Journal of Fixed Income 15 (2), 2005, pp. 63-75

A Multi-Factor Approach for Systematic Default and Recovery Risk

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Abstract

The following article develops a simultaneous multi-factor model for defaults and recoveries. Applying this model, risk parameters can be forecast using systematic and idiosyncratic risk factors and their implied correlations. The theoretical framework is accompanied by an empirical analysis in which a negative correlation between defaults and recoveries over the business cycle is observed. In the study, default and recovery rates are modeled by business cycle indicators and the properties of the economic and regulatory capital given these risk drivers are shown.

Modeling Default and Recovery Risk

Today's banks face the challenge of forecasting losses and loss distributions in relation to their credit risk exposures. It can be observed that most banks choose a modular approach which is in line with the current proposals of the Basel Committee on Banking Supervision [2004], where selected risk parameters such as default probabilities, exposures at default and recoveries given default are modeled in independent modules. However, the assumption of independence is questionable. Previous studies have shown that default probabilities and recovery rates given default are negatively correlated (e.g., Carey [1998], Hu/Perraudin [2002], Frye [2003], Altman et al. [2003] or Cantor/Varma [2005]). A failure to take these dependencies into account will lead to incorrect forecasts of the loss distribution and the derived capital allocation.

The present paper extends a model introduced by Frye [2000]. Modifications of the approach can be found in Pykhtin [2003] and Düllmann/Trapp [2004]. Our contribution is original with regard to the following three aspects. First, we develop a theoretical model for the default probabilities and recovery rates and show how to combine observable information with random risk factors. In comparison to the above mentioned models, our approach explains the default and the recovery rate by risk factors which can be observed at the time of the risk assessment. According to the current Basel proposal, banks can opt to provide their own recovery rate forecasts for the regulatory capital calculation. Thus, there is an immediate industry need for modeling.

Second, we show a framework for estimating the joint processes of all variables in the model. Particularly, the simultaneous model allows the measurement of the correlation between the defaults and recoveries given the information. In this model statistical tests for the variables and correlations can easily be conducted. An empirical study reveals additional evidence on the correlations between risk drivers of default and recovery. Note that Cantor/Varma [2003] essentially analyze the same dataset and identify seniority and security as the main risk factors explaining recovery rates. The present paper extends their approach by developing a framework for modeling correlations between factor-based models for default and recovery rates.

Third, the implications of our results on economic and regulatory capital are shown. Note that according to the current proposals of the Basel Committee only the forecast default probabilities and recovery rates but no correlation estimates enter the calculation of the latter. We demonstrate the effects of spuriously neglecting correlations in practical applications.

The rest of the paper is organized as follows. The theoretical framework is introduced in the second section (‘Model and Estimation’) for a model using historic averages as forecasts and a model taking time-varying risk factors into account. The third section (‘Data and Results’) includes an empirical analysis based on default and recovery rates published by Moody’s rating agency and macroeconomic indices from the Conference Board. Section four (‘Implications for Economic and Regulatory Capital’) shows the implications of the different models on the economic capital derived from the loss distribution and the regulatory capital proposed by the Basel Committee. Section five (‘Discussion’) concludes with a summary and discussion of the findings.

Model and Estimation

The Model for the Default Process

Our basic framework follows the approach taken by Frye [2000]. We assume that n_t firms of one risk segment are observed during the time periods t (t=1,…,T). For simplicity these firms are assumed to be homogenous with regard to the relevant parameters and a latent variable describes each obligor i*’s* (i=1,…, n_t) credit quality

$$S_{it} = w \cdot F_t + \sqrt{1-w^2} \cdot U_{it} \tag{1}$$

($w \in [0,1]$). $F_t \sim N(0,1)$ and $U_{it} \sim N(0,1)$ are independent systematic and idiosyncratic standard normally distributed risk factors. The Gaussian random variable S_{it} may be interpreted as the return on a firm’s assets and therefore w^2 is often called ‘asset correlation’.
A default event occurs if the latent variable crosses a threshold c

$$S_{it} < c \tag{2}$$

which happens with probability $\pi = \Phi(c)$ where $\Phi(.)$ is the standard normal cumulative density function. If an obligor is in default the indicator variable D_{it} equals one and zero otherwise:

$$D_{it} = \begin{cases} 1 & \text{obligor } i \text{ defaults in period } t \\ 0 & \text{else} \end{cases} \tag{3}$$

Conditional on the realization f_t of the systematic risk factor, default events are assumed to be independent between obligors, i.e., each firm defaults with the conditional default probability

$$\pi(f_t) = P(D_{it} = 1 | F_t = f_t) = \Phi\left(\frac{c - w \cdot f_t}{\sqrt{1 - w^2}}\right) \tag{4}$$

The Model for the Recovery

In modeling the recovery rate R_{it} of a defaulted obligor we follow Schönbucher [2003] and Düllmann/Trapp [2004] and use a logistic normal process:

$$R_{it} = \frac{\exp(\tilde{Y}_{it})}{1 + \exp(\tilde{Y}_{it})} \tag{5}$$

with the transformed recovery rate

$$\tilde{Y}_{it} = \mu + b \cdot X_t + Z_{it} \tag{6}$$

where $X_t \sim N(0,1)$, $Z_{it} \sim N\left(0,\delta^2\right)$ are independent systematic and idiosyncratic factors and μ and b are parameters. These idiosyncratic factors are independent from the idiosyncratic factors which drive the latent default variable. Compared to the normal distribution assumption for recovery rates used in Frye [2000] the chosen transformation has the advantage that recovery rates are bounded between 0% and 100%. Note that any other cumulative density function could be used. As a matter of fact, we estimated models using a standard normal transformation and received similar results.

