THE TOTAL CLAIMS DISTRIBUTIONS UNDER THE DIFFERENT CONDITIONS

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Abstract:
Compound Poisson Process is one of the standard models used for the actuarial risk theory. In this paper we examine the distribution of the number of claims in a fixed time period and also concerned the associated the total claim cost for the fixed period under the different conditions. Including the most popular cases, like Poisson distribution, Mixed Poisson claim number processes, and the mixture compound process and the cases with exponential claim sizes, and some other distributions are discussed.

1. Introduction

The total claims payable on a portfolio of insurance industry is often modeled as the random sum, or a compound distribution, in order to account for randomness in both frequency and severity of claims. Therefore one of the main problems is the evaluation of the distribution of aggregate claims on a portfolio of business. Computation of this probability distribution is extremely hard, but for a certain parametric claim frequency distributions, we are able to use recursive formula to find the probability distribution for total claims. See Panjer (1981), Sundt and Jewell (1981), Willmot, (1988) and Deng (1998, 99, 00).

Traditional models have usually assumed that claim severities are independent and identically distributed random variable. A major difficulty with this assumptions is that in the insurance industry, the total claims distribution may not be that perfect. The claims may results deductible, maximum benefit or inflation. The will give the more difficulty to evaluate the probability distribution of the total claims. The results we had before been not hold for those situations.

The purpose for this paper is to study families introduced by Deng (1999). Then outline those families by allowed one to entertain models with a deductible and maximum benefit per claims to find the total claim distribution under the different conditions.

2. Background and Notation

In the collective risk model the basic concept is that of a random process that generates claims for a portfolio of policies. Let N denote the number of claims produced by a portfolio of policies in a given time period. Let X_1 be the amount of the first claim, X_2 be the amount of the second claim and soon. Then

$$S = X_1 + X_2 + ... + X_N$$

represents the aggregate claims under the study. The number of claims, N, is a random variable and is associated with the frequency of claim. In addition, the individual claim amounts $X_1, X_2, ...$ are also random variables and they are the measurement of the severity of claims.

There are two fundamental assumptions:

- $X_1, X_2, ...$ are identically distributed random variable.
- $N, X_1, X_2, ...$ are independent random variables.

Hence, the probability distribution function of the aggregate claim variable S can be found by the following process.

$$F(x) = P(S \le x) = \sum_{n=0}^{\infty} P(S \le x \mid N = n)P(N = n)$$

$$= \sum_{n=0}^{\infty} P(X_1 + X_2 + ... + X_n \le x)P(N = n) \quad (2.1)$$

In terms of the convolution operation, we can write

$$P(X_1 + X_2 + ... + X_n \le x) = P * P * P ... * P(x)$$

$$= P^{*n}(x) \quad (2.2)$$

Where we use $P(x)$ denote the probability distribution of X_1. It is extremely hard to find F(x), because of the convolution operation.

Panjer's Results:

Panjer (1981) has shown that, if there exist constants a and b such that

$$p_n = p_{n-1}(a + \frac{b}{n}) \qquad (n=1, 2, \dots) \qquad (2.3)$$

Then

$$g(s) = p_1 f(s) + \int_0^s (a + b\frac{x}{s}) f(x) g(s-x) d\mu(x). \quad (2.4)$$

Deng' s results:

Deng (1998) considered the families of claim number distributions satisfies the recursion (1.1)

$$p_n = \frac{a}{n} p_{n-1} + \frac{b}{n(n-1)} p_{n-2} \qquad n= 2, 3, \dots (2.5)$$

where p_n denotes the probability that exactly n claims occur in the fixed time interval. She showed [1] the Poisson distribution and mixed Poisson distribution

$$p_n = P(N=n) = \frac{e^{-\lambda y_1}(\lambda y_1)^n}{n!} p + \frac{e^{-\lambda y_2}(\lambda y_2)^n}{n!} q \quad (2.6)$$

where Y is a discrete random variable which takes two values such that,

$$P(Y = y_1) = p$$

$$P(Y = y_2) = 1 - p = q.$$

are members of these families. Then,

$$F(x) = p_1 g(x) + \frac{bp_0}{2} g^{*(2)}(x) + \int_0^x a\frac{y}{x} F(x-y)g(y)dy + \int_0^x \int_0^{x-y} b\frac{yu}{x(x-y)} g(u)g(y)F(x-y-u)dudy \quad (2.7)$$

