Efficient Local Search Algorithms for the Vehicle Routing Problem with Time Windows

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1 Introduction

The purpose of this paper is to present new methods for solving the vehicle routing problem with time windows (VRPTW). VRPTW can be described as the problem of designing least cost routes from one depot to a set of geographically scattered points. The routes must be designed in such a way that each point is visited only once by exactly one vehicle within a given time interval; all routes start and end at the depot, and the total demands of all points on one particular route must not exceed the capacity of the vehicle.

2 The Problem Solving Methodology

Here we suggest a new three-phase approach for solving vehicle routing problems. In the first phase several initial solutions are created using route construction heuristics with different combinations of parameter values. In the second phase an effort is put to reduce the number of routes using a new ejection chain-based approach. In the third phase we use well-known Or-opt exchanges (Or, 1976) to minimize total traveled distance.

Two new construction heuristics were used to create the initial solutions, namely Hybrid Construction heuristic (HC) and Merge Heuristic (MH). They are depicted below followed by description of the five local search procedures B1, B2, B3, B4 and B5 that are based on HC and MH heuristics, route elimination procedure and Or-opt exchanges.

Hybrid construction heuristic borrows its basic ideas from the studies of Solomon (1987) and Russell (1995). Routes are build one at a time in sequential fashion and after k customers have been inserted into the route, the route is reordered using Or-opt exchanges. In the beginning a set of seed customers is identified by finding a set of geographically dispersed customers located far away from depot. The selected seed customers are then considered in different orders by the sequential insertion heuristic. Once the first customer is selected, the unrouted customers are examined one by one and the customer that minimizes the weighted combination of additional detour and waiting time is selected and inserted into the best feasible insertion place. In the experimental part the combination of HC heuristic, route elimination procedure and Or-opt exchanges is marked by B1.

Merge heuristic draws its basic concepts from the Savings heuristic of Clarke et al. (1964). Here the original measure of savings is modified to consider also changes in waiting times. Here we implemented parallel version of the Savings heuristic. Moreover, the customers in the combined route are reordered before evaluating the saving incurred by uniting the two routes. This reordering is performed using

a new within-route improvement heuristic O-opt, described in Bräysy (2001). The basic idea is to rebuild routes with cheapest insertion heuristic by using more than one customers to initialize routes. In the experimental part the combination of MH, route elimination procedure and Or-opt exchanges is marked by B2.

In addition to testing B1 and B2 heuristics independently, they were combined simply by running both of the methods once and taking the better out of the two results produced. In the experimental part we refer to this method by notation B3. The basic idea of B4 is the same as used in the implementation of B1. However, here we use the knowledge of the best solutions reported in the literature and the search is stopped once this number of vehicles is obtained. Also the number of parameter combinations tried is greater than in the B1 implementation. Finally we created fifth implementation B5 by examining all parameter values specified for B4, without prematurely stopping the search.

The basic idea of the route elimination procedure is to use a new type of neighborhood structure based on Ejection Chains (EC) proposed by Glover (1991, 1992). In the VRPTW context the basic idea is to remove first some customer c_i from route r_k and after the removal, to insert some other customer c_j currently served by route r_l into the partial route r_k. If the insertion of c_j is possible, an attempt is made to insert customer c_i in another route r_m. If a feasible insertion place can be found, the chain is completed and another customer c_{j+1} is selected to initialize another chain. In each phase within the ejection chain, one customer remains unrouted. The removal and insertion procedures are repeated until we can insert a customer into a neighboring route without the need to remove (eject) any customer. It is reasonable to assume, that the order of the customers on routes affect the number of feasible insertion places. Therefore we suggest that a reordering procedure should be attached to ejection chain procedure.

If simple insertion fails, a special type of insertion called intelligent reordering is tried. First the customer c_i originally served by the route r_e that we try to eliminate, is inserted to a insertion place that least increases the value of cost function and without considering time window constraints. Next we identify the first customer c_v whose time window constraints are violated. Roughly speaking there are two alternative ways to reorder the route and thus try to get the route feasible without needing to remove any customer. The simplest way is to consider the customers served before the customer c_v on the route and try to serve them after the c_v on the route. The other possibility is to reorder the customers served before customer c_v so that the duration of the partial route c_1,...,c_{v-1} is minimized. The reordering procedures are repeated with all the customers at whom the vehicle arrives too late.

If we are not able to eliminate a route, a special post-processing is applied. Here we insert as many customers as possible from neighboring routes r_n into the just examined route r_e to facilitate eliminating some other route. A route is considered as a neighboring route if it contains customers that are geographically close to a customer of the current route. Since we must keep the route r_e, it is reasonable to utilize its resources as well as possible.