If we observe a homogenous segment of borrowers, the transformed recovery rate is given by

$$\tilde{Y}_t = \frac{1}{n_t}\sum_{i=1}^{n_t}\tilde{Y}_{it} = \mu + b \cdot X_t + \frac{1}{n_t}\sum_{i=1}^{n_t} Z_{it} \tag{7}$$

with $Z_t = \frac{1}{n_t}\sum_{i=1}^{n_t} Z_{it}$ which is normally distributed with mean zero and variance δ^2 / n_t^2 . The variance converges for large n_t to zero:

$$\lim_{n_t \to \infty} \operatorname{Var}\left(\frac{1}{n_t}\sum_{i=1}^{n_t} Z_{it}\right) = 0 \tag{8}$$

Therefore, we approximate the average transformed recovery rate by

$$\tilde{Y}_t \approx Y_t = \mu + b \cdot X_t \tag{9}$$

which is driven only by a systematic risk factor and normally distributed $Y_t \sim N\left(\mu, b^2\right)$. The link between the recovery and default process is introduced by modelling the dependence of the two systematic risk factors. Since both F_t and X_t are marginally normal distributed we model their dependence by assuming that they are bivariately normal distributed with correlation parameter

ρ. Alternatively, a copula which is different from the Gaussian could have been assumed. It then follows that the average transformed recovery rate and latent default triggering variable have a correlation

$$\mathrm{Corr}(S_{it}, Y_t) = w \cdot \rho \tag{10}$$

The correlation equals one in the special case that a single systematic factor drives both the default events as well as the recoveries given these events.

A Multi-Factor Model Extension

So far, we presented a model for systematic risk in defaults and recoveries where systematic risk is driven by common factors which are not directly observable. These unobservable factors induce uncertainties into the forecasts of loss distributions. The higher their impact is, the more skewed ceteris paribus the resulting distributions are and the higher key risk measures such as the Value-at-Risk or the Conditional Value-at-Risk will be. Since the true parameters of the models are unknown, the severity of the impact must be estimated from observable data.

As an alternative to the models from above, we analyze a model, which has already been used in the context of default modeling. Examples are Rösch/Scheule [2004] and Hamerle/Liebig/Rösch [2003]. These models show that part of the cyclical fluctuations in default rates can be attributed to observable systematic risk factors. Once these factors are identified and incorporated into the model, a large part of uncertainty from unobservable factors can be explained. These types of models are also exhibited in Heitfield [2005] and are related to a concept broadly known as a point-in-time approach because losses are forecast based on information on the prevailing point of the business cycle.

In our extension, it is assumed that the default threshold for the factor model of the default process fluctuates through time. Alternatively, we could introduce a factor model with time-varying mean. This variation with time is introduced by K observable macroeconomic risk factors, such as GDP growth or interest rates. We assume that these state variables are observed in prior time periods and denote them by $\boldsymbol{z}_{t-1}^{D} = \begin{pmatrix} z_{t-1,1}^{D} & \cdots & z_{t-1,K}^{D} \end{pmatrix}$. As a result, the conditional default probability for each borrower within the risk segment is modified (compare Rösch [2003] and Heitfield [2005] who additionally condition default probabilities on firm-specific factors):

$$\pi^*(z_{t-1}, f_t) = P(D_{it} = 1 | z_{t-1}, f_t) = \Phi\left(\frac{\gamma_0 + \boldsymbol{\gamma}' \cdot \boldsymbol{z}_{t-1}^{D} - w^* \cdot f_t}{\sqrt{1 - w^{*2}}}\right) \tag{11}$$

where $\boldsymbol{\gamma} = \begin{pmatrix} \gamma_1 & \cdots & \gamma_K \end{pmatrix}'$ denotes a vector of exposures to the common observable factors and γ_0 a constant. The mean of this conditional default probability with respect to the unobservable standard normally distributed factor f_t is given by

$$\pi^*\left(\boldsymbol{z}_{t-1}^{D}\right) = \int_{-\infty}^{\infty} \pi^*\left(\boldsymbol{z}_{t-1}^{D}, f_t\right) d\Phi(f_t) = \Phi\left(c^* + \boldsymbol{\gamma}' \cdot \boldsymbol{z}_{t-1}^{D}\right) \tag{12}$$

In a similar way, we assume that the mean of the log-transformed systematic recovery rate depends on common macroeconomic factors $\boldsymbol{z}_{t-1}^{R} = \begin{pmatrix} z_{t-1,1}^{R} & \cdots & z_{t-1,L}^{R} \end{pmatrix}$. This vector may or may not contain factors which also describe the default process:

$$Y_t^* = \beta_0 + \boldsymbol{\beta}' \boldsymbol{z}_{t-1}^{R} + b^* \cdot X_t \tag{13}$$

where $\boldsymbol{\beta} = \begin{pmatrix} \beta_1 & \cdots & \beta_L \end{pmatrix}$ denotes a vector of exposures and β_0 the constant.

If models (12) and (13) hold, i.e., defaults and recoveries are driven by observable lagged systematic risk factors, it can be shown that their means are fluctuating with the change of the economy. Moreover, if these models hold, then model (4) and (9) with constant mean are misspecifications. As a consequence, fitting model (4) and (9) to observable data will have the effect that all time variation is captured in the estimates of the exposures to the unobservable random factors F_t and X_t. On the other hand, attributing time variation to observable factors will lead to lower parameter estimates for the influences of the unobservable factors, thereby reducing uncertainty with regard to the forecasts of the loss distributions. We will demonstrate these effects on the economic and regulatory capital below.

Model Estimation

Once the models are specified an algorithm for estimating the parameters from observable data is needed. Following work by Frye [2000] we choose the Maximum-Likelihood method. In extension to these studies, we suggest an ML-procedure which allows the joint estimation of all coefficients, including those of models (11) and (13) with observable factors.

Let us consider a realization f_t of the unobservable random factor F_t. Given this realization the default events are independent and the number of defaults $D_t = \sum_{i=1}^{n_t} D_{it}$ is conditionally binomial distributed with probability distribution

$$P(D_t = d_t \mid f_t) = \begin{cases} \binom{d_t}{n_t} \pi(f_t)^{d_t} \cdot [1 - \pi(f_t)]^{n_t - d_t} & d_t = 0,1,\ldots,n_t \\ 0 & \text{else} \end{cases} \tag{14}$$

with $\pi(f_t)$ as in (4). Note that the transformed recovery rate can also be modeled given a realization f_t. It holds that the random vector $(F_t, Y_t)'$ is normally distributed with

$$\begin{pmatrix} F_t \\ Y_t \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ \mu \end{pmatrix}, \begin{pmatrix} 1 & b\rho \\ b\rho & b^2 \end{pmatrix}\right].$$

