{\Lambda}(\bar{\chi}_l - \frac{\bar{\chi}_l^2}{2\bar{\chi}_l})]$ ,  $\forall l$ .

*Proof:* We see that  $\Phi_l(\mu)$  is undefined when  $\zeta_l(\mu) \leq 0$ . Therefore,  $\Phi_l(\mu) > 0$  if and only if  $\zeta_l(\mu) > 0$  and  $\mu\Lambda + \Omega\bar{\chi}_l < 0$ , which corresponds to

$$\mu < \min\left\{-\frac{\Omega}{\Lambda}\left(\bar{\chi}_l - \frac{\bar{\chi}_l^2}{2\bar{\chi}_l}\right), -\frac{\Omega}{\Lambda}\bar{\chi}_l\right\} = -\frac{\Omega}{\Lambda}\bar{\chi}_l, \quad (15)$$

and  $\Phi_l(\mu) \leq 0$  if and only if  $\zeta_l(\mu) > 0$  and  $\mu\Lambda + \Omega\bar{\chi}_l \geq 0$ , which corresponds to  $-\frac{\Omega}{\Lambda}\bar{\chi}_l \leq \mu < -\frac{\Omega}{\Lambda}\left(\bar{\chi}_l - \frac{\bar{\chi}_l^2}{2\bar{\chi}_l}\right)$ .

Furthermore, when  $\mu < -\frac{\Omega}{\Lambda}\left(\bar{\chi}_l - \frac{\bar{\chi}_l^2}{2\bar{\chi}_l}\right)$ , i.e.,  $\zeta_l(\mu) > 0$ , it is clear that  $\Phi_l(\mu)$  is continuous and we have

$$\Phi_l'(\mu) = -\frac{\Lambda\sqrt{\Omega\bar{\chi}_l^2\zeta_l(\mu)}}{\zeta_l(\mu)^2} < 0, \quad (16)$$

which concludes the proof.  $\blacksquare$

3) *Optimal Solutions:* The optimal solutions  $\lambda_{\text{tot}}^{\text{eq}}(\mu^*)$  and  $\lambda_l^{\text{eq}}(\mu^*)$ ,  $\forall l$ , can be achieved by finding the optimal dual variable  $\mu^*$  that satisfies the first constraint  $\sum_{l=1}^L \lambda_l^{\text{eq}}(\mu) = \lambda_{\text{tot}}^{\text{eq}}(\mu)$  of problem (11). Hence, we have the following result.

**Lemma 2.** If

$$\tau^{\text{mo}} \triangleq r > \frac{\Omega}{\Lambda}\bar{\chi}_1, \quad (17)$$

there exist a unique solution  $\mu^* \in (-\tau^{\text{mo}}, -\frac{\Omega}{\Lambda}\bar{\chi}_1)$  of  $\sum_{l=1}^L [\Phi_l(\mu)]^+ = \lambda_{\text{tot}}^{\text{eq}}(\mu)$  and a corresponding channel index

$1 \leq L^* \leq L$  such that  $\Phi_l(\mu^*) > 0$ ,  $\forall l \leq L^*$ , and  $\Phi_l(\mu^*) = 0$ ,  $\forall l > L^*$ . If  $\tau^{\text{mo}} \leq \frac{\Omega}{\Lambda}\bar{\chi}_1$ ,  $[\Phi_l(\mu)]^+ = \lambda_{\text{tot}}^{\text{eq}}(\mu) = 0$ ,  $\forall l$ .

*Proof:* From Lemma 1, we see that  $[\Phi_l(\mu)]^+$ ,  $\forall l \neq 1$ , is continuous, non-negative and non-increasing, and  $[\Phi_1(\mu)]^+$  is continuous, positive and strictly decreasing on  $(-\infty, -\frac{\Omega}{\Lambda}\bar{\chi}_1)$ . Therefore,  $\sum_{l=1}^L [\Phi_l(\mu)]^+$  is continuous, positive and strictly decreasing on  $(-\infty, -\frac{\Omega}{\Lambda}\bar{\chi}_1)$ . Define  $f(\mu) \triangleq \sum_{l=1}^L [\Phi_l(\mu)]^+ - \lambda_{\text{tot}}^{\text{eq}}(\mu)$ . If  $\tau^{\text{mo}} > \frac{\Omega}{\Lambda}\bar{\chi}_1$ ,  $f(\mu)$  is continuous and strictly decreasing on  $(-\tau^{\text{mo}}, -\frac{\Omega}{\Lambda}\bar{\chi}_1)$  according to (13). Furthermore, we have