In the same paper, she also consider another family of claim number distributions satisfies the following recursion relation.

$$p_n = (a + \frac{b}{n})p_{n-1} + (c + \frac{d}{n} + \frac{e}{n(n-1)})p_{n-2} \quad (2.8)$$

Where p_n denotes the probability that exactly n claim occur in the fixed time interval. Members of this family are, Binomial distribution, Mixed Binomial distribution [1], Negative binomial distribution, Mixed negative binomial distribution [1], Geometric distribution and mixed geometric distribution [1]. Then for this family, she gave the following recursive formula for continuous case.

$$F(x) = (c + \frac{d}{2} + \frac{e}{2})p_0 g^{*(2)}(x) + p_1 g(x) + \int_0^x (a + b\frac{y}{x})F(x-y)g(y)dy + \int_0^x \int_0^{x-y} \left(c + d\frac{y}{x} + e\frac{yu}{x(x-y)} \right) g(u)g(y)F(x-y-u)dudy \quad (2.9)$$

Of course, she also considered the discrete cases and found the results.

Willmot's results:

Willmot's (1988) studied the Panjer's family, and considered the deductible, reinsurance and maximum benefit problem. Then he got the results as follows.

$$g(s) = p_1(1-f_m)f_c(s) + (1-f_m)\sum_{n=1} p_n[a + b(1 - \frac{mn}{x} 0] f_m^n f_c(x - mn) + (a + b\frac{m}{x}) f_m g(x-m) + (1-f_m)\int_0^s (a + b\frac{x}{s}) f_c(x) g(s-x) d\mu(x). \quad (2.10)$$

Where m is the maximum benefit and

$$f_m = \int_m^\infty f(x)dx, \quad (2.11)$$

$$f_c(x) = \begin{cases} f(x)/(1-f_m), & 0 < x < m \\ 0 & otherwise \end{cases} \quad (2.12)$$

3. Deductible

There are many type of insurance agreements. The most popular one is deductibles. The inclusion of a deductible in a policy is usually done to serve one of two purposes. The first one is to lower administrative costs by removing small claims from the insurance process. The second is to provide some inducement to the insured to avoid incidents that leads claims. The simplest deductible is that the insurance pays nothing when the loss is below the deductible amount, but pays everything when the loss exceeds the deductible amount. This case usually is the first reason for the deductible. Let X be the loss amount with the probability distribution $F_X(x)$. Let Y be the amount of benefit, then

$$Y = \begin{cases} 0 & X < d \\ X & X \ge d \end{cases} \quad (3.1)$$

Then the probability distribution of the Y is given by

$$F_Y(y) = \begin{cases} 0 & y < d \\ \dfrac{F_X(y) - F_X(d)}{1 - F_X(d)} & y \ge d \end{cases} \quad (3.2)$$

and the density function of Y is given by

$$f_Y(y) = \begin{cases} 0 & y < d \\ \dfrac{f_X(y)}{1 - F_X(d)} & y \ge d \end{cases} \quad (3.3)$$

Without loss any general, we may assume that $d = 0$, and use notation $f_0 = P(X \le 0)$, therefore the probability distribution of Y is conditional distribution given $X > 0$, we introduce the notation $f_c(x) = f(x)/(1-f_0)$, when $x \ge 0$.

Now, the total claim S is given by $S = X_1 + X_2 + ... + X_N$ and the probability density function of S is

$$g(s) = \sum_{n=1}^{\infty} p_n g_n(s), \quad (3.4)$$

Where $p_n = P(N = n)$, the probability function of frequency and $g_n(s)$ is the probability density function of the total claim amount of exactly n claims. This probability density function can be found by conditional on the number of nonzero claims. Therefore,

$$g_n(s) = \sum_{k=1}^{n} \binom{n}{k} f_0^{n-k} (1 - f_0)^k f_c^{*k}(x) \qquad (3.5)$$

where $f_c^{*k}(x)$ is the k-fold convolution of $f_c(x)$ with itself.

