Modelling the Variation in Human Decision Making

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***Abstract*— This paper presents the results of the recent research on modelling the variation in human decision making. The relationship between the uncertainty introduced to the membership functions (mfs) of a fuzzy logic system (FLS) and the variation in FLS's decision making is explored using two separate methods. Initially uncertainty is introduced to a type-1 FLS by adding noise to its mfs and the effect on decision making is examined. Secondly an interval type-2 FLS is developed by representing the terms used in the FLS with interval type-2 fuzzy sets and the variation in decision making is studied using the FLS's interval outputs. The variations in ranking of umbilical acid-base assessments by six experts is compared to the simulation results from the developed FLSs. It is shown that there is a direct relationship between the variation in decision making and the uncertainty in the linguistic terms used, and the level of variation is proportional to the magnitude of uncertainty.**

I. Introduction

Type-1 Fuzzy logic systems (FLSs) are deterministic in the sense that for the same inputs the outputs are always the same. However, human experts exhibit a nondeterministic behaviour in decision making. Variation may occur among the decisions of a panel of human experts as well as in the decisions of an individual expert for the same inputs. The terms that are used in an FLS have different meanings for different experts and experts may arrive to different conclusions in their inferencing depending on environmental conditions or over time. Understanding the dynamics of the variation in human decision making could allow creating truly intelligent systems that cannot be differentiated from their human counterparts. Moreover, in application areas where having an expert constantly available is not possible, such systems can produce a span of decisions that may be arrived at by a panel of experts. This paper presents the results of the research that studies the relation between the variation in decision making of an FLS and the vagueness introduced to the FLS's mfs. A nondeterministic type-1 FLS (ndFLS) and an interval type-2 FLS (it2FLS) are developed to investigate how the mfs of the FLSs influence the variation in decision making. The investigation is focused on expert systems based on fuzzy logic, in the context of medical applications.

The term "nondeterministic type-1 Fuzzy Logic System" used in this paper refers to a type-1 FLS with variation introduced to its mfs. The mfs are still type-1 fuzzy sets and there is no change in other units that make up an FLS (i.e. fuzzifier, inference, output processor, etc). However whenever a rule is evaluated the mfs are updated to vary according to certain specified conditions. Other variations of this structure could be built by updating the mfs either each time the FLS is started or each time an mf is used. To the best of our knowledge, type-1 FLSs have not been used in this way before for creating nondeterministic FLSs.

Fuzzy systems usually employ type-1 fuzzy sets that represent uncertainty by numbers in the range [0,1] which are referred to as degrees of membership. Type-2 fuzzy sets are an extension of type-1 fuzzy sets with an additional dimension that represent the uncertainty about the degrees of membership. Type-2 fuzzy sets are useful in circumstances where it is difficult to determine the exact membership function (mf) for a fuzzy set. Type-1 mfs are precise in the sense that once they have been chosen all the uncertainty disappears. However, type-2 mfs are fuzzy themselves. The simplest type-2 fuzzy sets are interval type-2 fuzzy sets whose elements' degree of membership are intervals with secondary membership degree of 1.0.

FLSs consist of four main interconnected components: rules, fuzzifier, inference engine, and output processor. Rules are the heart of this structure. They are provided by the experts or can be extracted from numerical data. In either case they are expressed as a collection of IF-THEN statements. A typical rule is like:

IF arterial pH is *low* and venous pH is *low*
THEN acidemia is *severe*.

The linguistic terms in the rules, shown in italics above, and the inputs to and the outputs from the FLS are associated with fuzzy sets. Type-1 FLSs use type-1 fuzzy sets and an FLS which uses at least one type-2 fuzzy set is called a type-2 FLS. A general type-2 FLS is too complicated, inferencing and output processing are prohibitive [1]. A simplification approach is to use interval type-2 fuzzy sets. There are fast algorithms to compute the output of an interval type-2 FLS (it2FLS) [2].