$$\begin{aligned} f(-\tau^{\text{mo}}) &= \sum_{l=1}^L [\Phi_l(-\tau^{\text{mo}})]^+ - \lambda_{\text{tot}}^{\text{eq}}(-\tau^{\text{mo}}), \\ &= \sum_{l=1}^L [\Phi_l(-\tau^{\text{mo}})]^+ - 0 > 0, \end{aligned} \quad (18)$$

$$\begin{aligned} f\left(-\frac{\Omega}{\Lambda}\bar{\chi}_1\right) &= \sum_{l=1}^L \left[\Phi_l\left(-\frac{\Omega}{\Lambda}\bar{\chi}_1\right)\right]^+ - \lambda_{\text{tot}}^{\text{eq}}\left(-\frac{\Omega}{\Lambda}\bar{\chi}_1\right) \\ &= 0 - \lambda_{\text{tot}}^{\text{eq}}\left(-\frac{\Omega}{\Lambda}\bar{\chi}_1\right) < 0. \end{aligned} \quad (19)$$

Hence, there exists a unique  $\mu^*$  satisfying  $\sum_{l=1}^L [\Phi_l(\mu^*)]^+ = \lambda_{\text{tot}}^{\text{eq}}(\mu^*)$ . When  $\mu^* \in (-\tau^{\text{mo}}, -\frac{\Omega}{\Lambda}\bar{\chi}_1)$ , there must exist a channel index  $1 \leq L^* < L$  satisfying  $\max\{-\tau^{\text{mo}}, -\frac{\Omega}{\Lambda}\bar{\chi}_{L^*+1}\} < \mu^* < -\frac{\Omega}{\Lambda}\bar{\chi}_{L^*}$ , or  $L^* = L$  if  $-\tau^{\text{mo}} < \mu^* < -\frac{\Omega}{\Lambda}\bar{\chi}_L$ . From Lemma 1 we have  $[\Phi_l(\mu^*)]^+ > 0$ ,  $\forall l \leq L^*$  and  $[\Phi_l(\mu^*)]^+ = 0$ ,  $\forall l > L^*$ .

When  $\tau^{\text{mo}} \leq \frac{\Omega}{\Lambda}\bar{\chi}_1$ , in order to have  $\lambda_{\text{tot}}^{\text{eq}}(\mu) \geq 0$ , by (13) we must have  $\mu \geq -\tau^{\text{mo}} \geq -\frac{\Omega}{\Lambda}\bar{\chi}_1$ , which leads to  $[\Phi_l(\mu)]^+ = 0$ ,  $\forall l$  by Lemma 1.  $\blacksquare$

We further illustrate this lemma numerically in Fig. 2b in Section V. With this unique  $\mu^*$  and channel index  $L^*$ , we obtain the unique solution  $\lambda_{\text{tot}}^{\text{eq}}(\mu^*)$  and  $\lambda_l^{\text{eq}}(\mu^*)$ ,  $\forall l$  of problem (11). Substituting these values into (10) and (7), we can achieve  $s_l^*$  and  $p_l^*$ ,  $\forall l$ .

**Proposition 2.** With  $\mu^*$  and  $L^*$  from Lemma 2, the load balancing and pricing optimal solutions of operator's revenue maximization problem are unique such that  $s_l^* = \frac{\Phi_l(\mu^*)}{\lambda_{\text{tot}}^{\text{eq}}(\mu^*)}$ ,  $p_l^* = [p_l(s_l^*, \Phi_l(\mu^*))]^+$  from (7),  $\forall l \leq L^*$ , and  $s_l^* = 0$ ,  $p_l^* = 0$ ,  $\forall l > L^*$ .

4) *Algorithm:* The optimal solutions of the operator's revenue maximization problem provided by Proposition 2 is based on the assumption that  $\lambda_{k,l}^{\text{eq}} > 0$ ,  $\forall k, l$ , which does not always hold true. As a result, we propose Algorithm 1 to search for a class  $K^* \leq K$  such that the optimal solutions in Proposition 2 corresponds to  $\lambda_{k,l}^{\text{eq}} > 0$ ,  $\forall l \leq L^*$ ,  $k \leq K^*$ . We can consider  $K^*$  the largest class that can be supportable by the operator. We have the following property on which Algorithm 1 relies.