When we consider continuous claim amount distribution F(x), $x > 0$, let f(x) be the density function of F(x). Then the following results will be used

$$\int_0^x f(y) f^{*(n)}(x-y)dy = f^{*(n+1)}(x)$$

$$\int_0^x y f(y) f^{*(n)}(x-y) / f^{*(n+1)}(x)dy = \frac{x}{n+1}$$

Of course, it is not hard to see that

$$\int_0^{x-y} u f(u) f^{*(n-1)}(x-y-u) / f^{*(n)}(x-y)du = \frac{x-y}{n}$$

where $f^{*(n)}(x)$ is the n-fold convolution of $f(x)$ with itself.

Theorem 1. If the claim frequency distribution satisfies (2.5), then the density function (3.4) of the total claim amount is given by the following formula.

$$\begin{aligned} g(x) &= p_1(1 - f_0) f_c(x) + \\ &+ (p_2 - p_1 \frac{a}{2})(2 f_0 (1 - f_0) f_c(x) + (1 - f_0)^2 f_c^{*2}(x)) \\ &+ (1 - f_0) \sum_{n=2}^{\infty} (n+1) p_{n+1} f_0^n f_c(x) \\ &+ \frac{b}{2}(1 - f_0)^2 \sum_{n=1}^{\infty} p_n f_0^n f_c^{*2}(x) \\ &+ a p_1 f_0 (1 - f_0) f_c(x) \\ &+ (1 - f_0) \int_0^x a \frac{y}{x} g(x-y) f_c(y)dy \\ &+ (1 - f_0)^2 \int_0^x \int_0^{x-y} b \frac{yu}{x(x-y)} f_c(u) f_c(y) g(x-y-u)dudy \end{aligned} \qquad (3.6)$$

where all notations are same as we introduced before.

Proof: Substitute (3.4), (3.5) into the integral $\int_0^x a \frac{y}{x} g(x-y) f_c(y)dy$, we have the following.

$$\int_0^x a \frac{y}{x} g(x-y) f_c(y)dy = \int_0^x a \frac{y}{x} f_c(y) \sum_{n=2}^{\infty} p_{n-1} g_{n-1}(x-y)dy =$$

$$\sum_{n=2}^{\infty} p_{n-1} \int_0^x a \frac{y}{x} f_c(y) \sum_{k=1}^{n-1} \binom{n-1}{k} f_0^{n-1-k} (1 - f_0)^k f_c^{*k}(x-y)dy$$

$$= \sum_{n=2}^{\infty} \frac{a}{x} p_{n-1} \sum_{k=1}^{n-1} \binom{n-1}{k} f_0^{n-1-k} (1 - f_0)^k \int_0^x y f_c(y) f_c^{*k}(x-y)dy$$

$$= \sum_{n=2}^{\infty} \frac{a}{n} p_{n-1} \sum_{k=1}^{n-1} \binom{n}{k+1} f_0^{n-(1+k)} (1 - f_0)^{k+1-1} f_c^{*(k+1)}(x)$$

$$= \frac{\sum_{n=2}^{\infty} \frac{a}{n} p_{n-1} g_n(x) - a(\sum_{n=1}^{\infty} p_n f_0^n)(1 - f_0) f_c(x)}{1 - f_0} \qquad (3.7)$$

