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2009 EPL 85 14002

(http://iopscience.iop.org/0295-5075/85/1/14002)

A Letters Journal Exploring the Frontiers of Physics

January 2009

EPL, **85** (2009) 14002
doi: 10.1209/0295-5075/85/14002

www.epljournal.org

Effect of the strong magnetic field on the magnetic interaction between two non-magnetic particles migrating in a conductive fluid

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received 23 September 2008; accepted in final form 1 December 2008
published online 7 January 2009

PACS 47.65.Cb – Magnetic fluids and ferrofluids
PACS 75.47.-m – Magnetotransport phenomena; materials for magnetotransport

Abstract – This paper presents a theoretical analysis of the interaction between two non-magnetic particles migrating in a conductive fluid due to an imposed strong magnetic field. A repulsive force induced by the conductive fluid flow around the particles is first derived and calculated for a concrete example. From the numerical results, a counteracting behavior with the interparticle magnetic dipole-dipole attractive force is found to exist at a critical particle size which makes the particles controllable under strong magnetic fields (*e.g.*, $B > 10\,\mathrm{T}$). This renders a better understanding of recent experimental results and provides a basis to control the particle size distribution using strong magnetic fields for materials processing.

Introduction. – Magnetic fields have been widely used for the removal of particles from fluids [1] and the preparation of alloys with aligned or gradient particle distribution [2,3]. Fundamental in these applications is that the induced magnetic forces on the particles and the fluid can enhance the separation and change the interaction behavior between particles. Ferromagnetic or super-paramagnetic particles in a conductive fluid, such as Fe_3O_4 particles can be easily separated even in the nanometer range [1,4] due to an agglomeration behavior enhanced by the imposed magnetic field. For non-magnetic particles in a conductive fluid, however, a distinct behavior is observed. For instance, micrometer-sized non-metallic particles (*e.g.*, $< 10\,\mu$m) in liquid metal can agglomerate and form larger clusters which are easily removed by the applied magnetic field [5]. In other cases, the agglomeration of large non-magnetic particles (*e.g.*, several tens of μm) is restricted, resulting in a finer particle size distribution of primary crystals in the matrix [6] or peculiar particle alignments [7]. The latter phenomenon becomes more obvious in stronger magnetic fields [8,9]. These observations suggest the existence of a repulsive force in parallel with the magnetic dipole-dipole attractive force. Furthermore, this interaction force exhibits a clear dependence on the particle size and the magnetic flux density [8,9].

The magnetic dipole-dipole attractive force has been investigated and measured under different conditions [10,11]. However, the repulsive force between two non-ferromagnetic particles migrating in a conductive fluid has not yet been clearly explained. In the present letter, we propose and describe a phenomenon, which can cause a repulsive force between two migrating particles in a conductive fluid under a strong magnetic field. To make a clear illustration of such phenomenon, the related interparticle forces for Al_2O_3 particles in liquid aluminum are analysed and calculated.

Theory and numerical method. – In strong magnetic fields, even a non-magnetic particle can be magnetised and therefore considered as a magnetic dipole. Its orientation is always parallel to the external magnetic field. The interparticle magnetic dipole-dipole attractive force becomes significant and has a value comparable to the net gravity force $\Delta G = (\rho_P - \rho_M)g \cdot \pi d_P^3/6$, where ρ_P and ρ_M are densities of the particle and the conductive melt, respectively, and g is the acceleration of gravity. The attractive interaction between two magnetised particles is shown in fig. 1(a). For a non-magnetic particle of homogeneous magnetic properties with a