**Lemma 3.** Defining  $\Lambda(k) \triangleq \sum_{j=1}^k \Lambda_j$  and  $\Omega(k) \triangleq \sum_{j=1}^k \Lambda_j \theta_j$ ,  $\frac{\Omega(k)}{\Lambda(k)}$  is increasing in  $k \in \mathcal{K}$ .

*Proof:* Since  $\Omega(k+1) = \Omega(k) + \Lambda_{k+1}\theta_{k+1}$ ,  $\Lambda(k+1) = \Lambda(k) + \Lambda_{k+1}$ , we have

$$\frac{\Omega(k+1)}{\Lambda(k+1)} - \frac{\Omega(k)}{\Lambda(k)} = \frac{\Lambda_{k+1}(\Lambda(k)\theta_{k+1} - \Omega(k))}{\Lambda(k)(\Lambda(k) + \theta_{k+1})} > 0. \quad (20)$$

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**Algorithm 1** Optimal Pricing and Load-Balancing in the Monopoly Market
 

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- 1: The operator collects  $\bar{\chi}_l, \bar{\chi}_l^2, \forall l$  and  $\Lambda_k, \theta_k, \forall k$
  - 2:  $i \leftarrow \arg \max_{k \in \mathcal{K}} \left\{ r > \frac{\Omega(k)}{\Lambda(k)} \bar{\chi}_1 \right\}$
  - 3: update  $p_l^*(i), s_l^*(i)$  and  $L^*(i)$  by Prop. 2 with  $\Lambda(i), \Omega(i)$
  - 4: **while**  $p_l^*(i) \geq r - \theta_i \bar{\chi}_l$  for some  $l \leq L^*(i)$  **do**
  - 5:    $i \leftarrow i - 1$
  - 6:   repeat step 3
  - 7: **end while**
  - 8:  $K^* \leftarrow i, p_l^* \leftarrow p_l^*(K^*), s_l^* \leftarrow s_l^*(K^*), \forall l$
  - 9: Operator broadcasts  $p_l^*, s_l^*$  and  $\bar{T}_l(\lambda_l^{\text{eq}}(\mu^*)) \forall l$ , and all SUs join the network by Definition 1.
- 

**Proposition 3.** *If*

$$r > \theta_1 \bar{\chi}_1, \quad (21)$$

Algorithm 1 always returns a class  $K^* \geq 1$ .

*Proof:* We prove by contradiction. Assuming that Algorithm 1 cannot return any optimal class, which means in the worst case with a single class  $k = 1$ , we have  $p_1^*(1) \geq r - \theta_1 \bar{\chi}_1 > 0$  for some  $1 \leq l \leq L^*(i)$  with condition (21), which means  $\lambda_{1,l}^{\text{eq}} = 0$  according to Proposition 1, implying  $s_1^*(1) = 0$  by (10), leading to  $p_1^*(1) = 0$  for these  $l$ s according to Proposition 2, which is a contradiction. ■

*Remark 1.* i) Since  $\theta_1 \bar{\chi}_1$  is the smallest cost that can be experienced by a potential SU (without queuing delay and with zero price), condition (21) eliminates a trivial scenario, that is when a SU does not have any incentive to join the operator. ii) In line 1 of Algorithm 1, all parameters can be estimated by the existing method [20] and through some feedback mechanisms from the SUs. The algorithm then determines the largest supportable class  $i$  (line 2), where we can always find such a class  $i \geq 1$  with condition (21). According to Lemma 3, condition (17) is always satisfied with  $\frac{\Omega(j)}{\Lambda(j)}, \forall j \leq i$ . Thus, from Proposition 2, we can always obtain  $p_l^*(j), s_l^*(j), \forall j \leq i$  in line 3 of Algorithm 1. Thus, the algorithm keeps lowering this largest class until, if it is possible, there is a class  $K^*$  and the corresponding  $L^*$  satisfying condition  $p_l^* < r - \theta_k \bar{\chi}_l, \forall l \leq L^*, k \leq K^*$  (lines 3 to 8 of Algorithm 1), which induces  $\lambda_{k,l}^{\text{eq}} > 0, \forall l \leq L^*, k \leq K^*$  according to Proposition 1. iii) In line 3, Algorithm 1 has to compute  $\mu^*$  satisfying Lemma 2. This  $\mu^*$  can be found using a bisection method of which the complexity, which is considered a constant in most common cases, depends on chosen starting points and a tolerance [21]. Hence, Algorithm 1 has a complexity  $O(K)$ .