By the similar way, we are going to substitute (3.4), (3.5) twice into the integral $\int_0^x \int_0^{x-y} b \frac{yu}{x(x-y)} f_c(u) f_c(y) g(x-y-u)dudy$, we have the following

$$\int_0^x \int_0^{x-y} b \frac{yu}{x(x-y)} f_c(u) f_c(y) g(x-y-u)dudy$$

$$= \int_0^x \frac{by}{x(x-y)} f_c(y)dy \int_0^{x-y} u f_c(u) \sum_{n=3}^{\infty} p_{n-2} g_{n-2}(x-y-u)du$$

$$= \sum_{n=3}^{\infty} p_{n-2} \int_0^x \frac{by}{x(x-y)} f_c(y)dy \int_0^{x-y} u f_c(u) \sum_{k=1}^{n-2} \binom{n-2}{k} f_0^{n-2-k} (1 - f_0)^k f_x^{*k}(x$$

$$= \sum_{n=3}^{\infty} p_{n-2} \frac{b}{x} \sum_{k=1}^{n-2} \binom{n-2}{k} f_0^{n-2-k} (1 - f_0)^k \frac{1}{k+1} \int_0^x y f_c(y) f_x^{*(k+1)}(x-y)dy$$

$$= \sum_{n=3}^{\infty} p_{n-2} \frac{b}{n(n-1)} \sum_{k=1}^{n-2} \binom{n}{k+2} f_0^{n-(2+k)} (1 - f_0)^{k+2-2} f_c^{*(k+2)}(x)$$

$$= \frac{\sum_{n=3}^{\infty} p_{n-2} \frac{b}{n(n-1)} g_n(x)}{(1 - f_0)^2} - \frac{\sum_{n=1}^{\infty} p_n \frac{b}{n+1} f_0^{n+1} (1 - f_0) f_c(x)}{(1 - f_0)^2} - \frac{\sum_{n=1}^{\infty} p_n \frac{b}{2} f_0^n (1 - f_0)^2 f_c^{*2}(x)}{(1 - f_0)^2} \qquad (3.8)$$

Now combine (3.7) and (3.8), we have

$$(1-f_0)\int_0^x a\frac{y}{x}g(x-y)f_c(y)dy$$
$$+(1-f_0)^2\int_0^x\int_0^{x-y} b\frac{yu}{x(x-y)}f_c(u)f_c(y)g(x-y-u)dudy$$
$$=$$
$$\sum_{n=2}^{\infty}\frac{a}{n}p_{n-1}g_n(x)-a(\sum_{n=1}^{\infty}p_n f_0^n)(1-f_0)f_c(x)$$
$$+\sum_{n=3}^{\infty}p_{n-2}\frac{b}{n(n-1)}g_n(x)$$
$$-\sum_{n=1}^{\infty}p_n\frac{b}{n+1}f_0^{n+1}(1-f_0)f_c(x)$$
$$-\sum_{n=1}^{\infty}p_n\frac{b}{2}f_0^n(1-f_0)^2f_c^{*2}(x)$$
$$=$$
$$\sum_{n=3}^{\infty}(\frac{a}{n}p_{n-1}+\frac{b}{n(n-1)}p_{n-2})g_n(x)$$
$$-\sum_{n=2}^{\infty}(ap_n+\frac{b}{n}p_{n-1})f_0^n(1-f_0)f_c(x)$$
$$-ap_1f_0(1-f_0)f_c(x)-\sum_{n=1}^{\infty}p_n\frac{b}{2}f_0^n(1-f_0)^2f_c^{*2}(x)$$
$$=$$
$$\sum_{n=3}^{\infty}p_ng_n(x)-\sum_{n=2}^{\infty}(n+1)p_{n+1}f_0^n(1-f_0)f_c(x)+$$
$$\frac{a}{2}p_1g_1(x)-ap_1f_0(1-f_0)f_c(x)$$
$$-\sum_{n=1}^{\infty}p_n\frac{b}{2}f_0^n(1-f_0)^2f_c^{*2}(x)$$
$$=$$
$$g(x)-p_1g_1(x)-p_2g_2(x)$$
$$-\sum_{n=2}^{\infty}(ap_n+\frac{b}{n}p_{n-1})f_0^n(1-f_0)f_c(x)$$
$$+\frac{a}{2}p_1g_1(x)-ap_1f_0(1-f_0)f_c(x)$$
$$-\sum_{n=1}^{\infty}p_n\frac{b}{2}f_0^n(1-f_0)^2f_c^{*2}(x).$$

When we simplify, we have the result. In this proof, we use the fact $f_c(x)=0$, when $x<0$. Notice that if we let $f_0=0$, which is the case, there is no deduction, then our result will be simplify as follows

$$g(x)=p_1f_c(x)+(p_2-p_1\frac{a}{2})f_c^{*2}(x))$$
$$+\int_0^x a\frac{y}{x}g(x-y)f_c(y)dy$$
$$+\int_0^x\int_0^{x-y} b\frac{yu}{x(x-y)}f_c(u)f_c(y)g(x-y-u)dudy$$

Pay attention to the fact, in this case $f_c(x)=f(x)$, the density function of claim and $(p_2-p_1\frac{a}{2})=\frac{b}{2}p_0$.