The concept of type-2 fuzzy sets was introduced by Zadeh [3]. Mizumoto and Tanaka studied the set theoretic operations of type-2 fuzzy sets and properties of membership degrees of such sets [4]; and examined type-2 fuzzy sets under the operations of algebraic product and algebraic sum [5]. Karnik and Mendel obtained algorithms for performing union, intersection, and complement for type-2 fuzzy sets, and developed the concept of the centroid of a type-2 fuzzy set [1]. Dubois and Prade gave a formula for the composition of type-2 relations

as an extension of the type-1 sup-star composition for the minimum t-norm [6]. Karnik et al. presented a general formula for the extended sup-star composition of type-2 relations [7]. Hisdal studied rules and interval sets for higher-than-type-1 fuzzy logic [8]. Liang and Mendel developed the theory for different kinds of fuzzifiers for it2FLSs [9].

Diagnostic medicine, where systematic handling of perceptual uncertainties is crucial to success, is an important application domain for this study. UAB assessment of an infant immediately after delivery is an objective measure of labour, and can be used to audit assessment of labour performance. The acidity (pH), partial pressure of oxygen ($\mathrm{pO_2}$) and partial pressure of carbon dioxide ($\mathrm{pCO_2}$) in blood samples taken from the venous and arterial vessels in the clamped umbilical cord can be measured by a blood gas analysis machine. A parameter termed base deficit of extracellular fluid (BDecf) can be derived from the pH and $\mathrm{pCO_2}$ parameters [10]. An interpretation is made based on the pH and BDecf parameters from both arterial and venous blood. A type-1 FLS was previously developed for the UAB assessment, encapsulating the knowledge of leading obstetricians, neonatologists and physiologists gained over years of acid-base interpretation [11]–[13]. This FLS combines knowledge of the errors likely to occur in acid-base measurement, physiological knowledge of plausible results and statistical knowledge of a large database of results. The FLSs developed to carry out the research presented in this paper are an extension of the original type-1 FLS.

Preliminary investigations for determining the parameters that define the uncertainties resulting in variation in decision making were presented in [14]–[16]. First the motivation for this research and proposed method was explained [14]. Then a nondeterministic type-1 FLS (ndFLS) and an it2FLS used for modelling nondeterminism were presented and the effect of the magnitude of the uncertainty introduced to the centre point of the mfs was studied [15]. Next, the effects of different shapes of type-2 mfs on the variation in decision making of an it2FLS was investigated [16]. This paper presents the results of the further studies where the relation between the variation in decision making and the different mechanisms used to introduce vagueness to the mfs of an ndFLS is investigated and an efficient algorithm for ranking the interval outputs of the it2FLS is developed.

II. Theory

A. The Original Umbilical Acid-Base Assessment FLS

The original type-1 UAB assessment FLS is based on four main input variables; arterial pH ($\mathrm{pH_A}$), arterial base deficit ($\mathrm{BD_A}$), venous pH ($\mathrm{pH_V}$), and venous base deficit ($\mathrm{BD_V}$). Each of the four input parameters is assigned a linguistic variable and each is divided into three fuzzy terms, corresponding to meanings of *low*, *medium* and *high*. Three output fuzzy variables are used, severity of acidemia (*acidemia*), duration of acidemia (*duration*), and component of the acidemia (*component*). The acidemia variable has six terms in its term-set: *severe*, *significant*, *moderate*, *mild*, *normal* and *alkalotic*; the duration variable has three terms: *chronic*, *intermediate* and *acute* and the component variable has three terms in its term-set: *metabolic*, *mixed* and *respiratory*.

The rules for the FLS are obtained as a result of knowledge elicitation sessions with several leading clinicians skilled in umbilical cord blood acid-base analysis, and has been carefully refined to form a complete and consistent set of classifiers.

The Mamdani model of inference is used. The probabilistic family of operators is chosen, in which conjunction is defined as $(a * b)$, disjunction as $(a + b - a * b)$ and negation as $1 - a$. The min operator is used for implication. Centre-of-gravity (centroid) defuzzification is used to produce crisp values for each fuzzy output variable.