From the law of conditional expectation it follows that Y_t has conditional mean

$$\mu(f_t) = E(Y_t \mid f_t) = \mu + b \cdot \rho \cdot f_t \tag{15}$$

and conditional standard deviation

$$\sigma(f_t) = +\sqrt{\mathrm{Var}(Y_t \mid f_t)} = b \cdot \sqrt{1-\rho^2} \tag{16}$$

Hence, the joint density $g(.)$ of d_t defaults and a transformed recovery rate y_t given f_t is simply the product of the density of y_t and the probability of d_t, i.e.,

$$\begin{aligned} & g(d_t, y_t \mid f_t) \\ & = \frac{1}{\sigma(f_t) \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left\{-\frac{[y_t - \mu(f_t)]^2}{2 \cdot [\sigma(f_t)]^2}\right\} \cdot \binom{d_t}{n_t} \cdot \pi(f_t)^{d_t} \cdot [1-\pi(f_t)]^{n_t - d_t} \end{aligned} \tag{17}$$

Note, $g(.)$ depends on the unknown parameters of the default and the recovery process. Since the common factor is not observable we establish the unconditional density

$$\begin{aligned} & g(d_t, y_t) \\ & = \int_{-\infty}^{\infty} \frac{1}{\sigma(f_t) \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left\{-\frac{[y_t - \mu(f_t)]^2}{2 \cdot [\sigma(f_t)]^2}\right\} \cdot \binom{d_t}{n_t} \cdot \pi(f_t)^{d_t} \cdot [1-\pi(f_t)]^{n_t - d_t} \; d\Phi(f_t) \end{aligned} \tag{18}$$

Observing a time series with T periods leads to the final unconditional log-likelihood function

$$\begin{aligned} & l(\mu, b, c, w, \rho) \\ & = \sum_{t=1}^{T} \ln\left(\int_{-\infty}^{\infty} \frac{1}{\sigma(f_t) \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left\{-\frac{[y_t - \mu(f_t)]^2}{2 \cdot [\sigma(f_t)]^2}\right\} \cdot \binom{d_t}{n_t} \cdot \pi(f_t)^{d_t} \cdot [1-\pi(f_t)]^{n_t - d_t} \; d\Phi(f_t) \right) \end{aligned} \tag{19}$$

This function is optimized with respect to the unknown parameters. In the appendix we demonstrate the performance of the approach by Monte-Carlo simulations.

For the second type of models which include macroeconomic risk factors, we replace $\pi(f_t)$ from (4) by $\pi^*\left(z_{t-1}^D, f_t\right)$ from (11) and $\mu(f_t)$ from (15) by $\beta_0 + \boldsymbol{\beta}' \boldsymbol{z}_{t-1}^R + b \cdot \rho \cdot f_t$ and obtain the analogous log-likelihood $l(\beta_0, \boldsymbol{\beta}, b, \gamma_0, \boldsymbol{\gamma}, w, \rho)$.

Data and Results

The Data

The empirical analysis is based on the global corporate issuer default rates and issue recovery rates published by Moody's [2005]. In this data set, default rates are calculated as the ratio of defaulted and total number of rated issuers for a given period. According to Moody's [2004], a default is in essence recorded if

- Interest and/or principal payments are missed or delayed,
- Chapter 11 or Chapter 7 bankruptcy is filed, or
- Distressed exchange such as a reduction of the financial obligation occurs.

Most defaults are related to publicly traded debt issues. Therefore, Moody's defines a recovery rate as the ratio of the price of defaulted debt obligations after 30 days of the occurrence of a default event and the par value. The recovery rates are published for different levels of seniority such as total (Total), senior secured (S_Sec), senior unsecured (S_Un), senior subordinated (S_Sub), subordinated (Sub) and junior subordinated debt. We excluded the debt category junior subordinated from the analysis due to a high number of missing values.

In addition, the composite indices published by The Conference Board (www.tcb-indicators.org) were chosen as macroeconomic systematic risk drivers, i.e., the

- Index of 4 coincident indicators (COINC) which measures the current health of the U.S. economy. The index includes the number of employees on non agricultural payrolls, personal income less transfer payments, index of industrial production and manufacturing as well as trade sales.

- Index of 10 leading indicators (LEAD) which measures the future health of the U.S. economy. The index includes average weekly hours in manufacturing, average weekly initial claims for unemployment insurance, manufacturers' new orders of consumer goods and materials, vendor performance, manufacturer's new orders of non-defense capital goods, building permits for new private housing units, stock price index, money supply, interest rate spread of 10-year treasury bonds less federal funds and consumer expectations.

The indices are recognized as indicators for the U.S. business cycle. Note that for the analysis, growth rates of the indices were calculated and lagged by three months.

Due to a limited number of defaults in previous years, the compiled data set was restricted to a time period from 1985 to 2004 and split into an estimation sample (1985 to 2003) and a forecast sample (2004). Exhibit 1 and Exhibit 2 include descriptive statistics and Bravais-Pearson correlations for default rates, recovery rates and time lagged macroeconomic indicators of the data set. Note that default rates are negatively correlated with the recovery rates of different seniority classes and macroeconomic variables.

[*** Insert Exhibit 1 about here ***]

[*** Insert Exhibit 2 about here ***]

Exhibit 3 shows visually that both, the default and recovery rate fluctuate over time in opposite directions. This signals that default and recovery rates show a considerable share of systematic risk which can be explained by time varying variables.

[*** Insert Exhibit 3 about here ***]

Exhibit 4 contains similar graphs for the recovery rates of the different seniority classes. Note that the recovery rates increase with the seniority of a debt issue and show similar patterns over time. This indicates that they may be driven by the same or similar systematic risk factors.

[*** Insert Exhibit 4 about here ***]

Next to the business cycle and the seniority, it is plausible to presume that recovery rates depend on the industry, the collateral type, the legal environment, default criteria as well as the credit quality associated to an obligor. Exhibit 5 and 6 show the recovery rates for different industries and issuer credit ratings as published by Moody's [2004 and 2005]. Refer to these documents for a more detailed analysis of the properties of recovery rates.

[*** Insert Exhibit 5 about here ***]

[*** Insert Exhibit 6 about here ***]

Estimation Results

Based on the described data set, two models were estimated:

- Model without macroeconomic risk factors (equations (4) and (9)): throughout the following text we call this model a through-the-cycle model because the forecast default and recovery rate equal the historic average from 1985 to 2003;
- Model with macroeconomic risk factors (equations (11) and (13)); we call this model a point-in-time model because the forecast default and recovery rates fluctuate over time.

