Poverty Trap and Public Capital

Yoseph Y. Getachew
UNU-MERIT

Abstract

The paper develops a growth model of multiple equilibria. In the model, poverty trap is caused by the presence of negative externalities – side effects– during an economy's take off. The role of growth-maximizing public investment in this economy is more decisive than in ergodic ones. Depending on the economy's initial condition, even a temporary policy shock may bring a permanent growth miracle, particularly, if the shock is not too weak or followed by a negative offsetting/counterbalancing one.

1 Introduction

In traditional economic growth models, efficient practices predict efficient outcomes.[1] In a perfectly competitive economy, agents make perfectly rational decisions that unanimously lead to unique, high-income equilibrium outputs. However, a snapshot of the real world provides a different picture: the fact is that both efficient and inefficient economies coexist. Theoretically, deviations from the neoclassical benchmark could generate inefficient outcomes along with efficient ones. A number of papers, for instance, show that multiple equilibria are likely to rise when fertility is endogenous,[2] technologies are non-convex,[3] income inequality prevails and/or the capital market is imperfect,[4] etc.[5]

[1] Classic examples are Solow (1956), Romer (1986), and Lucas (1988).

[2] Becker et al. (1990).

[3] Azariadis and Drazen (1990).

[4] Galor and Zeira (1993) and Galor and Tsiddon (1997).

[5] See also Galor (1996), Azariadis (1996), and Azariadis (2006) for more models with multiple equilibria with subsistence consumption, impatient government, incomplete market, monopolistic competition in product or factor market, augmented human capital, externalities, and income distribution.

This paper presents other possibilities that could cause multiple equilibria, along with a qualitative analysis (using phase-diagrams) of the role of growth-maximizing public investment in poverty trap which are rarely addressed in the literature.[6] The study shows analytically how a poverty trap could arise due to side effects related to ongoing economic progress, and examines the role of growth-maximizing public investment in evading it.

It is well acknowledged while economic development has tremendous benefits (e.g., improved health and education quality), it also has downsides (e.g., a temporary increase in income inequality, pollution, skilled migration, and lower social cohesion).[7] Some of these phenomena may seriously damage further economic progress. For instance, a temporary increase in brain drain (skilled migration) during an economy's takeoff could potentially cause poverty trap, and the literature provides no reason why it should not. Sometimes referred as "migration humps, or temporary increases in emigration during a country's economic takeoff,"[8] this is both a notable and a theoretically and empirically supported phenomenon that is detrimental to economic growth.

We have thus developed a model that shows economic development not only as a source of positive learning-by-doing externality that enhances further productivity, but also as a source of negative externality that discourages it. In the model, therefore, production takes place in an environment where learning-by-doing externality prevails, in line with Arrow (1962), Frankel (1962), and Romer (1986), which complements individual production and hence promotes endogenous growth. However, along with this learning-by-doing externality, a negative development-related externality also exists, which deters individual and aggregate productivity. The development-related problem is a temporary phenomenon particularly assumed to arise at the initial stage of economic growth, and then decline. We focus on a *particular instance* of "migration hump"–type brain drain as the development-related problem.

In the model, individuals accumulate human capital via private and public inputs while using it for goods production. Production of final goods takes place using Newman and Read's (1961) production function. This is a *generalized* Cobb-Douglas (CD) production function, but for a certain value of its parameter it contains the popular CD function. It

[6] Galor and Tsiddon (1997) have done qualitative analysis (using phase-diagrams) on *technology*'s effect on poverty trap. Azariadis (2006, p. 32-34) presents an informal discussion of the role of public policy in economies characterized by multiple equilibria and poverty trap.

[7] See, e.g., Kuznets (1955) for a temporary increase in income inequality during an economy's take-off.

[8] Martin and Taylor (1996, p. 45).

is less restrained on technologies; specifically, it is a variable elasticity of substitution (VES) production function rather than a standard CD production function. We use certain parameters of the Newman-Read production function to denote the development-related problem, in particular the "migration hump" effect.

The dynamics of the economy described above, for a range of parameters, yields a multiplicity of growth paths with possibilities that the economy could converge to a low or high equilibrium depending on initial human capital wealth. In the model, economies that start out below a certain critical value of initial economic development (or initial human capital wealth) may converge to the low-income equilibrium (poverty trap). However, they could monotonically converge to the high-income equilibrium if their initial human capital wealth is beyond the threshold value. The multiplicity of growth paths is mostly related to changes in technological states. The hump-shaped brain drain "interchanges" increasing for decreasing returns to social input (aggregate human capital), which gives the production function a convex and concave curvature at the bottom and upper part respectively.[9]

This paper analyzes the role of growth-maximizing public investment in the poverty trap model. Although policy may not *necessarily* enable a country to evade poverty trap, its role in the economy described above is important.[10] In the model, a policy shock changes the threshold value of the poverty trap. But whether the change enables the economy to evade the trap depends on other exogenous factors, such as the history and technology of the country at stake. In general, the effect of public policy in evading poverty trap is undecided in the model. (1) On the one hand, a policy shock *could* result in a growth miracle. Optimal (in terms of growth-maximizing) public investment may create a growth miracle by increasing the human capital stock from slightly below to slightly above the critical threshold value that determines the long-run growth path of the economy. It could even abolish the poverty trap and thus create an opportunity for the economy to move to a unique and globally stable high-income steady state equilibrium.[11] (2) Alternatively, a policy shock could change the critical value, but not enough to evade the poverty trap. In other words, the change may not leave the country above the threshold value, and hence the economy will inevitably converge to a low-income

[9] See Azariadis and Drazen (1990) for more information on how interchanging increasing to decreasing returns to a social input could result in multiple equilibria.

[10] Azariadis (2006, p. 32-34) argues informally that policy and history matter much more in an environment with poverty trap than they do in one without, such as that is found in a traditional ergodic growth model.