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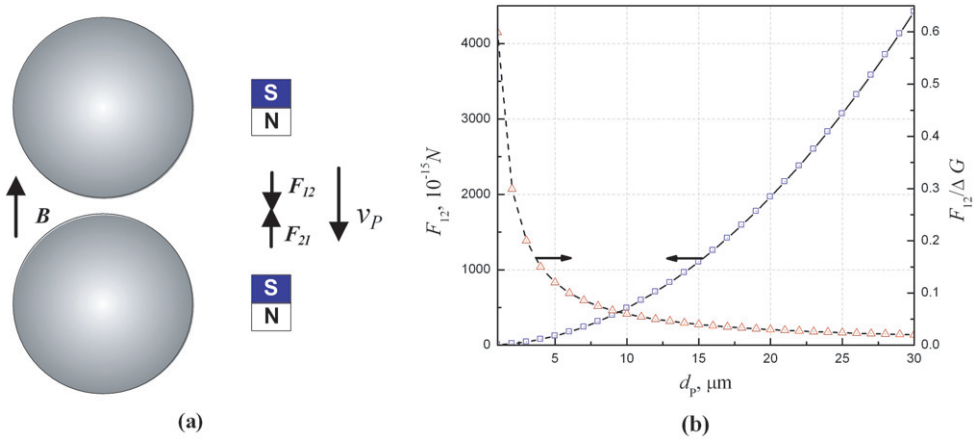

Fig. 1: (Colour on-line) Magnetisation dipole-dipole attractive forces F_{12} for two non-magnetic particles. (a) Schematic image for two magnetised particles. (b) F_{12} for two equal and attached Al_2O_3 particles in liquid aluminum ($l = d_P$ from eq. (2)) with different diameters in a strong magnetic field ($B = 12\,\mathrm{T}$) and a comparison with $\Delta G = (\rho_P - \rho_M)g \cdot \pi d_P^3/6$.

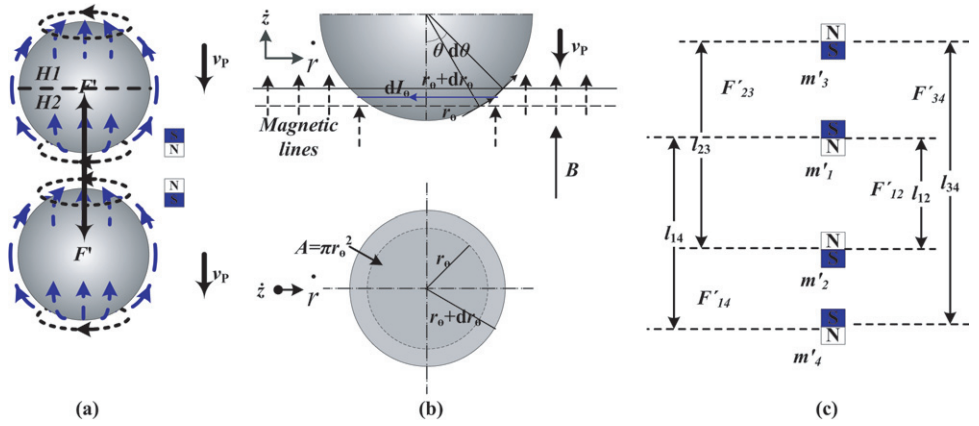

Fig. 2: (Colour on-line) Repulsive force due to the fluid flow around two non-magnetic particles in a conductive fluid; F'_{12}, F'_{23}, F'_{14}, F'_{34} are the forces between related dipoles.

diameter of d_P, its magnetic dipole moment m_P can be expressed by

$$m_P = \frac{\pi}{6} d_P^3 M, \tag{1}$$

where $M = \chi_P H$ (M is the volume density of the magnetic dipole moment and χ_P is the magnetic susceptibility of the particle) and H is the magnetic field intensity [12].

For two particles (with respective magnetic susceptibilities χ_{P1} and χ_{P2}) moving in tandem in a conductive fluid with a magnetic susceptibility χ_f, the magnetic dipole-dipole attractive force with a distance l (between the centers of dipoles which are the centers of the particles in this case) is given by [12]

$$F_{12} = -F_{21} = \frac{3\mu_0(1+\chi_f)m_1 m_2}{2\pi l^4};$$

when $\chi_f \ll 1$, we obtain

$$F_{12} = \frac{3\mu_0 m_1 m_2}{2\pi l^4} = \frac{\pi}{24\mu_0} \cdot \frac{d_1^3 d_2^3}{l^4} B^2 \chi_{P1}\chi_{P2} \left(l \geqslant \frac{d_1}{2} + \frac{d_2}{2}\right), \tag{2}$$

where m_1 and m_2 are the magnetic dipole moments of two magnetised particles defined by eq. (1), the velocity v_P and the magnetic flux density B are illustrated in fig. 1(a).