Note that within the credit risk community, a discussion on the correct definition of a through-the-cycle and point-in-time model exists in which the present article does not intend to participate. We use these expression as stylized denominations, being aware that other interpretations of these rating philosophies may exist. Compare Heitfield [2005] for a discussion.

Due to the limitations of publicly available data we use Moody's global default rates, total recoveries, and recoveries by seniority class. Exhibit 7 shows the estimation results for the through-the-cycle model (4) and (9) and Exhibit 8 for the point-in-time model (11) and (13) using the variables COINC and LEAD as explanatory variables. In the latter model we choose both variables due to their statistical significance.

[*** Insert Exhibit 7 about here ***]

[*** Insert Exhibit 8 about here ***]

First, consider the through-the-cycle model. Since we use the same default rates in each model, the estimates for the default process are similar across models, and consistent to the ones found in other studies (compare Gordy [2000] or Rösch [2005]). The parameter estimates for the (transformed) recovery process reflect estimates for the mean (transformed) recoveries and their fluctuations over time. Most important are the estimates for the correlation of the two processes which are positive and similar in size to the correlations between default rates and recovery rates found in previous studies. Note that this is the correlation between the systematic factor driving the latent default triggering variable 'asset return' S_{it} and the systematic factor driving the recovery process. Therefore, higher 'asset returns' (lower conditional default probabilities) tend to come along with higher recovery. A positive value of the correlation indicates negative association between defaults and recoveries. The default rate decreases while the recovery rate increases in boom years and vice versa in depression years.

Next, consider the point-in-time model. The default and the recovery process is driven by one macroeconomic variable in each model. The parameters of all macroeconomic variables show a plausible sign. The negative sign of the COINC index in the default process signals that a positive change of the index comes along with subsequent lower number of defaults. The positive signs of the variables in the recovery process indicate that higher recoveries follow a positive change in the variable. In addition, most variables are significant at the 10% level. The only exception is the parameter of the macroeconomic index LEAD for the senior subordinated recovery rate, which indicates only a limited exposure to systematic risk drivers. Note that the influence of the systematic random factor is reduced in each process by the inclusion of the macroeconomic variable.

While we do not mean to interpret these indices as risk drivers themselves, but rather as proxies for the (future) state of the economy, these variables are able to explain part of the previously unobservable systematic risk. The remaining systematic risk is reflected by the size of w and b and is still correlated but not explainable by our proxies.

Once the point estimates for the parameters are given, we forecast separately defaults and recoveries for year 2004. Exhibit 9 shows that the point-in-time model leads to forecasts for the default and recovery rates that are closer to the realized values than the ones derived from the through-the-cycle model.

[*** Insert Exhibit 9 about here ***]

Implications for Economic and Regulatory Capital

Since the main contribution of our approach lies in the joint modeling of defaults and recoveries, we now apply the forecast default rates, recovery rates for the year 2004 as well as their estimated correlation to a portfolio of 1,000 obligors. To keep things simple we take the senior secured class as an example and assume a credit exposure of one monetary unit for each obligor.

Exhibit 10 and Exhibit 11 compare two forecast loss distributions of the through-the-cycle model. To demonstrate the influence of correlation between the processes we compare the distribution which assumes independence to the distribution which is based on the estimated correlation between the default and recovery rate transformations of 0.7049. Economic capital or the credit portfolio risk is usually measured by higher percentiles of the simulated loss variable such as the 95-, 99-, 99.5- or 99.9- percentile (95%-, 99%-, 99.5%- or 99.9%-Value-at-Risk). It can be seen that these percentiles are considerably higher if correlations between default and recovery rates are taken into account. If we take the 99.9%-Value-at-Risk as an example, the percentile under dependence exceeds the percentile under independence by approximately 50 percent. In other words, if dependencies are not taken into account which is a common feature in many of today's credit risk models the credit portfolio risk is likely to be seriously underestimated.

[*** Insert Exhibit 10 about here ***]

[*** Insert Exhibit 11 about here ***]

The forecast default and recovery rates can be used to calculate the regulatory capital for the hypothetical portfolio. In the context of corporate credit exposures, the Basel Committee on Banking Supervision [2004] allows banks to choose one of the following options:

- Standardized approach: the regulatory capital is calculated based on the corporate issuer credit rating and results in a regulatory capital between 1.6% and 12% of the credit exposure. The regulatory capital equals 8% of the credit exposure if firms are unrated;
- Foundation Internal Ratings Based (IRB) approach: the regulatory capital is calculated based on the forecast default probabilities and a proposed loss given default for senior secured claims of 45% (i.e., a recovery rate of 55%) and for subordinated claims of 75% (i.e., a recovery rate of 25%).
- Advanced IRB approach: the regulatory capital is calculated based on the forecast default probabilities and forecast recovery rates.

For the through-the-cycle model the Standardized approach and the Foundation IRB approach result in a relatively close regulatory capital requirement (80.00 vs. 74.01). The reason for this is that the forecast default rate (0.0181) is close to the historic average which was used by the Basel Committee for the calibration of the regulatory capital to the current prevailing 8%. The Advanced IRB approach leads to a lower regulatory capital (70.08 vs. 74.01) due to a forecast recovery rate which is higher than the assumption in the Foundation IRB approach (57.39% vs. 55%). Note that the Foundation IRB approach’s recovery rate of 55% is comparable to the average recovery rate of the senior secured seniority class but is proposed to be applied to both the senior secured (unless admitted collateral is available) as well as the senior unsecured claims. This could indicate an incentive for banks to favor the Foundation approach over the Advanced IRB approach especially for senior unsecured credit exposures. Similar conclusions can be drawn for the Foundation IRB approach's recovery rate of 25% which will be applied for both senior subordinated as well as subordinated claims.

Exhibit 12 and Exhibit 13 compare the respective loss distributions with and without correlations using the point-in-time model.