All fuzzy sets are modeled with sigmoid membership functions - left-edge sets are modeled by a decreasing sigmoid:

$$\mu(x)=\frac{1}{1+e^{5(x-c)/w}} \tag{1}$$

right-edge sets are modeled by an increasing sigmoid:

$$\mu(x)=\frac{1}{1+e^{5(c-x)/w}} \tag{2}$$

and middle sets are modeled by a combination of two sigmoids:

$$\mu(x)=\frac{1}{(1+e^{15(x-c)/w-5})(1+e^{15(c-x)/w-5})} \tag{3}$$

where c is the centre point of the fuzzy set (the location of the $\mu = 0.5$ value of the *sigmoid* for left and right sets, or the location of the centre value for middle sets; w is the width parameter of the sigmoid, corresponding to the width at $\mu = 0.5$ for middle sets; and $\mu(x) = min(\mu(x), 1.0)$. Fig. 1 shows three type-1 sigmoidal functions.

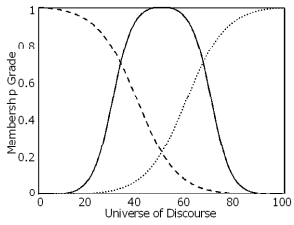

Fig. 1. Three type-1 sigmoidal mfs

B. The Possibility of Intrapartum Asphyxial Damage

In order to asses how well the FLS performs a comparison with the human experts is necessary. However, the output of a fuzzy system is usually arbitrarily scaled, making an absolute comparison between the fuzzy system output and target output difficult. In such a situation, a statistic which ignores the absolute values, but takes into account the ordering of results can be used. Spearman rank order correlation can determine the degree of association between two sets of rank-ordered data [17].

A target set of rankings was obtained by seeking the opinions of 6 human experts. The experts were given the pH and BD readings of 50 cases which were chosen from a database of 11068 cases to cover a wide range of categories. The clinicians were asked to rank these 50 results in order from

'worst' to 'best' in terms of likelihood of the infant having suffered asphyxial damage.

The original FLS produces a numeric index, which indicates the *health* of the infant as represented by its acid-base balance at birth, and is used in ranking the cases presented to the FLS. This health index was derived from the three outputs of the FLS by combining their centroids into a single index by:

$$condition{=}acidemia{+}\frac{component}{20}+\frac{duration}{10} \tag{4}$$

where the relative weighting of the three terms was determined empirically.

The Spearman rank order correlation coefficient used to evaluate the performance of the numeric interpretation output showed that the type-1 FLS modeled UAB assessment expertise successfully. However, plotting the rankings of the experts and the type-1 FLS on the same graph raised interesting observations. A graph of the six clinicians's rankings plotted against the type-1 FLS rankings is shown in Fig. 2. If perfect agreement had been found, a straight line through the origin to (50,50) would have been obtained. However, there is neither perfect agreement with the FLS nor among the experts. It can also be observed that at the extreme cases the experts tend to agree with each other and the FLS but in the cases that fall in the middle of the range, there is less agreement. The distribution presents the characteristic of an elliptic envelope around the diagonal line from (0,0) to (50,50).