[11] See Galor and Tsiddon (1997) for a similar effect of technology on poverty trap.

steady state equilibrium.

The proposition that brain drain could cause poverty trap is based upon two main premises that are largely supported by empirical evidence. The first is that there is a nonlinear relationship between migration and economic development. That is, economic development is one of the fundamental, driving forces of out-migration that rises at the initial stage of economic growth and then declines (see, e.g., Martin and Taylor 1996; Hatton and Williamson 2005; Chiswick et al. 2003).[12] Martin and Taylor (1996) state that migration humps, or temporary increases in emigration during a country's economic growth, are not new phenomena. Rather, they argue, migration humps have existed from the 19th century (Europe's period of industrialization) to the modern days of East Asian countries' growth miracles.[13]

It is largely believed that skilled migration constitutes the lion's share of total migration from developing to developed countries, particularly in recent times. According to Adams (2003), the vast majority of migrants from developing countries to the United States and the Organization for Economic Cooperation and Development (OECD) have a secondary or higher education. Hatton and Williamson (2006, p.328-329) calculated that the ratio of highly educated emigrants to total emigrants from poor nations to the OECD in 1990 averaged more than 14 to 1. Mishra (2007) found that many Caribbean countries have lost more than 70% of their skilled labor forces (12 years plus) due to emigration to the OECD. Docquier et al. (2007) state that, "Between 1990 and 2000, the stock of skilled immigrants in OECD countries increased by 64 percent. The rise was stronger for immigrants from developing countries (up 93 percent), especially from Africa (up 113 percent), Latin America and the Caribbean (up 97 percent)." Collier et al. (2004) documented that the last decade's "hemorrhage" of African human capital is accelerating.[14] The authors argued that Africa's financial capital flight (which, up to the late 1980s, reversed the human capital flight) is the new challenge in Africa's post-independence history.[15]

[12] See Ziesemer (2008) for a detailed survey of the literature on migration humps.

[13] The most common reasons mentioned in the literature as to why migration humps exist are "supply-push emigration" (Martin and Taylor 1996) and an increase in people's capabilities and aspirations during economic development (de Haas 2007).

[14] See also Fosu et al. (2004) for a discussion on Africa's challenge with respect to the flight of its human and other capital flight .

[15] Africa also experienced high growth rate in the 1990s. Ndulu et al. (2007) noted: "Since 1995, more than one-third of the countries in SSA are growing at average rates exceeding 5 percent annually. Several others have shown themselves to be capable of short spurts of high growth. The challenge for them is how to sustain such a pace for longer periods."

The second premise is that brain drain could be detrimental to the economy of either the home or the destination country. It could negatively affect the quantity and quality of human capital of those remaining at home, which, in turn, could hurt the economic growth of the home country. Bhagwati and Hamada (1974), in one of the earliest contributions on this topic, argued that the drain of highly skilled individuals is a loss to those left behind, and has negative implications for the income and welfare of the destination country. The negative impact of brain drain on the home country has also been emphasized in recent endogenous growth literature (Miyagiwa 1991; Haque and Kim 1995; Galor and Tsiddon 1997; Wong and Yip 1999; Beine et al. 2001). Wong and Yip (1999) argue that brain drain damages both the nonemigrants and the source country's economic growth. Haque and Kim (1995) developed a two-country endogenous growth model and showed that brain drain negatively affects the growth rate of the effective human capital of the emigrants' country and hence reduces its economic growth.

Recently, some studies have turned the issue of brain drain into brain gain by searching for some compensatory effect such as remittances, return migrations, or enhanced human capital accumulation. The main rationale of the latter is that when education is privately and endogenously determined, the possibility of migrating to a higher-wage country increases the average rate of return in the home country, which in turn increases domestic individual investment in human capital accumulation (Mountford 1997; Stark et al. 1998; Beine et al. 2001). However, the fact that much of the educational investment in developing countries is undertaken by the state (Fosu 2007) could undermine this compensatory effect. Moreover, some argue that the high probability of emigration could lead individuals to "under-invest" in education. Lien and Wang (2005) developed a model showing that when individuals choose education and language before migration, they can invest less in their human capital, depending on the substitution effect between language and education. "The result is a less educated, 'Americanized' population with better language skills and lower human capital" (Lien and Wang 2005, p. 154).

This paper is organized as follows: Section 2 describes the model, while Section 3 discusses the dynamics of aggregate variables, multiple equilibria and poverty trap. Section 4 analyzes the effect of optimal productive public investment in poverty trap. Section 5 concludes.

2 The Model

2.1 Preferences and Technologies

Suppose we have a continuum of homogeneous households, $i \in [0, 1]$, with overlapping generations. Each household i consists of an adult of generation t and a child of generation $t + 1$. The population size is thus constant and normalized to be one. Let, at the beginning, each household i of the initial generation be endowed with an initial human capital h_0.

When young, individuals accumulate human capital using both private and public input in a standard CD production technology. As adults, they use their accumulated human capital for final goods production. The government taxes income using a flat rate tax ψ in order to finance the public input, denoted by G_t, which is used to complement the accumulation of human capital.[16] During their active period, individuals allocate after-tax income between current consumption c_t and saving e_t to use for their children's education. The latter is incorporated in individuals' utility function as the "joy of giving." Therefore, altruistic individuals derive utility from consumption and giving, i.e., investing in the human capital of their offspring h_{t+1}.

The utility of an individual is thus defined as

$$u_t\left(c_t, e_t\right) \equiv \ln c_t + \beta \ln e_t \tag{1}$$

subject to

$$c_t + e_t = (1 - \psi) y_t \tag{2}$$

where y_t represents the individual's income.

