Correlations Induced in a Packet Stream by Background Traffic in a Multiplexed Environment*

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The authors present an analytic model that shows that the service requirements at the server for background classes of traffic can have a significant effect on a tagged class, impacting not only the variance, but more importantly, introducing slowly decaying correlations in the tagged stream. The model is general in that it incorporates first- and second-order statistics for all processes involved and focuses on the correlations introduced by multiplexing. The results show that correlations present in background classes, combined with differing service requirements for the different classes, have enormous impact on the departure stream of all classes involved.

1. Introduction

Over the past 20 years, IP networks have been extremely successful in carrying delay insensitive data. These networks have been relatively cheap to build, simple to manage, and scale very well. Recently, there has been much debate on the ability of IP networks to provide quality of service (QoS) in the Internet. In order to provide service comparable to flow based networks such as ATM, two issues must be addressed; efficient use of bandwidth, and ensuring QoS for a service class.

Efficient use of bandwidth can be ensured by incorporating traffic engineering. Fortz and Thorup [1] show that traffic engineering with open shortest path first (OSPF) performs within a few percent of what flow based networks (optimal general routing) can achieve. Pioro et al. [2] provide mechanisms to find weight systems that ensure effective traffic engineering in an OSPF network. Such approaches make *efficient use of bandwidth* within reach in an IP network.

Ensuring QoS for a service class in IP networks poses the same issues and considerations as in flow based networks, except that the traffic characteristics can vary more widely. In flow based networks, traffic streams share bandwidth with their own classes only, and hence it is reasonable to assume a certain homogeneity in terms of the arriving streams and their service requirements. In IP networks, bandwidth is usually shared between all arriving streams of all classes, and assumptions of homogeneity may not be realistic. In fact, we will show that common assumptions regarding outflows may not be valid.

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The problem of ensuring QoS for different service classes in IP networks has sparked renewed interest in the design and analysis of mechanisms originally conceived for flow based networks. The study of traffic distortion (jitter) in flow based networks is common in the literature [3–8], but the second-order per-stream effects of multiplexing using FIFO scheduling are not generally well known for multiple classes with different service distributions. Studies have shown minimal coupling for homogeneous and non-homogeneous traffic in a low utilization environment. However, these traffic characteristics may become distorted in a congested environment [9,10]. The problem may be particularly severe if one or more of the multiplexed streams is itself highly correlated.

Studies have shown that for a stable system, if the number of service classes goes to infinity, a departing tagged stream will not suffer distortion. However, in an IP environment running over an OSPF network, the maximum number of service classes (Per Hop Behaviors) in Differentiated Services Architecture [11]) is fairly small. Under such a situation, the assumption that the impact is negligible becomes questionable.

In our work, we present a Linear Algebraic Queueing Theory (LAQT) based model that captures the first- and second-order characteristics of the departure process in a heterogeneous environment. The contribution of this paper is the ability to completely characterize the tagged departure process in terms of both first- and second-order statistical properties. The model is also unique in that it incorporates correlations in the packet length distribution as well as the packet interarrival process for each class of traffic. In particular, the model allows us to study all aspects of multiplexed streams in heavy traffic, and answer the questions posed above.

In section 2, we present the analytical model used to capture both the marginals and the correlations in the departure stream. We present numerical results for several cases in section 3 and we close with a summary of our observations in section 4.