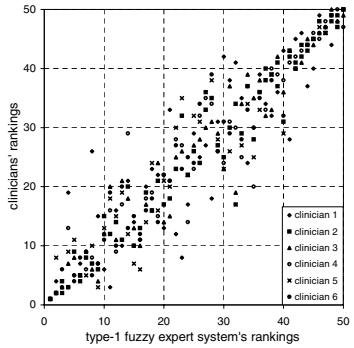

Fig. 2. Variation in rankings of 50 assessments - inter expert variation

C. Introducing Vagueness into mfs

In this study the relation between the variation in decision making and introducing vagueness to the mfs is explored in two stages. First, ndFLS is developed by changing the precise type-1 mfs of the original FLS by randomly varying them. The randomness is introduced using three different mechanisms. In the first two studies, new mfs are obtained by replacing the centre point (c), and the width (w) in (1-3) with ($c + \Delta\epsilon$) and ($w + \Delta\epsilon$) where Δ is a percentage of the universe of discourse of the variable that the mf belongs to and ϵ is a random number in [-1, 1]. The third method is to add white noise to the mfs. After obtaining $\mu(x)$ using (1-3), to each element of the array $\mu(x)$, $\Delta\epsilon$ is added where ϵ is a random number in [-1, 1] for each element and Δ is same as explained above. Each time a rule is processed in the inferencing module of the ndFLS, the mfs are updated resulting in slightly different type-1 mfs in comparison to their originals. In the experiments, to demonstrate the effect of the amount of the vagueness introduced to the mfs on the variation in decision making, the parameter Δ used to create the mfs with vagueness and the distance of the uniform band around the primary membership functions is varied between $1 - 5\%$ of their universe of discourse.

Secondly, an it2FLS is developed using interval type-2 mfs that are created by producing intervals around the original mfs which then can be referred to as the type-2 mfs' primary membership functions. Interval type-2 mfs are also created using three different mechanisms similar to the ones explained in the previous paragraph. The first two sets are created by shifting the centre point (c), and the width (w) of the original mfs by a Δ amount. And the third set is created by adding a uniform band around the primary mfs. After obtaining $\mu(x)$ using (1-3), to each element of the array $\mu(x)$, Δ is added where Δ is is a percentage of the universe of discourse of the variable that the mf belongs to. Fig. 3 shows three interval type-2 sigmoidal mfs, where c_i and c_i' ($i = 1, 2, 3$) are the centre point pairs for each type-2 mf ($c_1 = 40$, $c_2 = 50$, $c_3 = 60$, $w_1 = w_3 = 40$, $w_2 = 60$ and $\Delta = 1.0$). Fig. 4 shows three interval type-2 sigmoidal mfs which are obtained by varying the width of the primary mfs. Fig. 5 shows the mfs created by adding a uniform band around the original type-1 mfs.

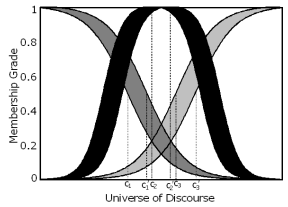

Fig. 3. Three interval type-2 sigmoidal mfs obtained by deviating centers

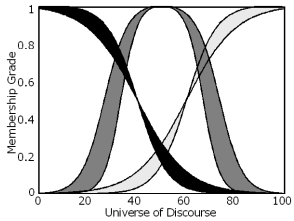

Fig. 4. Three interval type-2 sigmoidal mfs obtained by deviating widths

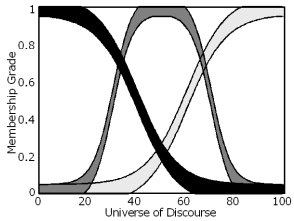

Fig. 5. Three type-2 mfs obtained by adding a uniform band

The three sets of interval type-2 mfs created as explained above contain all the possible uncertain type-1 mfs used in the ndFLS. The advantage of using ndFLS is its computational efficiency. Type-1 FLSs requires less computational power

in comparison to interval type-2 FLSs. However due to the uncertain nature of the mfs, although running the ndFLS several times produces a span of the possible outcomes, the range of all probable outputs cannot be established confidently. An interval type-2 FLS can be used to check the extreme behaviour of the corresponding ndFLS. Because the interval type-2 mfs used in the interval type-2 FLS contain all the vague mfs that can be produced during the running of the ndFLS.

D. Interval Type-2 Fuzzy Logic System (it2FLS)

The mechanisms of the it2FLS developed for this study is presented in Fig. 6.

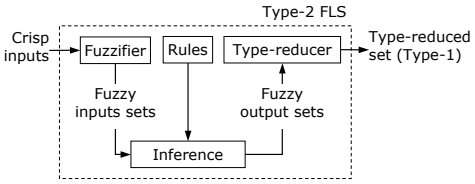

Fig. 6. Interval type-2 FLS (it2FLS)

After converting the rule set by associating the terms with interval type-2 mfs, the interference and defuzzification methods of the original type-1 FLS are updated to work with the type-2 fuzzy terms. The fuzzifier of the type-1 FLS, which turns the crisp input values into type-1 fuzzy input sets in order to compensate for the errors in readings of the blood gas analysis machine, is not changed. By Mendel's classification, the resulting FLS is a type-1 nonsingleton it2FLS because the inputs are type-1 fuzzy sets and all the antecedent and consequent sets of the rule base are interval type-2 fuzzy sets [1].