As previously mentioned, the human capital accumulation function of the offspring h_{t+1} is a function of public investment G_t and parental investment e_t. The accumulation function takes the standard CD form, with constant returns to scale in factors. Thus, for an individual born at time t, the human capital at $t + 1$ is given by

$$h_{t+1} = A\left(G_t\right)^{1-\eta}\left(e_t\right)^{\eta} + \chi \tag{3}$$

where $\chi > 0$ is a parameter that assures $h_{t+1} \neq 0$ even if parental investment on education is $e_t = 0$.

The government levies a flat-rate tax ψ on output Y_t, which is used to finance public investment. The government budget is balanced at all times as

[16] Note that in this paper, lower- and uppercase letters are used to denote individual and aggregate/average variables, respectively.

$$G_t = \psi Y_t \tag{4}$$

where G_t and Y_t represent public investment and aggregate income, respectively.

According to the above descriptions, an adult of period t solves the following problem, which is derived by substituting (2) into (1),

$$\underset{e_t}{Max} \ln\left((1-\psi)y_t - e_t\right) + \beta \ln e_t \tag{5}$$

taking as given ψ and G_t.

The first-order condition gives

$$e_t = a(1-\psi)y_t \tag{6}$$

where $a = \frac{\beta}{1+\beta}$. Equation (6) shows an individual's saving as a fraction of her after-tax income.

2.2 Goods Production

There are infinite numbers of competitive small firms owned by households. We suppose that production at firm level occurs using both private and social inputs in a Newman and Read (1961) production setting. At time t, output y_t is produced using individual and aggregate human capital inputs, denoted by h_t and H_t respectively. However, we model the latter to reflect positive learning-by-doing spillover, in line with Arrow (1962), Frankel (1962), and Romer (1986), along with a negative development-induced externality.

The Newman and Read (1961) production function is employed to capture these phenomena together. This is a *generalized* CD production function that contains the popular CD production function for a certain value of its parameter. It is less restrained in technologies in that it is a VES production function with a variable-factors income share. Although its use is relatively rare in the literature of economic growth, the Newman-Read production function offers a powerful analytical framework for economic development studies. In addition to its suitability for modeling a negative development-induced spillover (e.g., a temporary increase in skilled migration) along with a positive one (e.g., learning by doing), unlike the CD and the constant elasticity of substitution (CES) production functions, it is more suitable for income distribution studies[17]

[17] Getachew (2008) argued that the standard production functions such as CD and CES lack either flexibility in parameters or analytical tractability with regard to income distribution studies. However, the Newman-Read generalized CD function is both flexible in the values of the parameters and analytically tractable with respect to distribution studies. Getachew (2008) applied the Newman-Read production function in analyzing the effect of public capital on income inequality dynamics.

and gives a better approximation of the reality of the short-run behavior of factor shares.[18]

Thus, the income of an agent of generation t, in the Newman-Read production function, is defined as

$$y_t = (h_t)^{\alpha} (H_t)^{\beta} \exp\left(-\lambda(H_t) \ln H_t \ln h_t\right) \tag{7}$$

where y_t and h_t represent individual output and human capital, respectively; H_t is aggregate human capital, which is defined as $H_t \equiv \int_0^1 h_t \Gamma(h_t)$, where $\Gamma(h_t)$ is the distribution of wealth at time t. We assume:

$$H_t \geqslant 0; \;\; h_t \geqslant 0 \tag{A.1}$$

$$\lambda(H_t) > -1 \tag{A.2}$$

The exponential term $-\lambda(H_t) \ln H_t \leq 0$ in the Newman-Read function may represent, in general, the negative externalities that may arise in a country's economy (such as skilled "migration hump") during the economy's takeoff.[19] $\lambda(H_t)$, which is a fixed parameter (or simply λ) in the original Newman-Read function, is assumed here to be a function of aggregate human capital.

We use $\lambda(H_t) \ln H_t$ as a function of aggregate human capital to dictate a skilled migration hump, or a nonlinear relationship between brain drain and economic development, as the latter is denoted by an increase in H_t. Therefore, we shall assume $\lambda(H_t) \ln H_t$ to rise at the initial stage of economic development and then to decline as shown in Figure 1. [20]

[18] Empirical studies reveal that factor shares show large short-run fluctuations, but no long-run trend (e.g. Acemoglu 2003). The Newman-Read generalized CD production function contains a framework that provides a satisfactory approximation to this reality, particularly in contrast to the popular CD function. The latter imposes strict restrictions on relative factor share and on the elasticity of substitution between factors. In particular, the factor shares are constant and the factors' elasticity of substitution is equal to unity.

[19] This negative externality can be easily understood as quality and quantity reductions in *effective* human capital due to brain drain. For instance, if a home country produces human capital H_t, then we may define the effective human capital, after brain drain takes place, $\underline{H}_t \equiv H_t \exp(-\phi(H_t))$, where $\underline{H}_t$ and $\phi(H_t) \geqslant 0$ denote the effective human capital and the rate of brain drain. If we substitute this in a simple CD production function such as $y_t = (h_t)^{\alpha} (\underline{H}_t)^{\beta}$, then we get a production function *similar* to (7), $y_t = \exp(-\beta\phi(H_t)) (h_t)^{\alpha} (H_t)^{\beta}$.

[20] Note that migration hump is treated here in an *exogenous* manner. That is, the hump shape attributed to $\lambda(H_t) \ln H_t$ is an assumption based on an observation that has large empirical support.

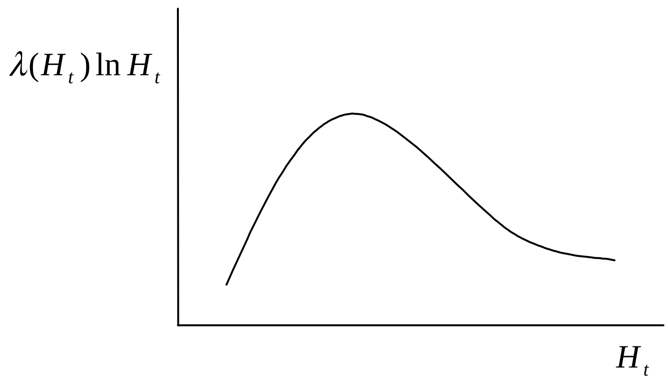

Figure 1: A negative development-induced externality.

