ERRATUM TO INFINITY-INNER-PRODUCTS ON A-INFINITY-ALGEBRAS, JOURNAL OF HOMOTOPY AND RELATED STRUCTURES, VOL. 3(1), 2008, PP.245–271

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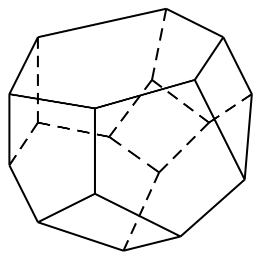
The following two items should be corrected in [T].

- The sign $(-1)^\varepsilon$ in Lemma 2.9 was falsely stated. Correctly, it should read

$$\varepsilon := (|a_1| + ... + |a_k|) \cdot (|m^*| + |a_{k+1}| + ... + |a_{k+l}| + |m|) + |m^*| \cdot (k + l + 1) + (k + 1) \cdot (l + 1).$$

- An edge in the pairahedron for $k = 2, l = 1$ (or for $k = 1, l = 2$) was misplaced on page 270. The pairahedron associated to $< \cdots >_{2,1}$ (and also associated to $< \cdots >_{1,2}$) consists of 4 square, 4 pentagons, and 3 hexagons. It is depicted below:



Acknowledgements. *I would like to thank Andrea Ferrario for pointing out the sign issue in Lemma 2.9, and Stefan Forcey and Jim Stasheff for bringing the misplaced edge to my attention.*

References

[T] T. Tradler, *Infinity-Inner-Products on A-Infinity-Algebras*, Journal of Homotopy and Related Structures, vol. 3(1), 2008, pp.245–271

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We now provide a proof that the sign ε given above is the correct sign for [T, Lemma 2.9]. (Note, that in the online version at arXiv:math/0108027v2, the corresponding lemma to [T, Lemma 2.9] is Lemma 3.9.)

Proof that the expression ε above satisfies the A_∞-bimodule equations for M^*.

Define the dual module structure as $(b'_{k,l}(a_1,\dots,a_k,m^*,a_{k+1},\dots,a_{k+l}))(m) := (-1)^\varepsilon\cdot m^*(b_{l,k}(a_{k+1},\dots,a_{k+l},m,a_1,\dots,a_k))$, where $\varepsilon := (|a_1|+\cdots+|a_k|)\cdot(|m^*|+|a_{k+1}|+\cdots+|a_{k+l}|+|m|)+|m^*|\cdot(k+l+1)+(k+1)\cdot(l+1)$. We need to calculate

$$x := \Bigg(\sum_{i=1}^{k}\sum_{j=1}^{k-i+1}(-1)^{\varepsilon_1}\cdot b'_{k-i+1,l}(a_1,\dots,m_i(a_j,\dots,a_{i+j-1}),\dots,m^*,\dots,a_{k+l})$$
$$+\sum_{i=0}^{k}\sum_{j=0}^{l}(-1)^{\varepsilon_2}\cdot b'_{k-i,l-j}(a_1,\dots,b'_{i,j}(a_{k-i+1},\dots,m^*,\dots,a_{k+j}),\dots,a_{k+l})$$
$$+\sum_{i=1}^{l}\sum_{j=1}^{l-i+1}(-1)^{\varepsilon_3}\cdot b'_{k,l-i+1}(a_1,\dots,m^*,\dots,m_i(a_{k+j},\dots,a_{k+i+j-1}),\dots,a_{k+l})\Bigg)(m),$$

and show that it vanishes, *i.e.* that $x=0$. If we abbreviate $a_{r,s}=|a_r|+\cdots+|a_s|$, then the signs in x are given as follows,

$$\begin{aligned}\varepsilon_1 &= i\cdot a_{1,j-1}+(j-1)\cdot(i+1)+k-i+1+l,\\ \varepsilon_2 &= (i+j+1)\cdot a_{1,k-i}+(k-i)\cdot(i+j+2)+k-i+l-j,\\ \varepsilon_3 &= i\cdot(a_{1,k+j-1}+|m^*|)+(k+j)\cdot(i+1)+k+l-i+1.\end{aligned}$$

In order to see that $x=0$, it will be enough to show that $(-1)^c\cdot x=0$, where we set

$$c := a_{1,k}\cdot(|m^*|+a_{k+1,k+l}+|m|)+|m^*|\cdot(k+l)+(k+1)\cdot(l+1).$$

Using the expression for ε for the dual module, we see that we can replace the individual expressions as follows. First,

$$\begin{aligned}&b'_{k-i+1,l}(a_1,\dots,m_i(a_j,\dots,a_{i+j-1}),\dots,m^*,\dots,a_{k+l})(m)\\ &\quad=(-1)^{\varepsilon_4}\cdot m^*(b_{l,k-i+1}(a_{k+1},\dots,a_{k+l},m,a_1,\dots,m_i(a_j,\dots,a_{i+j-1}),\dots,a_k)),\end{aligned}$$

where

$$\varepsilon_4\equiv(a_{1,k}+i)\cdot(|m^*|+a_{k+1,k+l}+|m|)+|m^*|\cdot(k-i+l)+(k-i)\cdot(l+1)\quad(mod\ 2).$$

Thus, we have a combined sign of,

$$\begin{aligned}\varepsilon_5 &:= c+\varepsilon_1+\varepsilon_4\\ &\equiv i\cdot(a_{k+1,k+l}+|m|+a_{1,j-1})+(l+j)\cdot(i+1)+l+k-i+1\quad(mod\ 2).\end{aligned}$$