The input, antecedent, consequent and implication operations use minimum t-norm and maximum t-conorm. The result of antecedent operations is an interval which is fed into the consequent. Firing of rules result in type-2 fuzzy sets which are combined into a single type-2 fuzzy set by minimum t-norm. Mendel [1] has established theoretical results to effectively determine the lower and upper bounds of the centroid of a type-2 set and has provided algorithms [2] for carrying out the necessary calculations. That is, the centroid is an interval, the mean value of the upper and lower bounds of which can be taken as a single crisp centroid, if required.

A type-2 FLS must reduce to a type-1 FLS when the uncertainty about the shape of mfs is zero. This was verified by running the it2FLS with 0% deviation in the papameters which were being varied to create the type-2 mfs.

An Efficient Ranking Method: The it2FLS produces for every input case an interval for the health measure which is used to rank the cases in terms of the perceived likelihood of having suffered brain damage due to lack of oxygen. Ranking of 50 UAB assessments done for every combination of the upper and lower bounds of the interval outputs results in 2^{50} rankings. This is computationally too expensive. The computational time required for ranking 20 cases is around an hour. By extension, this would mean 2^{30} hours would be needed for ranking 50 cases. The following two theorems are used to simplify the ranking process.

Theorem 2.1: Let $[l_i, u_i]$ $i = 1, \ldots, n$ be the n health index intervals sorted in ascending order according to l_i where l_i is the lower bound and u_i is the upper bound. If $1 \geq j \leq 50$ and $\forall k$ in $[1, j]$, $\max_k(u_k) < l_j$ then. The rank of the cases $1 \ldots j$ will never be higher then the ranks of the cases $j \ldots n$.

So the n cases can be divided into two separate groups for ranking and the same can be repeated for the cases $j \ldots n$ again which results in smaller groups to rank.

Theorem 2.2: Let $[l_i, u_i]$ $i = 1, \ldots, n$ be the n health index intervals where l_i is the lower bound and u_i is the upper bound. The minimum rank of case k is the position of l_k among all u_i except u_k. The maximum rank of case k is the position of u_k among all l_i except l_k.

Using theorem 1 the computation time required for ranking of 20 cases was decreased to a few minutes and for 50 cases to less then 30 mins for Δ between $(0.001\ to\ 0.005) \times length\ of\ the\ universe\ of\ discourse$. Using theorem 2, maximum and minimum rankings of 50 cases were determined in a few minutes.

Example: In table I, 20 UAB cases are presented with their upper and lower bound health indexes. Using theorem 1 the ranking problem can be divided into three ranking problems with $2^2 + 2^{14} + 2^3 = 16396$ combinations instead of a single ranking problem with $2^{20} = 1048576$ combinations to test.

TABLE I

RANKING OF 20 CASES USING THEOREM 1.

	Case ID	Health Index		Rank Range
		lower bound	upper bound	
1	P00865	11.075	11.075	1
2	P00862	16.075	25.725	
3	P09216	17.45	18.8	2-3
4	P03977	26.225	39.5	
5	P07320	28.675	47.925	
6	P09905	29.975	44.525	
7	P02370	31.55	32.425	
8	P03444	36.325	47.25	
9	P10297	39.225	51.525	
10	P00486	39.925	53.1	
11	P00136	47.85	57.8	
12	P06463	48.525	58.225	
13	P09308	49.45	59.85	
14	P10828	52.375	60.825	
15	P08722	55.05	63.0	
16	P06617	59.3	68.825	
17	P02844	63.825	70.45	4-17
18	P08070	74.5	83.625	
19	P06086	77.675	87.85	
20	P09151	81.2	88.075	18-20