We further assume that the production function in (7) exhibits diminishing (increasing) returns to scale with respect to individual (total) input(s):

$$0 < \alpha, \beta < 1, \text{ but } \alpha + \beta > 1 \tag{A.3}$$

2.3 Properties of Individual and Aggregate Production Function

As noted above, the production function used here is of a Newman and Read (1961) form. For certain values of its parameters, the Newman-Read production function satisfies the standard properties of production functions. As Solow (1957) noted, a particular functional form adopted for a production function is a matter of no great consequence as far as it possesses a positive partial derivative and the right curvature.

The first and the second derivative of the production function in (7) are positive and negative, respectively:

$$\frac{\partial y_t}{\partial h_t} = \frac{y_t}{h_t}(\alpha - \lambda(H_t)\ln H_t) > 0$$

$$\frac{\partial y_t}{\partial h_t \partial h_t} = \frac{y_t}{(h_t)^2}\left(\alpha - \lambda(H_t)\ln H_t - 1\right)\left(\alpha - \lambda(H_t)\ln H_t\right) < 0$$

assuming[21]

[21] Newman and Read (1961) show that their production function obeys neoclassical

$$\alpha > \lambda(H_t)\ln H_t;\ \beta > \lambda(H_t)\ln H_t \tag{A.4}$$

Therefore, at an individual level, and with respect to private factor human capital, the Newman-Read production function obeys the neoclassical rule in that it has a positive marginal productivity and a concave curvature.

In characterizing the properties of individual and aggregate production functions with respect to the social human capital H_t, we assume that the elasticity of the brain drain parameter $\lambda(H_t)$ to aggregate human capital H_t is sufficiently small, in a well-defined sense,

$$-\infty < \rho(H_t) \equiv \frac{H_t\lambda'(H_t)}{\lambda(H_t)} < \frac{\beta - \lambda(H_t)\ln H_t}{\lambda(H_t)\ln H_t \ln h_t} \tag{A.5}$$

where $\rho(H_t)$ denotes the elasticity of $\lambda(H_t)$ to H_t.

We establish the following two Lemmas to characterize the properties of individual and aggregate production functions with respect to the social human capital H_t.

Lemma 1 *Assumption (A.5) provides sufficient condition for an individual production function to have a positive marginal productivity,* $\frac{\partial y_t}{\partial H_t} > 0$*, with respect to the social input* H_t*.*

Proof. The first derivative of (7) with respective to H_t is given by

$$\frac{\partial y_t}{\partial H_t} = \frac{y_t}{H_t}\left[\beta - H_t\lambda'(H_t)\ln H_t \ln h_t - \lambda(H_t)\ln h_t\right]$$

Then, sufficient condition for $\frac{\partial y_t}{\partial H_t} > 0$ is

$$\beta > H_t\lambda'(H_t)\ln H_t \ln h_t + \lambda(H_t)\ln h_t$$

Rearranging the above gives

$$\rho(H_t) \equiv \frac{H_t\lambda'(H_t)}{\lambda(H_t)} < \frac{\beta - \lambda(H_t)\ln H_t}{\lambda(H_t)\ln H_t \ln h_t}$$

■

Aggregate income is simply derived by aggregating (7),[22]

rules (positive marginal productivity and concavity) if the parameter $\lambda > -1$. See also assumption (A.2).

[22] Note that $(h_t)^\alpha \exp(-\lambda(H_t)\ln h_t \ln H_t) = (h_t)^{\alpha-\lambda(H_t)\ln H_t}$. To see this, let $x = (h_t)^\alpha \exp(-\lambda(H_t)\ln h_t \ln H_t)$. Then, $\ln x = \ln (h_t)^\alpha - \lambda(H_t)\ln H_t \ln h_t = (\alpha - \lambda(H_t)\ln H_t)\ln h_t$. Thus, $x = (h_t)^{\alpha-\lambda(H_t)\ln H_t}$.

$$
\begin{aligned}
Y_t &= \int_0^1 (h_t)^{\alpha} (H_t)^{\beta} \exp\left(-\lambda(H_t) \ln h_t \ln H_t\right) \\
&= (H_t)^{\beta} (H_t)^{\alpha - \lambda(H_t) \ln H_t}
\end{aligned}
$$

Alternatively,

$$
Y_t = (H_t)^{\alpha + \beta - \lambda(H_t) \ln H_t} \tag{8}
$$

Therefore, equation (8) denotes aggregate production function in the economy. The following Lemma characterizes its property with respect to the social human capital H_t.

Lemma 2 *Assumptions (A.4) and (A.5) provide sufficient conditions for the aggregate production function (8) to have a positive marginal productivity with respect to its factor input H_t, or $\frac{\partial Y_t}{\partial H_t} > 0$.*

Proof. The first derivative of (8) with respect to H_t is given by

$$
\frac{\partial Y_t}{\partial H_t} = \frac{Y_t}{H_t} \left(\alpha + \beta - 2\lambda(H_t) \ln H_t - H_t \lambda'(H_t) \ln^2 H_t\right)
$$

Then, sufficient condition for $\frac{\partial Y_t}{\partial H_t} > 0$ is

$$
\alpha + \beta > H_t \lambda'(H_t) \ln^2 H_t + 2\lambda(H_t) \ln H_t
$$

But, from (A.4) and Lemma 1, we have $\alpha > \lambda(H_t) \ln H_t$ and $\beta > H_t \lambda'(H_t) \ln^2 H_t + \lambda(H_t) \ln H_t$, respectively. ■

The second derivative of the individual and aggregate production function might be positive or negative. In fact, at this point, deviation from the neoclassical benchmark is required to generate multiple equilibria and hence poverty trap.