[*** Insert Exhibit 12 about here ***]

[*** Insert Exhibit 13 about here ***]

It can be observed that the economic capital - expressed as Value-at-Risk - is considerably lower for the point-in-time model than for the through-the-cycle model. The reasons are twofold. First, the inclusion of macroeconomic variables leads to a lower forecast of the default rate (1.62%), a higher forecast of the recovery rate (61.59%) for 2004 and therefore to lower expected losses. Second, the exposure to unknown random systematic risk sources is reduced by the inclusion of the observable factors. This leads to less uncertainty in the loss forecasts and therefore to lower variability (measured, e.g., by the standard deviation) of the forecast distribution. Moreover, the regulatory capital is the lowest for the Advanced IRB approach which takes both the forecast default and recovery rate into account.

We also notice another important effect. Again, the economic capital, measured by the higher percentiles of the credit portfolio loss increases if the estimated correlation between the default and recovery rates are taken into account, but the increase is not that dramatic as in the through-the-cycle model, although the correlation between risk factors of defaults and recoveries has slightly increased. The inclusion of macroeconomic factors renders the systematic unobservable factors less important and diminishes the impact of correlations between both factors. To the extent that recoveries and defaults are not exposed at all to unobservable random factors, the correlations between these factors are negligible for loss distribution modeling. Exhibit 14 shows this effect. We assumed constant exposure of b=0.5 to the recovery factor and varied the exposure to the systematic factor for the defaults (asset correlation) for given correlation between the systematic factors. The benchmark case is a correlation of zero between the factors. Here we notice a reduction of economic capital from 44 (i.e., 4.4% of total exposure) for an asset correlation of 0.1 to 13 (1.3%) when the asset correlation is zero. In the case of a correlation between the factors of 0.8 the Value-at-Risk is reduced from 61 (6.1%) to 13 (1.3%). Thus, the higher the correlation of the risk factors, the higher the economic capital gains are from lowering the implied asset correlation by the explanation with observable factors.

[*** Insert Exhibit 14 about here ***]

Discussion

The empirical analysis resulted in the following insights:

1. Default events and recovery rates are correlated; Based on an empirical data set a
    a. Positive correlation between the default events and
    b. Negative correlation between the default events and recovery rates

   was found.
2. The incorporation of the correlation between the default events and recovery rates increases the economic capital. As a result most banks underestimate their economic capital when they do not account for this correlation.
3. Correlations between defaults decrease when systematic risk drivers, such a macroeconomic indices are taken into account. In addition, the impact of correlation between defaults and recoveries decreases.
4. As a result, the uncertainty of forecast losses and the economic capital measured by the percentiles decreases when systematic risk drivers are taken into account.
5. The economic as well as the regulatory capital charge for a given period depends on the forecast of the default and recovery rates.

Note that most empirical studies on recovery rates (including this article) are based on publicly available data provided by the rating agencies Moody's or Standard and Poor's and naturally lead to similar results. The data sets of the rating agencies are biased in the sense that only certain exposures are taken into account. Typically, large U.S. corporate obligors in capital intensive industries with one or more public debt issues and high credit quality are included. Thus, the findings can not automatically be transferred to other exposure classes (e.g., residential mortgage or credit card exposures), countries, industries or products.

Moreover, the data is limited with regard to the number of exposures and periods observed. Note that our assumption in (8) of a large number of firms is crucial since it leads to the focus on the mean recovery. If idiosyncratic risk can not be fully diversified the impact of systematic risk in our estimation may be overstated. Due to the data limitations we cannot draw any conclusions about the cross-sectional distribution of recoveries which is often stated to be U-shaped (see, e.g., Schuermann [2003]). In this sense, our results call for more detailed analyses, particularly with borrower-specific data which possibly includes financial ratios or other obligor characteristics and to extend our methodology to a panel of individual data. As a result, we would like to call upon the industry, i.e., companies, banks and regulators for feedback and a sharing of their experience.

In spite of these limitations, the present paper provides a robust framework, which allows creditors to model default probabilities and recovery rates based on certain risk drivers and simultaneously estimate interdependences between defaults and recoveries. It can be applied to different exposure types and associated information levels. Contrary to competing models, the presence of market prices such as bond or stock prices is not required.

Appendix: Results of Monte-Carlo Simulations

In order to prove the reliability of our estimation method a Monte-Carlo simulation was set up which comprises four steps:

- Step 1: Specify model (1) and model (9) with a given set of population parameters w, c, b, μ, and ρ.
- Step 2: Draw a random time series of length T for the defaults and the recoveries of a portfolio with size N from the true model.
- Step 3: Estimate the model parameters given the drawn data by the Maximum-Likelihood method.
- Step 4: Repeat Steps 2 and 3 for several iterations.

We use 1,000 iterations for different parameter constellations and obtain 1,000 parameter estimates which are compared to the true parameters. The portfolio consists of 10,000 obligors. The length of the time series T is set to T=20 years. We fix the parameters at w=0.2, μ=0.5, and b=0.5 and set the correlations between the systematic factors to 0.8, 0.1, and -0.5. In addition, we analyze three rating grades A, B, and C where the default probabilities and thresholds c in the grades are:

- A: $\pi = 0.005$, i.e., c=-2.5758
- B: $\pi = 0.01$, i.e., c=-2.3263
- C: $\pi = 0.02$, i.e., c=-2.0537

[*** Insert Exhibit 15 about here ***]

Exhibit 15 contains the results from the simulations. The numbers without brackets contain the average of the parameter estimates from 1,000 simulations. The numbers in round (.)-brackets represent the sample standard deviation of the estimates (which serve as an approximation for the unknown standard deviation). The numbers in square [.]-brackets give the average of the estimated standard deviations for each estimate derived by Maximum-Likelihood theory. It can be seen in each constellation that our ML–approach for the joint estimation of the default and recovery process works considerably well: the averages of the estimates are close to the originally specified parameters. Moreover, the estimated standard deviations reflect the limited deviation for individual iterations. The small downward bias results from the asymptotic nature of the ML-estimates and should be tolerable for practical applications.