## V. NUMERICAL RESULTS

In this section, we apply the analytical results to numerically illustrate Algorithms 1.

First, 15 classes of SUs are considered, represented by  $(\theta_1, \theta_2, \dots, \theta_{15}) = (0.2, 0.4, \dots, 3)$ . Furthermore,  $\Lambda_k$  follows a uniform distribution on  $[0, 3]$  for  $k = 1, \dots, 15$ . Second, we consider a shared-use operator that has five channels.

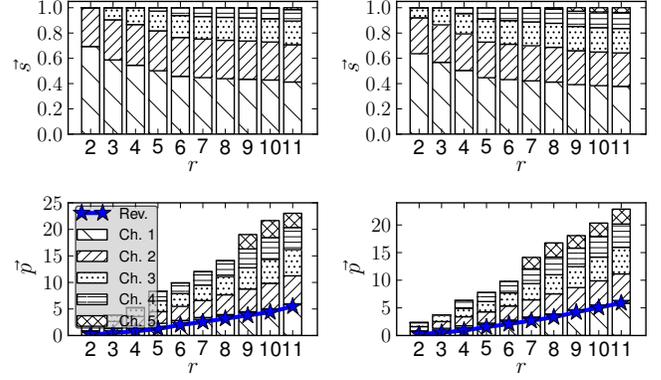


Fig. 1. The optimal load balancing and pricing solutions, and revenues of  $\mathcal{O}_1$  in the setting ExpErl (left column plots) and UniExp (right column plots).

In the first setting termed ExpErl,  $X$  has the exponential distribution with probability density function (pdf)  $f_X(x) = \mu_X e^{-\mu_X x}$ , whereas  $Y$  and  $Z$  have the Erlang distributions with pdfs  $f_{Y_l}(y) = \mu_{\text{on}}^2 y e^{-\mu_{\text{on}} y}$  and  $f_{Z_l}(z) = \mu_{\text{off}}^2 z e^{-\mu_{\text{off}} z}$ ,  $\forall l$ , respectively. We set  $\mu_X$  to 1 (i.e.  $\bar{X} = 1$ ) and the PU activities from channel 1 to 5 are set to  $(\mu_{\text{on}}, \mu_{\text{off}}) = (1.5, 0.5), (1.2, 0.8), (1.0, 1.0), (0.8, 1.2)$  and  $(0.5, 1.5)$ . In the second setting termed UniExp,  $X$  is uniformly distributed on  $[0.1, 1.9]$  (i.e.  $\bar{X} = 1$ ), whereas  $Y$  and  $Z$  have exponential distributions with  $f_{Y_l}(y) = \mu_{\text{on}} e^{-\mu_{\text{on}} y}$  and  $f_{Z_l}(z) = \mu_{\text{off}} e^{-\mu_{\text{off}} z}$ ,  $\forall l$ , respectively, where the PU activities from channel 1 to 5 are  $(\mu_{\text{on}}, \mu_{\text{off}}) = (1.4, 0.6), (1.3, 0.7), (1.0, 1.0), (0.7, 1.3)$  and  $(0.4, 1.6)$ . The PU channels in both settings models the increasing PU occupancy, i.e., light to heavy PU traffic. From [8],  $\bar{\chi}_l$  and  $\bar{\chi}_l^2$  of ExpErl are (1.25, 1.54, 2.0, 2.85, 5.0) and (3.52, 5.73, 10.33, 22.37, 72.38), respectively; and those of UniExp are (1.33, 1.66, 2.0, 2.5, 4.0) and (2.7, 4.64, 7.08, 11.69, 32.32), respectively, for  $l = 1, \dots, 5$ . For all channels, the second moments of case ExpErl are greater than those of UniExp. Finally, for the exclusive-use operator, we simply set  $\bar{X} = 1$  to illustrate the same bandwidth for all channels of  $\mathcal{O}_1$  and  $\mathcal{O}_2$ .