3 Dynamics, Multiple Equilibria, and Poverty Traps

3.1 Dynamics of Individual and Aggregate Human Capital

In this section we characterize the dynamics of the economy described in previous section. We derive aggregate human capital dynamics using the individual and aggregate production functions, and determine whether multiple equilibria and poverty trap exist. But before we start dealing with aggregate variables, we must first derive individuals' capital accumulation function that is related to their optimal behavior by substituting (4) and (6) into (3), and using (7) and (8),[23]

[23] See Appendix A for details on the derivation.

$$h_{t+1} = A\psi^{\eta}\left(a(1-\psi)\right)^{1-\eta}\left(H_t\right)^{(\alpha+\beta-\lambda(H_t)\ln H_t)\eta+\beta(1-\eta)}\left(h_t\right)^{\alpha(1-\eta)}$$
$$\exp\left(\left(-\lambda(H_t)\ln h_t \ln H_t\right)(1-\eta)\right)+\chi \tag{9}$$

Thus, equation (9) shows an individual's human capital accumulation function, which is associated to her optimal behavior. We simply aggregate (9), in determining the economy's human capital dynamic equation,[24]

$$H_{t+1} = \Omega\left(H_t\right)^{\alpha+\beta-\lambda(H_t)\ln H_t} + \chi \tag{10}$$

where Ω is defined

$$\Omega \equiv A\left(\psi\right)^{\eta}\left(a(1-\psi)\right)^{1-\eta} \tag{10$'$}$$

Equation (10) thus determines the dynamics of the economy, which we characterize in detail below.

3.2 Multiple Equilibria and Poverty Trap

The dynamics of (10) are characterized by the existence of multiple equilibria. For a range of parameters, equation (10) yields multiple growth paths that lead to three steady states, of which two are stable and one is unstable. Of the stable steady states, one is characterized by low-income equilibrium (or poverty trap); the other by high-income equilibrium.

In characterizing (10) and showing the existence of multiple equilibria, we follow Galor and Tsiddon (1997). We thus take the first derivative of (10) to get

$$\frac{\partial H_{t+1}}{\partial H_t} = \Omega\left(H_t\right)^{\alpha+\beta-\lambda(H_t)\ln H_t-1}\left(\alpha+\beta-2\lambda(H_t)\ln H_t - H_t\lambda'(H_t)\ln^2 H_t\right) \tag{11}$$

Following the Lemmas, we define the term in the big bracket of equation (11) as

$$\alpha+\beta-2\lambda(H_t)\ln H_t - H_t\lambda'(H_t)\ln^2 H_t \in (0,\infty) \tag{12}$$

We make the following two additional assumptions which are related to a negative development-related externality:

$$\lim_{H_t\to 0}\alpha+\beta-\lambda(H_t)\ln H_t > 1 \tag{A.6}$$

$$\lim_{H_t\to \infty}\alpha+\beta-\lambda(H_t)\ln H_t < 1 \tag{A.7}$$

[24] See Appendix B for details on the derivation.

Then, together with earlier assumptions (A.1)–(A.5) and hence (12), (A.6) and (A.7) assure that equation (10) becomes a well-behaved non-convex function. (A.6) in particular creates increasing returns to scale at a lower level of human capital related to the existence of low-income equilibrium, whereas (A.7) assures the existence of a high-income steady state equilibrium rather than a divergence one: $\lim_{H_t \to \infty} H_{t+1} = H$. The following proposition summarizes the existence of multiple equilibria and hence poverty trap, in the economy described in equation (10).

Proposition 3 *Given assumptions (A.1)–(A.7), (12), and* $\chi > 0$*, the economy described in (10) is characterized by multiple steady-state equilibria satisfying the following conditions:*

1. $\lim_{H_t \to 0} \frac{\partial H_{t+1}}{\partial H_t} = 0$

2. $\lim_{H_t \to \infty} \frac{\partial H_{t+1}}{\partial H_t} = 0$

3. $H_{t+1} > H_t$ for some values of H_t.

Proof. It follows from the continuity of (10), Figure 2, assumptions (A.1)–(A.7) and the intermediate value theorem (see also Galor and Tsiddon 1997, section 2.5). Equation (10) is continuous at H_t by definition. Therefore, from the intermediate value theorem, the third condition is satisfied. The first and second conditions are satisfied from assumptions (A.6), (A.7) and equation (12).

Together a sufficiently small χ, the first condition assures the existence of a low-income equilibrium, as shown in Figure 2, H_1. The second condition, together with the third, assures the existence of a high-income equilibrium (also shown in Figure 2, H_3). ■

Thus, Figure 2 shows multiple equilibria, with three steady states for a given level of Ω. There is a *low-income stable* steady state H_1, a *threshold unstable* steady state H_2 and a *high-income stable* steady state equilibrium H_3. The second steady state equilibrium, $H_2 \in (H_1, H_3)$, creates a critical point for the low- and high-income steady state equilibria to be realized. The low steady state equilibrium H_1 is similar to what the literature refers to as a "poverty trap." For a given range of technological parameters, both the low and high steady states H_1 and H_3, respectively, are self-reinforcing. Given a technological level Ω, an initial human capital H_0 smaller (greater) than the threshold level human capital H_2 converges to the low steady state H_1(high-income steady state H_3) in the long run.