When particles are moving relative to the fluid (fig. 1(a)), *e.g.* because of gravity or convection, the fluid flow around the particles is generated as shown in fig. 2(a) by the blue arrows if the flow pattern is symmetric. The flow of the conductive fluid around the particles cuts the magnetic lines and induces a local voltage. The local voltage can generate a local circling current in the flowing conductive fluid as indicated in fig. 2(a) by the dashed circles with arrows. According to Faraday's law, such a circling current can be considered as a magnetic dipole. Therefore, a new interaction between a couple of tandem particles occurs and can be considered as four dipoles interacting as illustrated in fig. 2(c).

In practice, the flow pattern of the conductive fluid is much more complex and the fluid flow equations cannot be solved analytically [13]. In order to describe the interparticle force due to the flow of the fluid around the

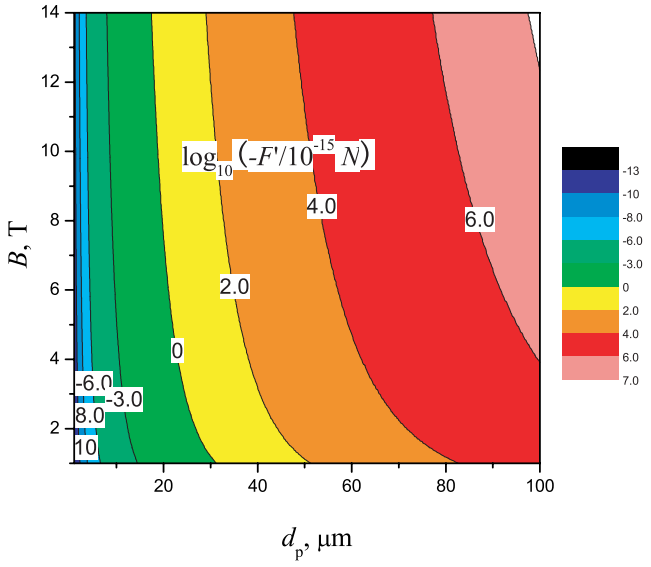

Fig. 3: (Colour on-line) Interparticle force F' with different particle size and magnetic flux density B.

particles 1 and 2, it is necessary to make some assumptions in addition to the symmetric flow pattern: 1) only a laminar flow (in the present case, the velocity is in the magnitude of 10^{-4} m/s and $\mathrm{Re} \ll 1$) boundary layer with a thickness of δ [13] is influenced and involved in cutting the magnetic lines; 2) the velocity v_L of the conductive fluid close to the surface of the particle can be obtained from the Navier-Stokes equations (consistent with the creeping flow model when $\mathrm{Re} \ll 1$) [13].

The following procedure is introduced to calculate the interparticle force. In a time span $\mathrm{d}t$, in which the particle moves from position S_1 to S_2 (see upward of fig. 2(b)), the change of magnetic flux $\mathrm{d}\phi$ of S_1 for a static magnetic field in the time span $\mathrm{d}t$ can be calculated by eq. (3) according to Faraday's law:

$$\mathrm{d}\phi = \mathrm{d}(B \cdot A) \cong B \cdot 2\pi r_\theta \, \mathrm{d}r_\theta. \tag{3}$$

An annulus with radius r_θ of the fluid around the particle is shown schematically in the lower part of fig. 2(b). The arrow indicates the current direction. According to the geometric relation in fig. 2(b), the following equations are established: $r_\theta = r_P \sin\theta$, $\mathrm{d}r_\theta = r_P \cos\theta \, \mathrm{d}\theta$ and $\mathrm{d}t = r_P \sin\theta \, \mathrm{d}\theta / v_P$, where r_P is the radius of the particle. Therefore, the induced local voltage ε_θ on this differential stripe of conductive fluid can be derived from the change of magnetic flux:

$$\varepsilon_\theta = -\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{B\mathrm{d}A}{\mathrm{d}t} = 2\pi r_P v_P B \cos\theta. \tag{4}$$