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Exhibits

Exhibit 1: Descriptive statistics of variables

Variable	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Default Rate	0.0176	0.0144	0.0382	0.0052	0.0103	0.6849	2.2971
Recovery Rate (Total)	0.4208	0.4300	0.6170	0.2570	0.0902	0.2883	3.0464
Recovery Rate (S_Sec)	0.5794	0.5725	0.8360	0.3570	0.1379	0.2631	2.0440
Recovery Rate (S_Un)	0.4481	0.4450	0.6280	0.2310	0.1158	-0.1816	2.2725
Recovery Rate (S_Sub)	0.3703	0.3695	0.5190	0.2030	0.0984	-0.1868	1.7668
Recovery Rate (Sub)	0.2987	0.3245	0.4620	0.1230	0.1117	-0.2227	1.7387
COINC	0.0215	0.0245	0.0409	-0.0165	0.0160	-0.9365	3.0335
LEAD	0.0130	0.0154	0.0336	-0.0126	0.0151	-0.4568	1.9154

Exhibit 2: Bravais-Pearson correlations of variables

Variable	Default Rate	Total	S_Sec	S_Un	S_Sub	Sub	COINC	LEAD
Default Rate	1.00	-0.67	-0.72	-0.72	-0.53	-0.34	-0.75	-0.47
Recovery Rate (Total)		1.00	0.78	0.68	0.72	0.29	0.32	0.54
Recovery Rate (S_Sec)			1.00	0.66	0.48	0.37	0.33	0.55
Recovery Rate (S_Un)				1.00	0.56	0.42	0.49	0.48
Recovery Rate (S_Sub)					1.00	0.24	0.20	0.40
Recovery Rate (Sub)						1.00	0.41	0.17
COINC							1.00	0.28
LEAD								1.00

Exhibit 3: Moody's default rate vs. recovery rate

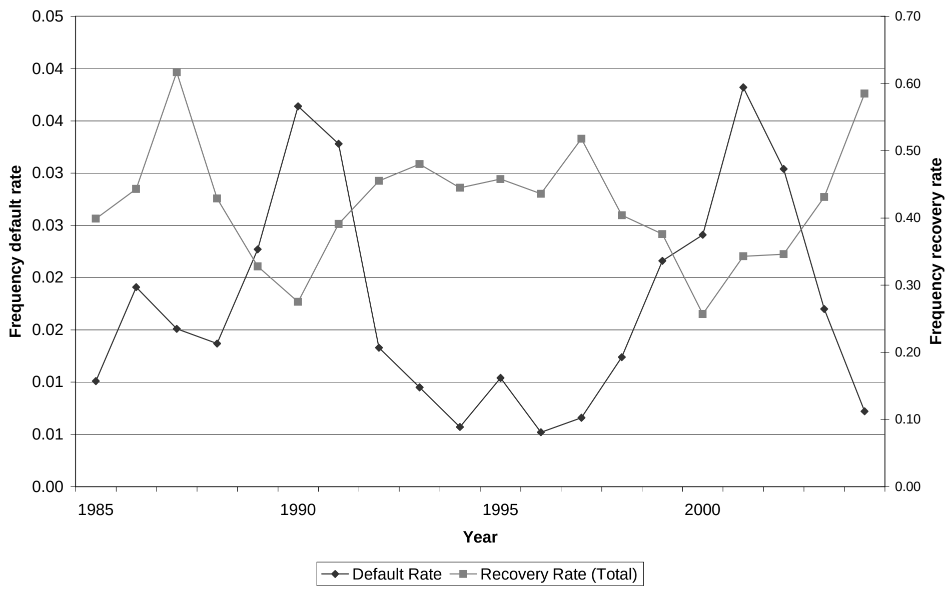

Exhibit 4: Moody's recovery rates by seniority class

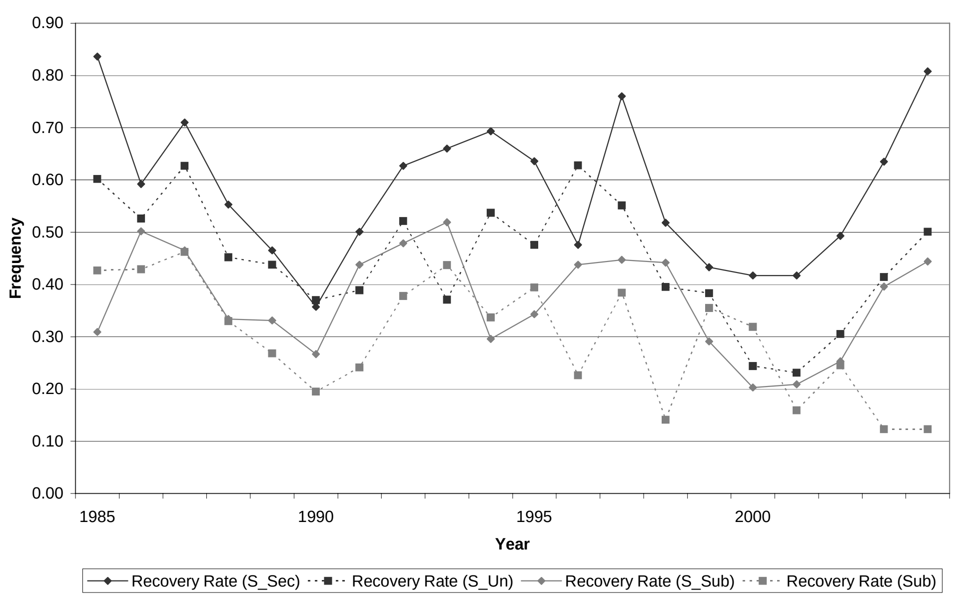

Exhibit 5: Recovery rates for selected industries (Moody's [2004])

Industy	Recovery Rate (1982-2003)
Utility-Gas	0.515
Oil	0.445
Hospitality	0.425
Utility-Electric	0.414
Transport-Ocean	0.388
Media, Broadcasting and Cable	0.382
Transport-Surface	0.366
Finance and Banking	0.363
Industrial	0.354
Retail	0.344
Transport-Air	0.343
Automotive	0.334
Healthcare	0.327
Consumer Goods	0.325
Construction	0.319
Technology	0.295
Real Estate	0.288
Steel	0.274
Telecommunications	0.232
Miscellaneous	0.395

Exhibit 6: Recovery rates for selected issuer credit rating categories (Moody's [2005])

Issuer Credit Rating	Recovery Rate (1982-2004)
Aa	0.954
A	0.498
Baa	0.433
Ba	0.407
B	0.384
Caa-Ca	0.364

Exhibit 7: Parameter estimation results for the through-the-cycle model

Annual default and recovery data from 1985 to 2003 is used for estimation; standard errors are in parentheses; *** significant at 1% level, ** significant at 5% level, * significant at 10% level