This result is exactly same as Deng's result (1998).

4. **Maximum Benefits Payable**

In the last section, we were dealing with claims of the deductible. Without loss the generality, we may assume that d=0. In this section, we are going to consider the maximum benefits payment. This looks like the similar problem, but it is more difficulty to deal with.

Now consider the loss random variable X with the probability density function $f(x)$, the maximum benefits payable is m. Then the payment variable Y can be introduce as follow.

$$Y=\begin{cases}X & X\le m\\ m & X>m\end{cases}\qquad(4.1)$$

Then the probability distribution of the Y is given by

$$F_Y(y)=\begin{cases}\dfrac{F_X(y)}{F_X(m)} & y\le m\\ 1 & y>m\end{cases}\qquad(4.2)$$

and the density function of Y is given by

$$f_Y(y)=\begin{cases}\dfrac{f_X(y)}{F_X(m)} & y\le m\\ 0 & y>m\end{cases}\qquad(4.3)$$

introduce the notation $f_m=P(X>m)$, therefore the probability distribution of Y is conditional distribution given $X\le m$ is given by $f_c(x)=f(x)/(1-f_m)$, when $0<x<m$.

Now, the total claim S is given by

$$S=X_1+X_2+...+X_N$$

and the probability density function of S is

$$g(s)=\sum_{n=1}^{\infty}p_ng_n(s),\qquad(4.4)$$

Where $p_n=P(N=n)$, the probability function of frequency and $g_n(s)$ is the probability density function of the total claim amount of exactly n claims. This probability density function can be found by conditional on the number of nonzero claims. Therefore,

$$g_n(s) = \sum_{k=1}^{n} \binom{n}{k} f_m^{n-k} (1 - f_m)^k f_c^{*k} [x - m(n-k)]$$

(4.5)

where $f_c^{*k}(x)$ is the k-fold convolution of $f_c(x)$ with itself. Of course, when $f_m = 0$, which mean there is no maximum limit of the benefit,

$$g_n(x) = f_c^{*n}(x) = f^{*n}(x).$$

Now we are interesting to find the probability density function of the total claim under the maximum benefit is m. we have the following result.

Theorem 2. If the claim frequency distribution satisfies (2.5), then the density function (4.4) of the total claim amount is given by the following formula.

$$g(x) = p_1(1 - f_m) f_c(x) +$$

$$+ (p_2 - p_1 \frac{a}{2})(2 f_m (1 - f_m) f_c(x - m) +$$

$$+ (1 - f_m)^2 f_c^{*2}(x))$$

$$+ (1 - f_m) \sum_{n=2}^{\infty} (n+1) p_{n+1} f_m^n f_c(x - mn)$$

$$+ \frac{b}{2}(1 - f_m)^2 \sum_{n=1}^{\infty} p_n f_m^n f_c^{*2}(x - mn)$$

$$+ a p_1 f_m (1 - f_m) f_c(x - m) + \frac{am}{x} f_m g(x - m)$$

$$+ \frac{bm^2}{x(x-m)} f_m^2 g(x - 2m)$$

$$+ (1 - f_m) \int_0^x a \frac{y}{x} g(x - y) f_c(y) dy$$

$$+ (1 - f_m)^2 \int_0^x \int_0^{x-y} b \frac{yu}{x(x-y)} f_c(u) f_c(y) g(x - y - u) du dy$$

(4.6)

where all notations are same as we introduced before. Notice that the in fact, $f_c(x) = 0$, when $x < 0$. We used this fact a lot in the proof and also pay attention to the fact $f_c(x) = 0$, when $x > m$. So the infinite summation is in fact a finite summation.