If we define R_θ as the resistance of this differential stripe of the conductive fluid, the current in this stripe can be obtained by eq. (5) according to Ohm's law:

$$\mathrm{d}I_\theta = \frac{\varepsilon_\theta}{R_\theta}. \tag{5}$$

For the electron conductive fluid, the resistance R_θ of this differential stripe of fluid can be calculated by the method described in [14]:

$$R_\theta = \frac{1}{2\pi\sigma_M r_P \sin\theta \, \mathrm{d}\theta} \ln\left(\frac{r_\theta + \delta\sin\theta}{r_\theta}\right). \tag{6}$$

Therefore, the current in this differential stripe can be obtained by the following equation:

$$\mathrm{d}I_\theta = \frac{\varepsilon_\theta}{R_\theta} = \frac{4\pi^2 r_P^2 v_P B \sigma_M \sin\theta\cos\theta \, \mathrm{d}\theta}{\ln\left(\dfrac{r_\theta + \delta\sin\theta}{r_\theta}\right)}. \tag{7}$$

Then the magnetic dipole moment $\mathrm{d}m'_\theta$ becomes:

$$\mathrm{d}m'_\theta = \pi r_\theta^2 \cdot \mathrm{d}I_\theta = \frac{4\pi^3 r_P^4 v_P B \sigma_M \sin^3\theta\cos\theta \, \mathrm{d}\theta}{\ln\left(\dfrac{r_\theta + \delta\sin\theta}{r_\theta}\right)}, \tag{8}$$

where the thickness of the liquid involved in cutting the magnetic lines can be estimated by using the simple boundary layer relationship [13] $\delta = 2r_P/\sqrt{\mathrm{Re}} = 2r_P/\sqrt{2\rho_M v_P r_P/\eta_M}$ and η_M is the viscosity of the conduction melt.

As shown in fig. 2, the flow of the fluid around the particle surface can be divided into two parts, *i.e.* two hemispheres, *H1* and *H2*, which can be considered as separated dipoles. Thus, the interparticle interaction caused by the fluid flow involves four dipole-dipole forces (fig. 2(c)). In the present case, eq. (2) is applied to calculate the force for two adjacent particles in line (fig. 2(a)) if the distances between the centers of the related dipoles are l_{12}, l_{34}, l_{14}, l_{23} (in fig. 2(c)). By integrating eq. (8), the magnetic dipole moment m' is obtained:

$$m' = \int_{\theta=0}^{\frac{\pi}{2}} \mathrm{d}m'_\theta = \frac{\pi^3 r_P^4 v_P B \sigma_M}{\ln\left(\dfrac{r_P + \delta}{r_P}\right)}. \tag{9}$$

The center of the magnetic dipole can be determined by the following equation:

$$z^* = \int_{\theta=0}^{\frac{\pi}{2}} r_P \cos\theta \, \mathrm{d}m'_\theta \Bigg/ \int_{\theta=0}^{\frac{\pi}{2}} \mathrm{d}m'_\theta \, . \tag{10}$$

For each dipole, the magnetic dipole moments m'_1, m'_2, m'_3, m'_4 and their center positions z_1^*, z_2^*, z_3^*, and z_4^* can be calculated through eqs. (9) and (10). The distances of the dipoles for two particles with radius r_1 and r_2 shown in fig. 2(c) are $l_{12} = r_1 + r_2 - z_1^* - z_2^*$, $l_{34} = r_1 + r_2 + z_3^* + z_4^*$, $l_{14} = r_1 + r_2 - z_1^* + z_4^*$, $l_{23} = r_1 + r_2 - z_2^* + z_3^*$.