Parameter	Total	S_Sec	S_Un	S_Sub	Sub
c	-2.0942***	-2.0951***	-2.0966***	-2.0942***	-2.0940***
	(0.0545)	(0.0550)	(0.0546)	(0.0544)	(0.0549)
w	0.2194***	0.2212***	0.2197***	0.2191***	0.2210***
	(0.0366)	(0.0369)	(0.0367)	(0.0366)	(0.0369)
μ	-0.3650***	0.2976**	-0.2347*	-0.5739***	-0.8679***
	(0.0794)	(0.1284)	(0.1123)	(0.0998	(0.1235)
b	0.3462***	0.5598***	0.4898***	0.4351***	0.5384***
	(0.0562)	(0.0908)	(0.0795)	(0.0706)	(0.0873)
ρ	0.6539***	0.7049***	0.7520***	0.5081**	0.3979*
	(0.1413)	(0.1286)	(0.1091)	(0.1799)	(0.2013)

Exhibit 8: Parameter estimation results for the point-in-time model

Annual default and recovery data from 1985 to 2003 is used for estimation; standard errors are in parentheses; *** significant at 1% level, ** significant at 5% level, * significant at 10% level.

Parameter	Total	S_Sec	S_Un	S_Sub	Sub
γ_0	-1.9403***	-1.9484***	-1.9089***	-1.9232***	-1.9040***
	(0.0524)	(0.05210)	(0.0603)	(0.05660)	(0.0609)
γ_1	-8.5211***	-8.1786***	-10.0782***	-9.2828***	-10.1399***
	(1.8571)	(1.7964)	(2.2618)	(2.0736)	(2.2884)
	COINC	COINC	COINC	COINC	COINC
w	0.1473***	0.1522***	0.1485***	0.1483***	0.1508***
	(0.0278)	(0.0286)	(0.0276)	(0.0277)	(0.0279)
β_0	0.4557***	0.1607	-0.5576***	-0.6621***	-1.1883***
	(0.0867)	(0.1382)	(0.1635)	(0.1194)	(0.1845)
β_1	7.4191*	11.1867*	15.0807**	7.2136	14.9625**
	(4.1423)	(6.4208)	(6.1142)	(6.0595)	(6.8940)
	LEAD	LEAD	COINC	LEAD	COINC
b	0.3063***	0.4960***	0.4260***	0.4071***	0.4820***
	(0.0513)	(0.0838)	(0.0691)	(0.0673)	(0.0279)
ρ	0.6642***	0.7346***	0.6675***	0.4903**	0.1033
	(0.1715)	(0.1520)	(0.1481)	(0.2088)	(0.2454)

Exhibit 9: Forecasts and realizations for year 2004 (through-the-cycle versus point-in-time)

Parameter	Total	S_Sec	S_Un	S_Sub	Sub
Default Rate					
Forecast TTC	0.0181	0.0181	0.0180	0.0181	0.0181
Forecast PIT	0.0162	0.0162	0.0160	0.0162	0.0162
Realization	0.0072	0.0072	0.0072	0.0072	0.0072
Recovery Rate					
Forecast TTC	0.4097	0.5739	0.4416	0.3603	0.2957
Forecast PIT	0.4381	0.6159	0.4484	0.3867	0.3014
Realization	0.5850	0.8080	0.5010	0.4440	0.1230

Exhibit 10: Loss distributions for the through-the-cycle model (S_Sec)

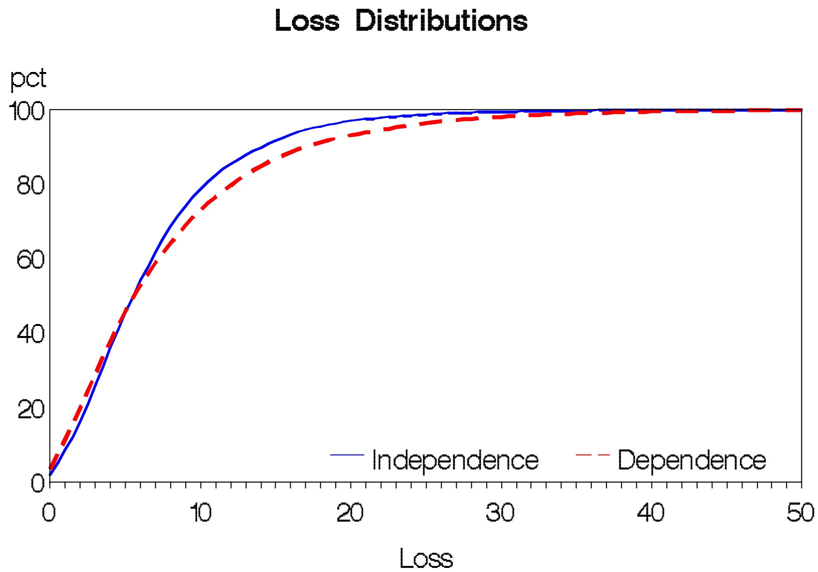

Exhibit 11: Descriptive statistics of loss distributions for the through-the-cycle model

Portfolios contain 1,000 obligors with an exposure of one monetary unit each, 10,000 random samples were drawn for each distribution with and without correlation between systematic factors

	Mean	Std.dev.	Med	95	99	99.5	99.9	Basel II Capital (Standardized)	Basel II Capital (Foundation IRB)	Basel II Capital (Advanced IRB)
Independent Factors	7.82	5.59	6.53	18.55	27.35	31.92	39.02	80.00	74.01	70.08
Correlated Factors	8.73	7.59	6.62	23.81	36.04	42.43	58.75	80.00	74.01	70.08

Exhibit 12: Loss distributions for the point-in-time model (S_Sec)