2. Basic Model Description

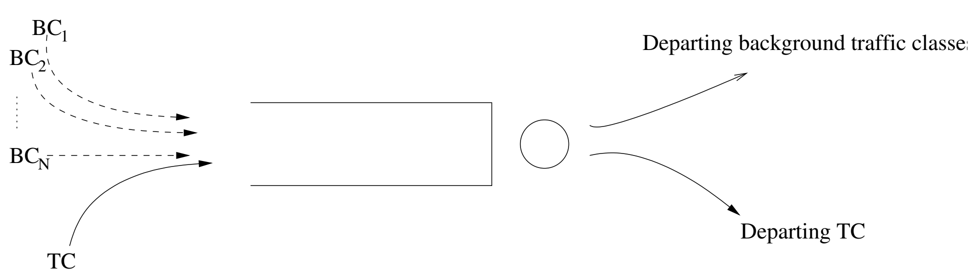

Figure 1. Diagram of the shared buffer under consideration

Figure 1 represents a server receiving multiplexed traffic. Numerous flows are multiplexed into a single queue, and are serviced according to a first come first serve scheduling. We assume that there are N background classes BC_1, BC_2,. . ., BC_N and a single tagged class TC. The departing flows eventually will be separated in the network according to the routed path of each flow. An exact Markov state space representation of the departure process of a single stream requires $\Theta((N + 1)^K)$ global states, where N is the number of arrival streams, and K is the size of the buffer. Obviously, the state space explosion makes it impossible to perform an exact analysis unless N and K are small. However, low bit loss requirements for high speed networks allow us to assume an infinite buffer. Also, since traffic distortion has been observed to take place during times of high congestion [9,10], we are particularly interested in behavior under heavy traffic (i.e., the server is always busy). The heavy traffic model derived in section 2.4 has only $\Theta(N)$ global states, yet allows us to completely study the impact of background traffic on the first- and second-order statistics of a tagged stream departing from a multiplexer.

2.1. Matrix Exponential Process

In this section, we review the construction of the Linear Algebraic Queueing Theory (LAQT) model that we use to study the second order statistics of an isolated departure stream [12,13]. A matrix exponential (ME) distribution [14] is defined as a probability distribution whose density can be written as

$$f(t) = \boldsymbol{p} \exp\left(-\boldsymbol{B}t\right) \boldsymbol{B}\boldsymbol{e}', \qquad t \geq 0, \tag{1}$$

where $\boldsymbol{p}$ is the starting operator for the process, $\boldsymbol{B}$ is the process rate operator, and $\boldsymbol{e'}$ is a summing operator consisting of a vector of all 1's. The n^{th} moment of the matrix exponential distribution is given by $E[X^n] = n!\boldsymbol{p}\boldsymbol{V}^n\boldsymbol{e}'$, where $\boldsymbol{V}$ is the inverse of $\boldsymbol{B}$. The class of matrix exponential distributions is identical to the class of distributions that possess a rational Laplace-Stieltjes transform. As such, it is more general than continuous phase type distributions.

The joint density function of the first k-successive intervals between events describes a matrix exponential process (MEP):

$$f_k(x_1, \ldots, x_k) = \boldsymbol{p}\text{exp}(-x_1\boldsymbol{B})\boldsymbol{L} \ldots \text{exp}(-x_k\boldsymbol{B})\boldsymbol{L}\boldsymbol{e'}. \tag{2}$$

The matrix $\boldsymbol{L}$ is the event generator matrix. Examples for such processes are a Poisson process ($\boldsymbol{B}$=[λ], $\boldsymbol{L}$=[λ]), a renewal process ($\boldsymbol{L} = \boldsymbol{B}\boldsymbol{e'}\boldsymbol{p}$), and a Markov Arrival Processes (MAP) ($\boldsymbol{B} = -\boldsymbol{D}_0$, $\boldsymbol{L} = \boldsymbol{D}_1$). Note that $\boldsymbol{B}$ and $\boldsymbol{L}$ are not limited to being Markovian rate matrices, so every MAP is an MEP, but not vice versa, see also [15]. We assume the process to be covariance stationary, so that $\boldsymbol{p}$ is the stationary vector for the process at embedded event points (i.e., $\boldsymbol{p}\boldsymbol{V}\boldsymbol{L} = \boldsymbol{p}$). The expression for the lag-k covariance, the covariance between the first interval and the k^{th} is

$$cov[X_1, X_k] = \boldsymbol{p}\boldsymbol{V}(\boldsymbol{V}\boldsymbol{L})^k\boldsymbol{V}\boldsymbol{e'} - (\boldsymbol{p}\boldsymbol{V}\boldsymbol{e'})^2. \tag{3}$$

Hence, the auto-correlation at lag-k, r[k] can be found by dividing $cov[X_1, X_k]$ by the variance

$$var[X] = 2\boldsymbol{p}\boldsymbol{V}^2\boldsymbol{e'} - (\boldsymbol{p}\boldsymbol{V}\boldsymbol{e'})^2. \tag{4}$$

Finally, the marginal process is matrix exponential with density given in equation (1).