3 Computational Experiment

To analyze the performance of our algorithm, an experiment was conducted over 56 VRPTW problem instances partitioned in six data sets denoted R1, C1, RC1, R2, C2 and RC2 (Solomon, 1987). The problems vary in fleet size, vehicle capacity, travel time, spatial and temporal customer distribution (position, time window density, time window width, and service time). The algorithms were implemented in JAVA and the computational experiments were conducted using a Pentium 200 MHz computer. The experimental results are presented in Table 1.

	R1			**R2**			**RC1**			**RC2**		
Author	NV	TD	TT	NV	TD	TT	NV	TD	TT	NV	TD	TT
S	13.6	1437	<0.1	3.3	1402	<0.1	13.5	1597	<0.1	3.9	1682	<0.1
RT	12.6	1197	68	3.1	954	245	12.4	1370	65	3.6	1140	195
TBGGP	12.3	1220	230	3.0	1013	337	11.9	1381	188	3.4	1199	193
C-WC	12.5	1242	42	2.9	995	40	12.4	1409	22	3.4	1140	28
GTA	12.4	1211	210	3.0	960	210	11.9	1388	210	3.3	1149	210
LS	12.2	1250	226	2.8	1017	34	11.9	1413	155	3.3	1205	36
HG1	11.9	1228	275	2.7	967	352	11.6	1393	242	3.3	1144	363
HG2	12.0	1226	431	2.7	1034	477	11.5	1407	516	3.3	1176	469
HG3	12.4	1201	44	2.9	945	44	12.0	1356	44	3.3	1140	44
B1	12.3	1254	13	2.8	1053	7	11.9	1428	9	3.3	1250	3
B2	12.3	1259	5	3.0	1022	5	12.0	1407	3	3.4	1269	4
B3	12.2	1253	17	2.8	1040	11	11.9	1408	12	3.3	1245	7
B4	12.0	1296	11	2.7	1070	28	11.5	1462	13	3.3	1272	2
B5	12.0	1240	118	2.7	1017	105	11.5	1418	95	3.3	1200	44

Table 1: Comparison of the solution quality and computational burden. Notations NV, TD and TT indicate the average number of vehicles, average total distance and total time consumption in minutes respectively with respect to four problem groups of Solomon. One must note that the time consumption is normalized to Sun Sparc 50/10 by using Dongarra's (1998) factors. In addition if several runs are required to get the reported results, the computational times are multiplied by this number to see the real computational burden.

The authors in Table 1 are: S: Solomon (1987), RT: Rochat et al. (1995), TBGGP: Taillard et al. (1997), C-WC: Cordone et al. (1998), GTA: Gambardella et al. (1999), LS: Liu et al. (1999), HG1–HG2: Homberger et al. (1999), HG3: Gehring et al. (1999), B1–B5: this paper.

The first observation from Table 1 is that none of the methods is able to dominate all other methods in all problem groups. Considering the solution quality and time consumption, our approaches seem to be most efficient. Especially the solution quality of B5 appears to be good. It dominates all other approaches except HG1, HG2, HG3 and GTA in all four problem groups. Also HG1 and HG2 are dominated in one and HG3 and GTA are dominated in three cases respectively. We are aware of only two other methods by Chiang et al. (1997) and Cordeau et al. (2000) that are able to dominate B5 in one and three cases respectively. However, we had problems in identifying their computational burden and therefore they are not considered here. In addition one must note that these competing methods use stochastic approaches and the results are often the best ones over multiple runs. If the average performance of the methods is considered, our approach clearly outperforms them.

4 Conclusions

We proposed two new construction heuristics that were used to design five new methods for solving VRPTW. The proposed methods use a new three-phase approach. In the first phase an initial solution is created using one of the two construction heuristics. Then a special route-elimination operator, based on a new type of ejection chains is used to minimize the number of routes. Finally, in the third phase the created solutions are improved in terms of distance using Or-opt exchanges. The computational testing of the proposed methods was carried out with 56 test problems of Solomon (1987).

The proposed local search methods were compared with the best previous local search and metaheuristic methods found in the literature. In terms of solution quality, the best performing methods were B5

and methods of Homberger et al. (1999) and Gehring et al. (1999). To our best knowledge B4 and B5 are the only methods that can consistently produce the lowest known cumulative number of vehicles, 406 for Solomon's test problems.

The methods proposed in this paper are very efficient approaches for solving vehicle routing problems with time windows, producing results that outperform recent local search approaches and are on par with the best metaheuristic approaches. Due to their deterministic nature they have better reliability than the best competing approaches. In addition they seem to be very fast and robust, yielding every time feasible solutions of acceptable quality to all types of well-known test problems.

5 References

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