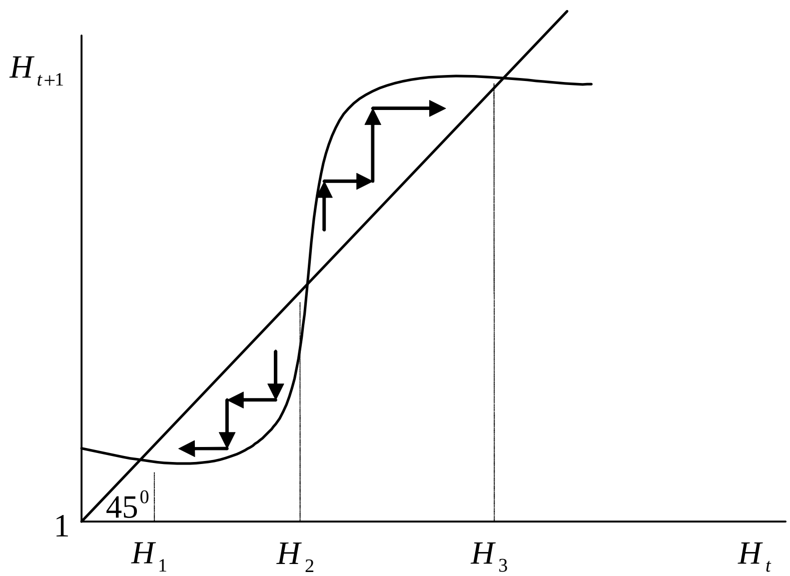

Figure 2: Multiple Equilibria and Poverty Trap

4 Public Investment and Threshold Externality

4.1 Does Public Policy Matter for Long Run Growth?

Is there any role for public policy in this economy? Does growth-maximizing public investment in this economy lead a country to evade the poverty trap? Growth-maximizing public investment in this economy is the tax-rate level that maximizes the technological parameter Ω shown in equation (10), where Ω is defined in (10′).[25] The tax rate that maximizes Ω is given by,

$$\frac{\partial \Omega}{\partial \psi} = 0 \Rightarrow \psi^* = \eta \tag{13}$$

where ψ^* denotes the growth-maximizing tax-rate. In this economy, a policy shock has an effect on the threshold point. Even a temporary policy shock may bring permanent consequences.

In general, two different possibilities are presented in relation to a change in public policy towards growth-maximizing level of public investment. First, such policy change in an economy described in equation (10) may abolish the poverty trap and thus create an opportunity for the economy to move to a *unique* and globally stable high-income steady state equilibrium as shown in Figure 3.[26] The figure illustrates a possible effect of a policy shock via a change in tax rate from a suboptimal to growth-maximizing level of public investment, denoted by ψ^s and ψ^* respectively, where the curves associated with the suboptimal and growth-maximizing policies are represented by $f(H_t, \Omega^s)$ and $f(H_t, \Omega^*)$ respectively. The shock moves the curve defined in (10) upward in the direction of the arrow. As a result, the economy changes from a non-ergodic economy with multiple steady states to an ergodic one with a unique steady state. The shock not only ensures that the economy evades a poverty trap but also creates a new, higher equilibrium point (compare H_3^s and H_3^*).

Second, a policy shock (a change in public policy towards growth-maximizing level of public investment) in an economy described in (10) may change the threshold value, but whether this change leads the economy out of the poverty trap depends on other factors, such as initial income/capital of the economy at stake. If the economy's initial capital is situated near enough to the threshold value, the policy shock may create a growth miracle by increasing the capital stock from slightly below

[25] Recall that Ω constitutes both exogenous technological and public investment parameters.

[26] Galor and Tsiddon (1997) show that a technical progress could have a similar effect.

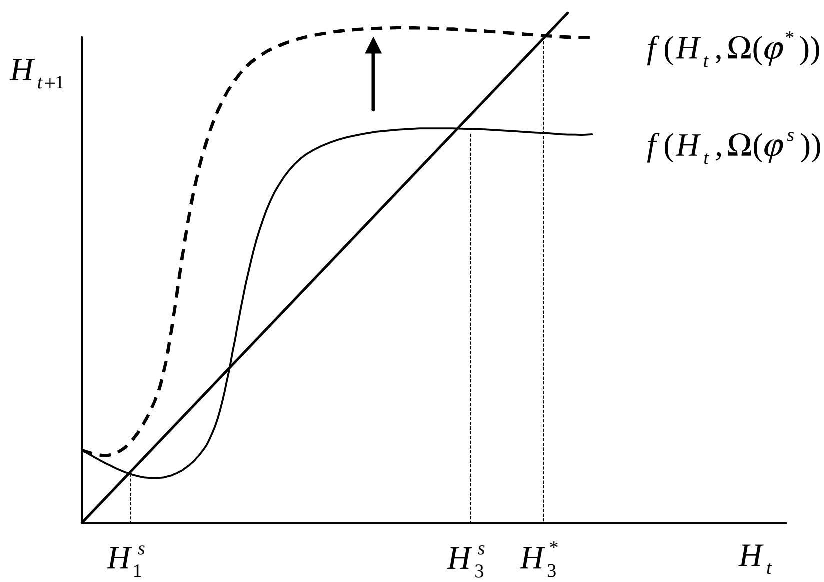

Figure 3: A shock that may abolish a poverty trap and hence result in a growth miracle

