Developments in Mac Lane Set Theory

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This talk reports on the status of recent work in "Mac Lane set theory" (**ML**). Two years ago in [2], S. Mac Lane called for a categorical set theory with a global membership relation $A \in B$ between the sets. In contrast to elementary topos theory with its many different (and useful) interpretations, the idea was to axiomatize categories of *sets* specifically. Mac Lane's proposal was for the category Sets of sets and functions to have a subcategory of distinguished monomorphisms $A \hookrightarrow B$, called *inclusions*, which partially order the objects. In joint work with Butz, Simpson, Streicher and the author, some progress toward this goal has recently been made, using the methods of algebraic set theory developed by Joyal and Moerdijk [1]. This approach employs an axiomatic notion of *small maps* in a category, which serve to limit comprehension in a new and flexible way. The axioms of **ML** are a variant of those recently given by Simpson [3] for categorical models of **IZF**.

An apparently new result in this connection is that the axiom scheme of (full) Replacement is conservative over bounded intuitionistic Zermelo set theory.

References

[1] A. Joyal and I. Moerdijk, *Algebraic Set Theory*, LMS Lecture Notes 220, Cambridge University Press, 1995.

[2] S. Mac Lane, "Foundations by using categories with elements," Annual Meeting of the Association for Symbolic Logic, Urbana, June 2000.

[3] A. Simpson, "Elementary axioms for categories of classes," *14th IEEE Symposium on Logic in Computer Science*, pp. 77–85, 1999.