III. RESULTS

A. Varying the Center Points

Fig. 7 and 8 show the variation in rankings as the deviation in the centre point of the mfs used in the ndFLS is increased from 1% to 5% of the universe of discourse of the variable they are associated to. These plots show the results of running the ndFLS 100 times. The resulting ranks of the 50 cases are plotted as three dimensional histograms where horizontal axises show the ranks by the ndFLS versus the ranks by the original FLS and the third dimension shows the number of times the corresponding case is ranked by ndFLS at the

corresponding position. Fig. 9 and 10 show the variation in rankings as the shift in the centre point of the mfs is increased from 1% to 5% of their universe of discoure to obtain the type-2 mfs used in the it2FLS. These figures show the maximum and minimum rank of each case versus the corresponding rank decided by the original FLS.

B. Varying the Widths

Fig. 11 and 12 show the variation in ndFLS's rankings as the deviation in mfs' width is increased from 1% to 5% of their universe of discourse. Fig. 13 and 14 show the variation in rankings as the shift in widths is increased from 1% to 5% for creating the type-2 mfs used in the it2FLS. In comparison to the centre points results, the variation in ranking increases less rapidly with the increasing the shift in widths of the mfs. However, in comparison to the uniform bands introduced around type-1 mfs, the behaviour is similar.

C. Adding White Noise or Uniform Band

Fig. 17 and 18 show the variation in the rankings as the thickness of the band introduced around the original type-1 mfs is increased from 0.1 to 0.5. Fig. 15 and 16 show the variation in the rankings as the white noise added to the type-1 mfs is increased from [-0.1, 0.1] to [-0.5, 0.5]. In comparison to the effect of the centre points, the variation in ranking increases less rapidly with the increasing thickness of the bands. However, in comparison to similar amount of changes in the widths of the original type-1 mfs, the behaviour is very similar.

IV. Discussions

It can be observed from these trials that there is a direct relationship between the variation in the rankings and the uncertainty in the mfs used in the ndFLS and it2FLS. As the vagueness in the linguistic terms used is increased, the variation in decision making is observed to increase. Another important observed feature is in the nature of the variation, the extreme UAB cases are ranked most of the time the same but in the cases that fall in the middle of the range, there is less agreement which results in a cloud of data bounded in an elliptic envelop along the diagonal. This is in parallel to the behaviour exhibited by the panel of experts presented in Fig. 2. Variation of all three parameters have resulted in similar behaviour. However, mfs created by shifting the centre point of the original type-1 mfs have caused greater variation in the rankings in comparison to the other two trials which were created by varying the width and adding white noise or a uniform band.

Most success of fuzzy logic is in fuzzy logic control, but this success has not yet been carried over to modelling human reasoning - Zadeh's Computing with words Paradigm [18]. In this paper, the work done on modelling the variation in human expert opinion is presented. Specifically, the relation of the uncertainty in the mfs used in the FLSs and the variation in decision making is explored. It is shown that it is possible to capture the variation in human decision making using an ndFLS or it2FLS. This kind of nondeterministic behaviour can be modeled by using uncertain mfs or type-2 mfs which can be derived from type-1 mfs associated to the terms used in the rule base of an FLS. The variation in decision making by an FLS can be controlled using the level of uncertainty in its mfs. This can be used in creating intelligent systems that can mimic their human counterparts better. An example of the major benefits that this may provide may be in application areas where having an expert constantly available is not possible. Such systems can produce a span of decisions that may be arrived at by a panel of experts.

The research on understanding and modelling the dynamics of variation in human decision making is ongoing. In future, the possibility of modelling nondeterminism using a general type-2 FLS will be investigated. A general type-2 FLS is too complicated, inferencing and output processing are prohibitive [1]. One simplification approach is to use interval type-2 fuzzy sets, which was done in developing the it2FLS. The main aim of the future work is to develop other approximation methods which simplifies the inferencing and output processing of general type-2 FLSs.