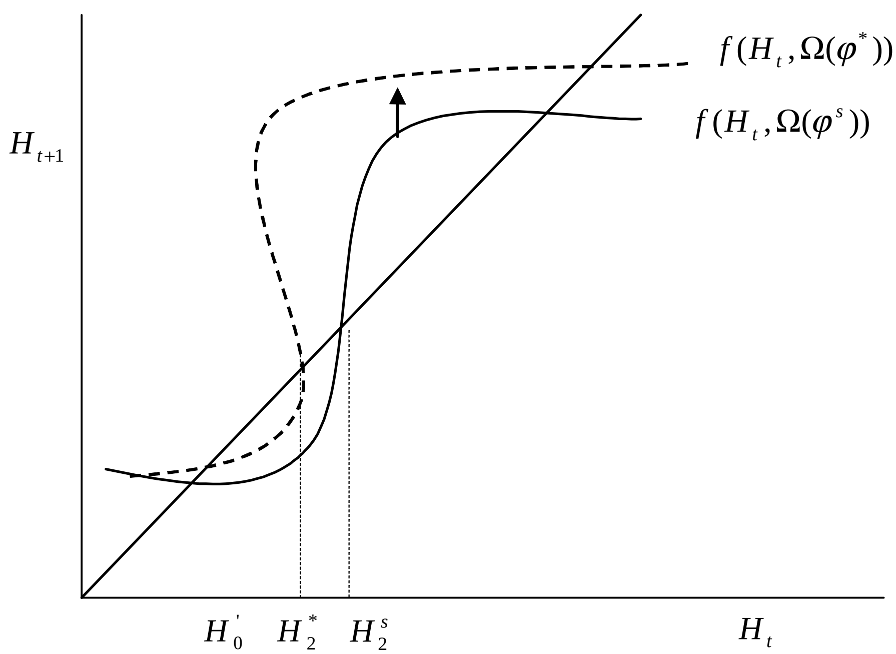

Figure 4: A policy shock that does not necessarily generate a growth miracle

to slightly above the threshold value. However, if the initial income is sufficiently lower than the threshold value, the policy change may not lead the country to evade poverty trap. Figure 4 shows that a policy shock moves the curve in equation (10) upward. However, this time the shock brings the desired growth miracle *conditionally.* That is, the miracle may happen *only if* the initial human capital stock H_0 is located in between the new and the old threshold levels, i.e., $H_0 \in (H_2^s, H_2^*)$. However, if the initial human capital is sufficiently low (smaller than H_2^*), for instance if it is located at H_0', then the economy converges to a low-income steady state equilibrium despite the policy shock.

5 Conclusion

This paper presented a model showing economic development not only as a source of positive learning-by-doing externality that has enhanced further productivity, but also as a source of negative externality that has discouraged it. The presence of negative externalities (e.g., skilled "migration humps" or temporary increases in brain drain during an economy's takeoff) can result in a poverty trap. In the model, a change in public policy towards growth-maximizing level of public investment was found to be crucial but indeterminate with respect to poverty trap. Depending on some initial conditions, however, such policy shock could create a threshold externality that leads economies to evade poverty trap.

A Individual Human Capital Dynamics

To derive the individual's human capital accumulation equation, substitute (4) and (6) into (3), to get

$$\begin{aligned} h_{t+1} &= A\left(G_t\right)^{\eta}\left(e_t\right)^{1-\eta}+\chi \\ &= A\left(Y_t\psi\right)^{\eta}\left(a(1-\psi)y_t\right)^{1-\eta}+\chi \end{aligned}$$

Then, substitute (7) and (8) into the above to get

$$h_{t+1}=A\left(\left(H_t\right)^{\alpha+\beta-\lambda(H_t)\ln H_t}\,\psi\right)^{\eta}\left(\frac{a(1-\psi)\left(h_t\right)^{\alpha}\left(H_t\right)^{\beta}}{\exp\left(-\lambda(H_t)\ln h_t\ln H_t\right)}\right)^{1-\eta}+\chi$$

After rearranging, we get equation (9), of Section 3,

$$\begin{aligned} h_{t+1} = {} & A\psi^{\eta}\left(a(1-\psi)\right)^{1-\eta}\left(H_t\right)^{(\alpha+\beta-\lambda(H_t)\ln H_t)\eta+\beta(1-\eta)}\left(h_t\right)^{\alpha(1-\eta)} \\ & \exp\left(\left(-\lambda(H_t)\left(1-\eta\right)\ln h_t\ln H_t\right)\right)+\chi \end{aligned} \tag{9}$$

B Aggregate Human Capital Dynamics

To get the economy's human capital accumulation function H_{t+1}, at $t+1$, we simply aggregate (9), i.e.,

$$\begin{aligned} H_{t+1} = {} & A\psi^{\eta}\left(a(1-\psi)\right)^{1-\eta}\left(H_t\right)^{(\alpha+\beta-\lambda(H_t)\ln H_t)\eta+\beta(1-\eta)} \\ & E\left[\left(h_t\right)^{\alpha(1-\eta)}\exp\left(\left(-\lambda(H_t)\left(1-\eta\right)\ln h_t\ln H_t\right)\right)\right]+\chi \end{aligned} \tag{Ap.1}$$

Since

$$E\left[\left(h_t\right)^{\alpha(1-\eta)}\exp\left(\lambda(H_t)\left(1-\eta\right)\ln h_t\ln H_t\right)\right]=\left(H_t\right)^{(\alpha-\lambda(H_t)\ln H_t)(1-\eta)} \tag{Ap.2}$$

From combining (Ap.1) and (Ap.2), we get

$$
\begin{aligned}
H_{t+1} = & A\,(\psi)^{\eta}\,(a(1-\psi))^{1-\eta} \\
& (H_t)^{(\alpha+\beta-\lambda(H_t)\ln H_t)\eta+\beta(1-\eta)+(\alpha-\lambda(H_t)\ln H_t)(1-\eta)} + \chi \\
= & A\,(\psi)^{\eta}\,(a(1-\psi))^{1-\eta}\,(H_t)^{\alpha+\beta-\lambda(H_t)\ln H_t} + \chi
\end{aligned}
$$

We may rewrite the last equation to get equation (10), of Section 3,

$$
H_{t+1} = \Omega\,(H_t)^{\alpha+\beta-\lambda(H_t)\ln H_t} + \chi \tag{10}
$$

where

$$
\Omega \equiv A\,(\psi)^{\eta}\,(a(1-\psi))^{1-\eta} \tag{10'}
$$

References

ACEMOGLU, D. (2003): "Labor- and Capital-Augmenting Technical Change," *Journal of the European Economic Association*, 1(1), 1–37.

ADAMS, RICHARD H., J. (2003): "International Migration, Remittances, and the Brain Drain: A Study of 24 Labor-Exporting Countries," The World Bank, Policy Research Working Paper Series: 3069.