Fig. 1 illustrates a sample of the optimal load balancing and pricing solutions, and the optimal revenues of the monopolist  $\mathcal{O}_1$  in both settings ExpErl and UniExp when  $r$  is varied. From observation, we note that when  $r$  is small, only those channels  $l$ s with low value  $\bar{\chi}_l$  and  $\bar{\chi}_l^2$  are activated with  $s_l > 0, p_l > 0$ ; and the setting ExpErl, with greater channels' variability, has less activated channels than those of UniExp (e.g. when  $r = 2$ , only channels 1 and 2 of ExpErl and channels 1, 2 and 3 of UniExp are active). For these activated channels, it is clear that the channel with low value  $\bar{\chi}_l$  and  $\bar{\chi}_l^2$  will have high value  $s_l$ . When  $r$  increases, the number of activated channels, the optimal prices and revenues also increase in both settings. Furthermore, we observe that when  $r$  keeps increasing, the load balancing solution converges to a fixed distribution.

Fig. 2a indicates the largest class value  $\theta_{K^*}$  that is supportable by  $\mathcal{O}_1$  using Algorithm 1. In both settings, it can be seen that the higher value  $r$ , the higher class- $K^*$  SUs that can be admitted into the monopoly network. We also observe that the

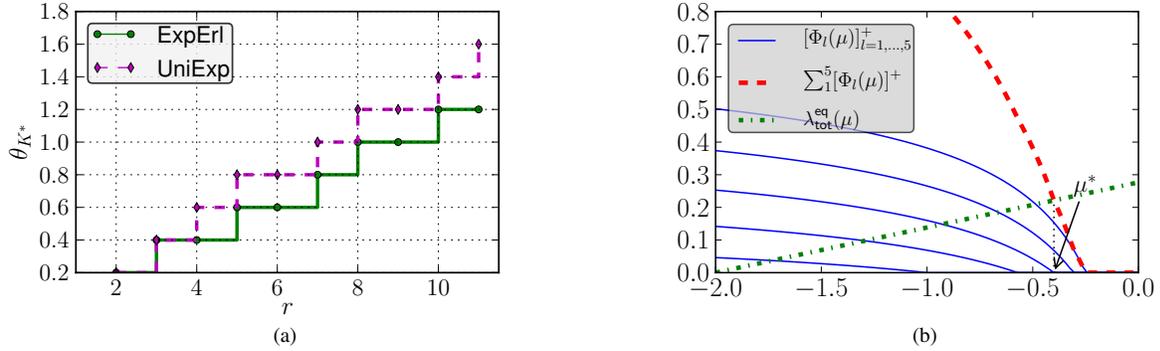


Fig. 2. (a) The largest class value  $\theta_{K^*}$  that can be supportable by  $\mathcal{O}_1$  using Algorithm 1, (b) Numerical illustration of Lemma 2 in the setting ExpErl with  $r = 2$ . The lines of  $[\Phi_l(\mu)]^+$  are continuous and decreasing to 0 at  $-\frac{\alpha}{\Lambda} \bar{\lambda}_l$ ,  $l = 1, \dots, 5$ , respectively. The functions  $\sum_{l=1}^5 [\Phi_l(\mu)]^+$  and  $\lambda_{\text{tot}}^{\text{eq}}(\mu)$  intersect at a unique  $\mu^*$  that corresponds to  $L^* = 2$ .

UniExp setting with lower channels' variability can support higher class SUs than ExpErl does.

Fig. 2b illustrates Lemma 2 in the setting ExpErl when  $r = 2$ . With the unique value  $\mu^*$ , we can see that the corresponding index  $L^* = 2$  where  $[\Phi_l(\mu)]^+ = 0$  for  $l = 3, 4, 5$  and  $[\Phi_l(\mu)]^+ > 0$  for  $l = 1, 2$ , which leads to  $s_l > 0$ ,  $l = 1, 2$ , in the top left graph of Fig. 1 with  $r = 2$ .

## VI. CONCLUSION

Dynamic spectrum access control is traditionally separated by pricing and load balancing aspects. However, we perform a joint optimization in a monopoly market. We first address the constraint in the operators' revenue optimization problem, that is the heterogeneous multi-class SUs' equilibrium behavior, which can be decomposed into smaller problems revealing their convex structures. By using this information in the next step, we provide the unique optimal pricing and load balancing solutions. Finally, we propose a low-complexity algorithm for the operator' revenue maximization. We will consider the oligopoly markets where more than two operators compete for SUs' customers in the future work.

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