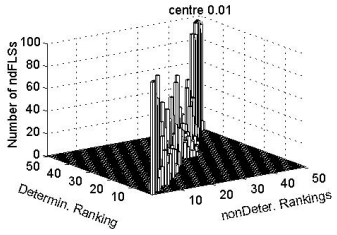

Fig. 7. ndFLS rankings, 1% centre deviation

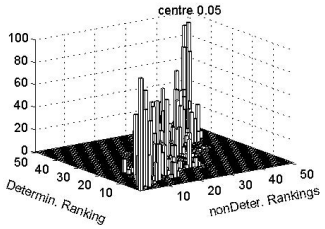

Fig. 8. ndFLS rankings, 5% centre deviation

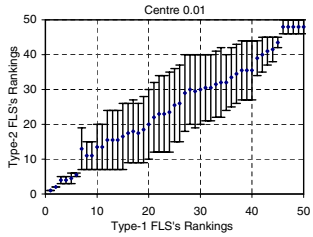

Fig. 9. it2FLS rankings, 1% centre shift

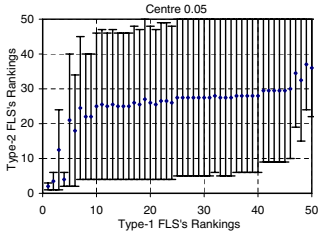

Fig. 10. it2FLS rankings, 5% centre shift

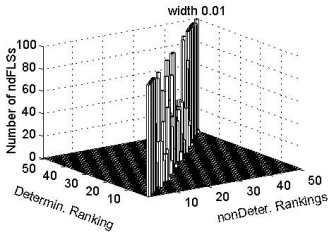

Fig. 11. ndFLS rankings, 1% width deviation

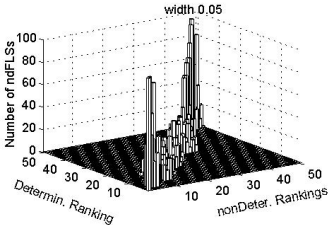

Fig. 12. ndFLS rankings, 5% width deveviation

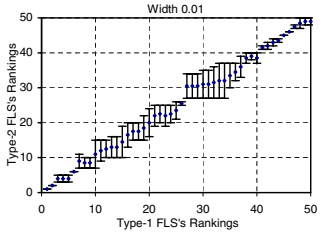

Fig. 13. it2FLS rankings, 1% width shift

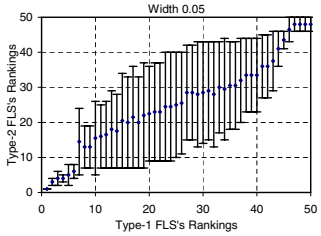

Fig. 14. it2FLS rankings, 5% width shift

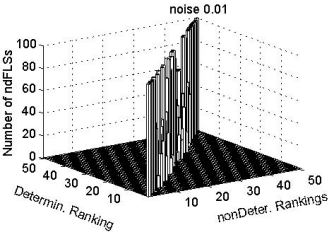

Fig. 15. ndFLS rankings, 1% noise

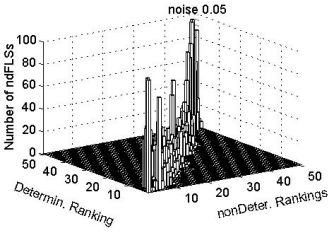

Fig. 16. ndFLS rankings, 5% noise

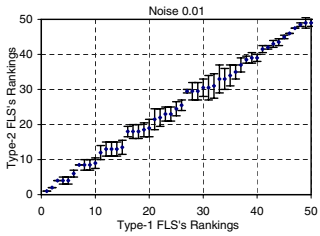

Fig. 17. it2FLS rankings, 1% band

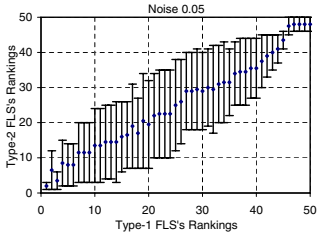

Fig. 18. it2FLS rankings, 5% band

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