ARROW, K. J. (1962): "The Economic Implications of Learning by Doing," *Review of Economic Studies*, 29(June), 155–173.

AZARIADIS, C. (1996): "The Economics of Poverty Traps: Part One: Complete Markets," *Journal of Economic Growth*, 1(4), 449–496.

——— (2006): "The Theory of Poverty Traps: What Have We Learned?," in *Poverty Traps*, ed. by S. Bowles, S. N. Durlauf, and K. Hoff, pp. 17–40. New York: Russell Sage Foundation; Princeton and Oxford: Princeton University Press.

AZARIADIS, C., AND A. DRAZEN (1990): "Threshold Externalities in Economic Development," *Quarterly Journal of Economics*, 105(2), 501–526.

BECKER, G. S., K. M. MURPHY, AND R. TAMURA (1990): "Human Capital, Fertility, and Economic Growth," *Journal of Political Economy*, 98(5), S12–37.

BEINE, M., F. DOCQUIER, AND H. RAPOPORT (2001): "Brain Drain and Economic Growth: Theory and Evidence," *Journal of Development Economics*, 64(1).

BHAGWATI, J. N., AND K. HAMADA (1974): "The Brain Drain, International Integration of Markets for Professionals and Unemployment: A Theoretical Analysis," *Journal of Development Economics*, 1(1), 19–42.

CHISWICK, B. R., T. J. HATTON, M. D. BORDO, A. M. TAYLOR, AND J. G. WILLIAMSON (2003): "International Migration and the Integration of Labor Markets," in *Globalization in historical perspective*, pp. 65–117. NBER Conference Report series. Chicago and London: University of Chicago Press.

COLLIER, P., A. HOEFFLER, AND C. PATTILLO (2004): "Africa's Exodus: Capital Flight and the Brain Drain as Portfolio Decisions," *Journal of African Economies*, 13, ii15–54.

DE HAAS, H. (2007): "Turning the Tide? Why Development Will Not Stop Migration," *Development and Change*, 38(5), 819–841.

DOCQUIER, F., O. LOHEST, AND A. MARFOUK (2007): "Brain Drain in Developing Countries," *World Bank Economic Review*, 21(2).

FOSU, A. K. (2007): "Fiscal Allocation for Education in Sub-Saharan Africa: Implications of the External Debt Service Constraint," *World Development*, 35(4), 702–713.

Fosu, A. K., P. Krishnan, and L. Ndikumana (2004): "Africa and the World Economy: A Focus on Capital – An Overview," *Journal of African Economies*, 13(2), ii1–ii14.

Frankel, M. (1962): "The Production Function in Allocation and Growth: A Synthesis," *The American Economic Review*, 52(5), 996–1022.

Galor, O. (1996): "Convergence? Inferences from Theoretical Models," *Economic Journal*, 106(437), 1056–1069.

Galor, O., and D. Tsiddon (1997): "The Distribution of Human Capital and Economic Growth," *Journal of Economic Growth*, 2(1), 93–124.

Galor, O., and J. Zeira (1993): "Income Distribution and Macroeconomics," *The Review of Economic Studies*, 60(1), 35–52.

Haque, N. U., and S.-J. Kim (1995): "'Human Capital Flight': Impact of Migration on Income and Growth," *International Monetary Fund Staff Papers*, 42(3), 577–607.

Hatton, T. J., and J. G. Williamson (2005): *Global Migration and the World Economy: Two Centuries of Policy and Performance*. Cambridge and London: MIT Press.

Kuznets, S. (1955): "Economic Growth and Income Inequality," *The American Economic Review*, 45(1), 1–28.

Lien, D., and Y. Wang (2005): "Brain Drain or Brain Gain: A Revisit," *Journal of Population Economics*, 18(1), 153–163.

Lucas, Robert E., J. (1988): "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22(1), 3–42.

Martin, P. L., and E. Taylor (1996): "The Anatomy of a Migration Hump," in *Development strategy, employment and migration: insights from models*, ed. by E. Taylor. Organisation for Economic Cooperation and Development, Paris.

Mishra, P. (2007): "Emigration and Brain Drain: Evidence from the Caribbean," *B.E. Journal of Economic Analysis and Policy: Topics in Economic Analysis and Policy*, 7(1), 1–42.

Miyagiwa, K. (1991): "Scale Economies in Education and the Brain Drain Problem," *International Economic Review*, 32(3), 743–759.

Mountford, A. (1997): "Can a Brain Drain Be Good for Growth in the Source Economy?," *Journal of Development Economics*, 53(2), 287–303.

Ndulu, B. J., L. Chakraborti, L. Lijane, V. Ramachandran, and J. Wolgin (2007): *Challenges of African Growth: Opportunities, Constraints, and Strategic Directions*. Washington D.C.: The World Bank.

Newman, P. K., and R. C. Read (1961): "Production Functions with Restricted Input Shares," *International Economic Review*, 2(1), 127–133.

Romer, P. M. (1986): "Increasing Returns and Long-run Growth," *Journal of Political Economy*, 94(5), 1002–1037.

Solow, R. M. (1956): "A Contribution to the Theory of Economic Growth," *The Quarterly Journal of Economics*, 70(1), 65–94.

——— (1957): "Technical Change and the Aggregate Production Function," *The Review of Economics and Statistics*, 39(3), 312–320.

Stark, O., C. Helmenstein, and A. Prskawetz (1998): "Human Capital Depletion, Human Capital Formation, and Migration. A Blessing in a 'Curse'?," Institute for Advanced Studies, Economics Series: 55.

Wong, K.-y., and C. K. Yip (1999): "Education, Economic Growth, and Brain Drain," *Journal of Economic Dynamics and Control*, 23(5-6), 699–726.

Ziesemer, T. (2008): "Growth with Endogenous Migration Hump and the Multiple, Dynamically Interacting Effects of Aid in Poor Developing Countries," United Nations University, UNU-MERIT Working Paper: 2008-057.