Proof: We first consider the integration $\int_0^x a \frac{y}{x} g(x - y) f_c(y) dy$. We substitute (4.4) and (4.5) into this integration and have the following result.

$$\int_0^x a \frac{y}{x} g(x - y) f_c(y) dy$$

$$= \int_0^x a \frac{y}{x} f_c(y) \sum_{n=2}^{\infty} p_{n-1} g_{n-1}(x - y) dy$$

$$=$$

$$\sum_{n=2}^{\infty} p_{n-1} \int_0^x a \frac{y}{x} f_c(y) \sum_{k=1}^{n-1} \binom{n-1}{k} f_m^{n-1-k} (1 - f_m)^k f_c^{*k}(x - y - m(n-k)) dy$$

$$=$$

$$\sum_{n=2}^{\infty} \frac{a}{x} p_{n-1} \sum_{k=1}^{n-1} \binom{n-1}{k} f_m^{n-1-k} (1 - f_m)^k \int_0^{x-m(n-k)} y f_c(y) f_c^{*k}(x - m(n-k) - y) dy$$

$$=$$

$$\sum_{n=2}^{\infty} \frac{a}{x} p_{n-1} \sum_{k=1}^{n-1} \binom{n-1}{k} f_m^{n-(1+k)} (1 - f_m)^{k+1-1} \frac{x - m(n-1-k)}{k+1} f_c^{*(k+1)} [x - m(n -$$

$$=$$

$$\sum_{n=2}^{\infty} \frac{a}{n} p_{n-1} \sum_{k=1}^{n-1} \binom{n}{k+1} f_m^{n-(1+k)} (1 - f_m)^{k+1-1} f_c^{*(k+1)} \{x - m[n - (k+1)]\} -$$

$$\sum_{n=2}^{\infty} \frac{a}{x} p_{n-1} m \sum_{k=1}^{n-1} \binom{n-1}{k+1} f_m f_m^{(n-1)-(1+k)} (1 - f_m)^{k+1-1} f_c^{*(k+1)} \{x - m - m[(n-1) -$$

$$= \frac{\sum_{n=2}^{\infty} \frac{a}{n} p_{n-1} g_n(x) - a(\sum_{n=1}^{\infty} p_n f_m^n)(1 - f_m) f_c(x - mn)}{1 - f_m}$$

$$- \frac{\sum_{n=2}^{\infty} \frac{a}{x} p_{n-1} m f_m g_{n-1}(x - m)}{1 - f_m}$$

$$= \frac{\sum_{n=2}^{\infty} \frac{a}{n} p_{n-1} g_n(x) - a(\sum_{n=1}^{\infty} p_n f_m^n)(1 - f_m) f_c(x - mn)}{1 - f_m} -$$

$$- \frac{\frac{a}{x} m f_m g(x - m)}{1 - f_m} \qquad (4.7)$$

By the similar way, we can show that we have the following

$$\int_0^x \int_0^{x-y} b \frac{yu}{x(x-y)} f_c(u) f_c(y) g(x - y - u) du dy$$

$$= \frac{\sum_{n=3}^{\infty} p_{n-2} \frac{b}{n(n-1)} g_n(x)}{(1 - f_m)^2}$$

$$- \frac{\sum_{n=1}^{\infty} p_n \frac{b}{n+1} f_m^{n+1} (1 - f_m) f_c(x - mn)}{(1 - f_m)^2}$$

$$-\frac{\sum_{n=1}^{\infty} p_n \frac{b}{2} f_m{}^n (1-f_m)^2 f_c^{*2}(x-mn)}{(1-f_m)^2}$$

$$-\frac{\frac{bm^2}{x(x-m)} f_m^2 g(x-2m)}{(1-f_m)^2} \qquad (4.8)$$

Combine the (4.7), (4.8) and simplify, we then complete the proof.

Remark: If we let m=0, then this will be same as the case one. When we substitute m=0 into formula (4.6). The result will be exactly same as the theorem 1.

5. References

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