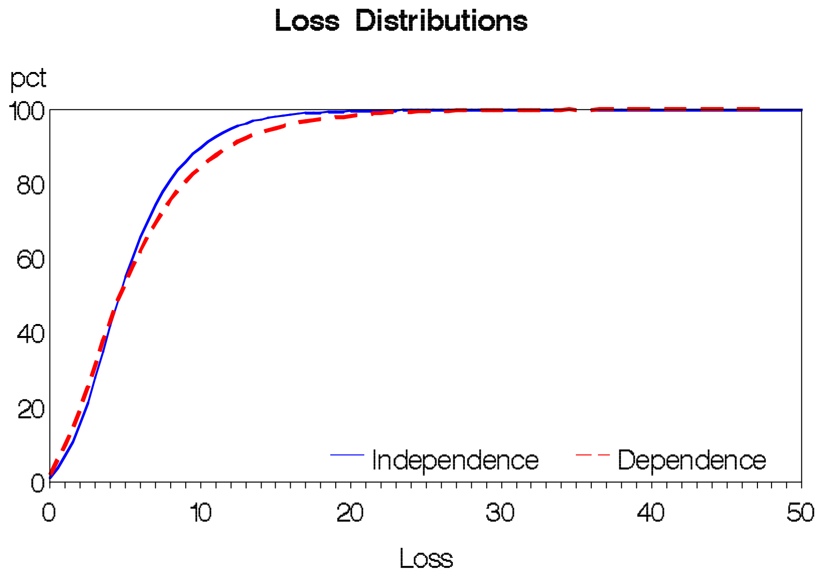

Exhibit 13: Descriptive statistics of loss distributions for the point-in-time model

Portfolios contain 1,000 obligors with an exposure of one monetary unit each, 10,000 random samples were drawn for each distribution with and without correlation between systematic factors

	Mean	Std.dev.	Med	95	99	99.5	99.9	Basel II Capital (Standardized)	Basel II Capital (Foundation IRB)	Basel II Capital (Advanced IRB)
Independent Factors	6.33	3.61	5.64	13.10	18.01	20.43	25.77	80.00	71.16	60.74
Correlated Factors	6.78	4.71	5.64	16.03	22.78	25.60	31.77	80.00	71.16	60.74

Exhibit 14: Economic capital gains from decrease in implied asset correlation for correlated risk factors

Exhibit shows 99.9-percentiles of loss distributions for the senior secured seniority class depending on asset correlation and correlation of systematic risk factors. Portfolio contains 1,000 obligors each with default probability of 1%, exposure of one monetary unit, and expected recovery of 50%.

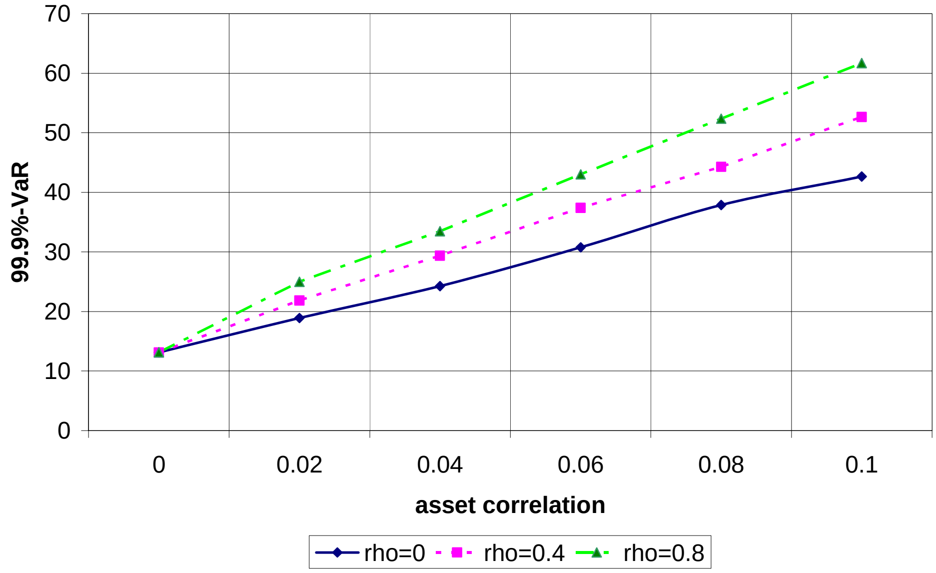

Exhibit 15: Results from Monte-Carlo simulations

Grade	ρ	c	w	μ	b	ρ
A	0.8	-2.5778	0.1909	0.4991	0.4784	0.7896
		(0.0495)	(0.0338)	(0.1112)	(0.0776)	(0.1085)
		[0.0468]	[0.0317]	[0.1070]	[0.0756]	[0.0912]
	0.1	-2.5789	0.1936	0.4970	0.4824	0.1139
		(0.0484)	(0.0336)	(0.1154)	(0.0788)	(0.2269)
		[0.0475]	[0.0322]	[0.1079]	[0.0763]	[0.2185]
	-0.5	-2.5764	0.1927	0.5048	0.4826	-0.4956
		(0.0492)	(0.0318)	(0.1116)	(0.0798)	(0.1923)
		[0.0472]	[0.0320]	[0.1078]	[0.0763]	[0.1697]
B	0.8	-2.3287	0.1927	0.4999	0.4852	0.7951
		(0.0480)	(0.0327)	(0.1104)	(0.0774)	(0.0920)
		[0.0460]	[0.0306]	[0.1084]	[0.0765]	[0.0856]
	0.1	-2.3291	0.1906	0.4927	0.4831	0.0861
		(0.0472)	(0.0306)	(0.1105)	(0.0778)	(0.2330)
		[0.0456]	[0.0305]	[0.1080]	[0.0764]	[0.2152)]
	-0.5	-2.3305	0.1900	0.4988	0.4805	-0.4764
		(0.0479)	(0.0324)	(0.1115)	(0.0806)	(0.1891)
		[0.0453]	[0.0303]	[0.1074]	[0.0759]	[0.1703]
C	0.8	-2.0536	0.1935	0.4972	0.4855	0.7915
		(0.0489)	(0.0315)	(0.1104)	(0.0804)	(0.0956)
		[0.0448]	[0.0297]	[0.1080]	[0.0763]	[0.0843]
	0.1	-2.0542	0.1943	0.5030	0.4851	0.1067
		(0.0580)	(0.0382)	(0.1168)	(0.0782)	(0.2374)
		[0.0448]	[0.0298]	[0.1085]	[0.0770]	[0.2128]
	-0.5	-2.0554	0.1923	0.4998	0.4833	-0.4898
		(0.0510)	(0.0359)	(0.1085)	(0.0852)	(0.1815)
		[0.0443]	[0.0295]	[0.1076]	[0.0766]	[0.1656]