Finally, the total interparticle force caused by the conductive fluid flow can be evaluated referred to eq. (2) by

$$F' \cong \frac{3\pi^5\mu_0}{512}\left(-\frac{1}{l_{12}^4} - \frac{1}{l_{34}^4} + \frac{1}{l_{14}^4} + \frac{1}{l_{23}^4}\right)\left[\frac{1}{\ln\left(\frac{d_P+2\delta}{d_P}\right)}\right]^2 \times d_P^8 B^2 \sigma_M^2 v_P^2. \tag{11}$$

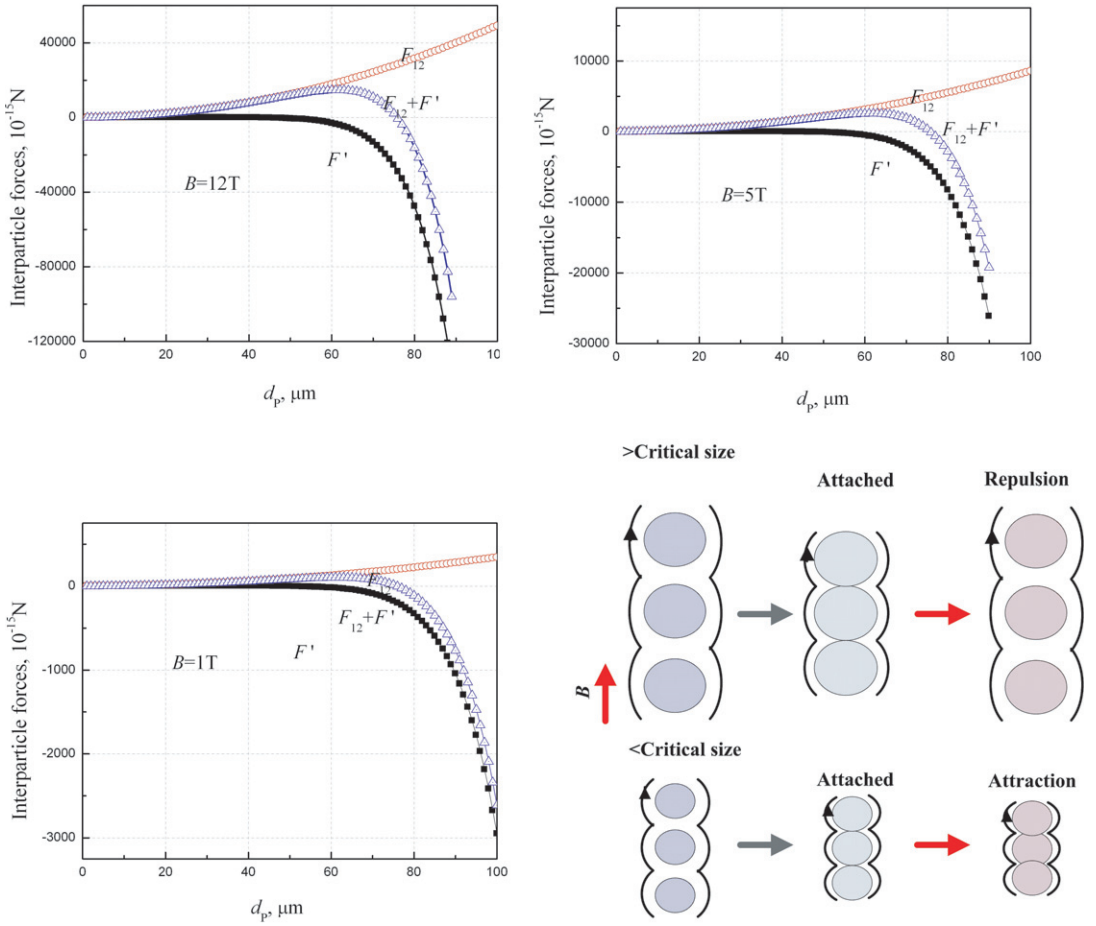

Fig. 4: (Colour on-line) Interparticle forces due to magnetic field in a conductive fluid. F_{12} and F' are the attractive and repulsive forces for two Al_2O_3 particles in liquid aluminum.