Next,

$$\begin{aligned}&b'_{k-i,l-j}(a_1,\dots,b'_{i,j}(a_{k-i+1},\dots,m^*,\dots,a_{k+j}),\dots,a_{k+l})(m)\\ &=(-1)^{\varepsilon_6}\cdot(b'_{i,j}(a_{k-i+1},\dots,m^*,\dots,a_{k+j}))(b_{l-j,k-i}(a_{k+j+1},\dots,a_{k+l},m,a_1,\dots,a_{k-i}))\\ &=(-1)^{\varepsilon_6+\varepsilon_7}\cdot m^*(b_{j,i}(a_{k+1},\dots,a_{k+j},b_{l-j,k-i}(a_{k+j+1},\dots,a_{k+l},m,a_1,...,a_{k-i}),a_{k-i+1},\dots,a_k)),\end{aligned}$$

where

$$\begin{aligned}\varepsilon_6 &\equiv a_{1,k-i}\cdot(i+j+1+a_{k-i+1,k+l}+|m^*|+|m|)\\ &\quad +(i+j+1+a_{k-i+1,k+j}+|m^*|)\cdot(k-i+l-j+1)\\ &\quad +(k-i+1)\cdot(l-j+1) \qquad (mod\ 2),\\ \varepsilon_7 &\equiv a_{k-i+1,k}\cdot(|m^*|+a_{k+1,k+l}+l-j+k-i+1+|m|+a_{1,k-i})\\ &\quad +|m^*|\cdot(i+j+1)+(i+1)\cdot(j+1) \qquad (mod\ 2),\end{aligned}$$

so that

$$\begin{aligned}\varepsilon_8 &:= c+\varepsilon_2+\varepsilon_6+\varepsilon_7\\ &\equiv (l-j+k-i+1)\cdot a_{k+1,k+j}+j\cdot(l-j+k-i)+j+i \quad (mod\ 2).\end{aligned}$$

Finally,

$$\begin{aligned}&b'_{k,l-i+1}(a_1,\dots,m^*,\dots,m_i(a_{k+j},\dots,a_{k+i+j-1}),\dots,a_{k+l})(m)\\ &=(-1)^{\varepsilon_9}\cdot m^*(b_{l-i+1,k}(a_{k+1},\dots,m_i(a_{k+j},\dots,a_{k+i+j-1}),\dots,a_{k+l},m,a_1,\dots,a_k)),\end{aligned}$$

where

$$\varepsilon_9\equiv a_{1,k}\cdot(|m^*|+a_{k+1,k+l}+i+|m|)+|m^*|\cdot(k+l-i)+(k+1)\cdot(l-i) \quad (mod\ 2).$$

so that

$$\begin{aligned}\varepsilon_{10} &:= c+\varepsilon_3+\varepsilon_9\\ &\equiv i\cdot a_{k+1,k+j-1}+(j-1)\cdot(i+1)+l-i+1+k \quad (mod\ 2).\end{aligned}$$

We see that

$$\begin{aligned}&(-1)^c\cdot x\\ &=\sum_{i=1}^{k}\sum_{j=1}^{k-i+1}(-1)^{\varepsilon_5}\cdot m^*(b_{l,k-i+1}(a_{k+1},\dots,a_{k+l},m,a_1,\dots,m_i(a_j,\dots,a_{i+j-1}),\dots,a_k))\\ &+\sum_{i=0}^{k}\sum_{j=0}^{l}(-1)^{\varepsilon_8}\cdot m^*(b_{j,i}(a_{k+1},\dots,a_{k+j},b_{l-j,k-i}(a_{k+j+1},\dots,a_{k+l},m,a_1,...,a_{k-i}),a_{k-i+1},\dots,a_k))\\ &+\sum_{i=1}^{l}\sum_{j=1}^{l-i+1}(-1)^{\varepsilon_{10}}\cdot m^*(b_{l-i+1,k}(a_{k+1},\dots,m_i(a_{k+j},\dots,a_{k+i+j-1}),\dots,a_{k+l},m,a_1,\dots,a_k))\\ &=m^*\Bigg(\sum_{i=1}^{l}\sum_{j=1}^{l-i+1}(-1)^{\varepsilon_{10}}\cdot b_{l-i+1,k}(a_{k+1},\dots,m_i(a_{k+j},\dots,a_{k+i+j-1}),\dots,a_{k+l},m,a_1,\dots,a_k)\\ &+\sum_{j=0}^{l}\sum_{i=0}^{k}(-1)^{\varepsilon_8}\cdot b_{j,i}(a_{k+1},\dots,a_{k+j},b_{l-j,k-i}(a_{k+j+1},\dots,a_{k+l},m,a_1,...,a_{k-i}),a_{k-i+1},\dots,a_k)\\ &+\sum_{i=1}^{k}\sum_{j=1}^{k-i+1}(-1)^{\varepsilon_5}\cdot b_{l,k-i+1}(a_{k+1},\dots,a_{k+l},m,a_1,\dots,m_i(a_j,\dots,a_{i+j-1}),\dots,a_k)\Bigg)\\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad=m^*(0)=0.\end{aligned}$$

Here we used the fact that the $\{b_{k,l}\}_{k,l\geq 0}$ define an A_∞-bimodule structure on M to see that this expression vanishes. This completes the proof of the lemma. □