Results and discussion. – In recent experimental results [6–9], it was noted that the interparticle interaction depends on the particle size. Aimed to verify our theoretical analysis, the interparticle force calculations for Al_2O_3 particles in liquid aluminum are conducted in the present letter for different particle diameters.

For spherical Al_2O_3 particles migrating in a liquid aluminum with small Reynolds number, the velocity of the particle can be obtained from Stokes' law $v_P = d_P^2(\rho_P - \rho_M)g/18\eta_M$. The densities of Al_2O_3 and liquid aluminum at 933 K are 3970 kg/m^3, and 2370 kg/m^3, respectively. The viscosity η_M of the liquid aluminum at the considered temperature is 1.25×10^{-3}Pa$\cdot$s [5].

When two particles are too near to each other or attached, their boundary layers would overlap and make the conditions more complex. However, to simplify the calculation, we assume the above equations are suitable for this case. Therefore, the dipole-dipole distances l_{12}, l_{34}, l_{14}, l_{23} for two attached Al_2O_3 particles in liquid aluminum can be obtained from the above calculation and equations with the results of $l_{12} = 7d_P/15$, $l_{34} = 23\,d_P/15$ and $l_{14} = l_{23} = d_P$.

The interparticle forces can now be estimated for Al_2O_3 particles migrating in liquid aluminum. The magnetic susceptibilities of Al_2O_3 and liquid aluminum at 933 K are -1.81×10^{-5} and 1.323×10^{-5} (SI unit) [5]. The electrical conductivity of liquid aluminum at the considered temperature (933 K) is $4.132 \times 10^6(\Omega \cdot \mathrm{m})^{-1}$ [5]. As illustrated in fig. 1(b), the attractive force for two Al_2O_3 particles attached to each other *i.e.* with the dipole-dipole distance $l = d_P$, has values comparable to the gravity force particularly for smaller-sized particles under a magnetic flux density $B = 12$ T. This means the attractive force is strong enough to enhance the agglomeration and form large clusters.

On the other hand, for two attached Al_2O_3 particles the interparticle force F' induced by the conductive fluid flow is estimated in fig. 3 by eq. (11). In a strong magnetic field the absolute value of F' increases dramatically with the diameter if the particles are large enough. This is the reason why the agglomeration of large particles in conductive fluid can be limited and the particles repulse each other in a strong magnetic field. This phenomenon is verified by the recent experimental observations [6,8]

as described in first section. However, under a weak magnetic field, *e.g.*, 1 T, the repulsive force is neglectable in the Al melt-Al_2O_3 system (fig. 3). Therefore, the inter-particle interaction due to the applied strong magnetic field includes two effects, *i.e.* the magnetisation dipole-dipole attraction and the repulsion caused by the flow of the conductive fluid around the particles. A comparison between them is estimated in fig. 4. It is clear that there is a critical particle size (fig. 4) for the total interparticle force changing from attractive to repulsive for an Al melt-Al_2O_3 system. However, the magnitudes of the forces drop dramatically with the decrease of the magnetic flux density and are neglectable compared with other forces, *e.g.* gravity force or convection (fig. 1(b) and fig. 4). This means that under a strong magnetic field, particles become controllable by the magnetic field and those smaller than the critical size would tend to be attracted to each other, while those larger than the critical size tend to be repelled.

Conclusion. – The force due to the flow of the conductive fluid around the particles which influences the mutual movement of the particles is presented in this paper. This phenomenon only exists under very specific conditions which include strong magnetic fields (*e.g.* $B > 10\,\mathrm{T}$) and low Reynolds numbers. Nevertheless, it becomes dominant if the particle size is larger than a critical value. With the understanding of this phenomenon we can explain the behavior of particle movement and segregation in a strong magnetic field as reported in the literature. A counteracting behavior in the particle interaction is clearly indicated. With this analysis a basis can be provided to control the particle size distribution using strong magnetic fields in materials processing. In addition, the potentiality to position particles as well as to control the particle size distribution may lead to novel material synthesis methods, *e.g.* for the preparation of gradient materials.

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