2.2. Combining Different Processes

All arrival streams (tagged and all backgrounds) are assumed to be MEP processes. Similarly, all service time distributions follow MEP processes. The arrival and service processes for each class are concurrently active and are statistically independent of one another. These spaces are combined into the system space by using the *Kronecker product* whose operator is denoted as "$\otimes$". An operator that operates on a particular space is embedded in the system space by taking the Kronecker product of it with the identity operator from the other spaces. For example, the progress rate operator of tagged arrivals, denoted as $\boldsymbol{B}_t$, is joined with the identity operator for the tagged service, background arrival and background service process, $\boldsymbol{I}_s^t$, $\boldsymbol{I}_b$, and $\boldsymbol{I}_s^b$ to form $\boldsymbol{B}_t \otimes \boldsymbol{I}_s^t \otimes \boldsymbol{I}_b \otimes \boldsymbol{I}_s^b$ which is written as $\widehat{\boldsymbol{B}}_t$. Similarly, $\boldsymbol{B}_b$ is embedded in the product space by $\widehat{\boldsymbol{B}}_b = \boldsymbol{I}_t \otimes \boldsymbol{I}_s^t \otimes \boldsymbol{B}_b \otimes \boldsymbol{I}_s^b$. Each process embedded into the system space is differentiated from the non-embedded process by the use of hats, thus the product space used to model the system is sometimes referred to as *hat space*. We prefer to use the more compact hat notation rather than explicitly using Kronecker products in our equations. Please see [16] for details.

2.3. Notation

We will be using the following symbols (after embedding into the system space)

$\widehat{\boldsymbol{B}}_t$	$\widehat{\boldsymbol{L}}_t$	:	Process rate and event rate matrix for the arrival process of tagged packets
$\widehat{\boldsymbol{B}}_{bi}$	$\widehat{\boldsymbol{L}}_{bi}$	:	Process rate and event rate matrix for the arrival processes of background packet streams $i, i = 1, 2, \ldots, N$
$\widehat{\boldsymbol{B}}_s^t$	$\widehat{\boldsymbol{L}}_s^t$	:	Process rate and event rate matrix for the service process of tagged packets
$\widehat{\boldsymbol{B}}_s^{bi}$	$\widehat{\boldsymbol{L}}_s^{bi}$	:	Process rate and event rate matrix for the service processes of background packet streams $i, i = 1, 2, \ldots, N$

2.4. Tagged Departure Process

Building upon the model presented in [12] and assuming that in heavy traffic the server is always busy, we model the arrivals as a count process. Realizing that the order in which different classes enter the queue is identical to the order they depart from the queue, we generate arrivals only when they are needed to enter into service, greatly reducing the space complexity. Following Lipsky [14], we define the embedding operators $\boldsymbol{H}_t$ and $\boldsymbol{H}_{bi}$ as

$$\boldsymbol{H}_t = (\widehat{\boldsymbol{B}}_t + \Sigma_{i=1}^N \widehat{\boldsymbol{B}}_{bi})^{-1} \widehat{\boldsymbol{L}}_t \tag{5}$$

and

$$\boldsymbol{H}_{bi} = (\widehat{\boldsymbol{B}}_t + \Sigma_{i=1}^N \widehat{\boldsymbol{B}}_{bi})^{-1} \widehat{\boldsymbol{L}}_{bi} \tag{6}$$

These operators transform the arrival components of the system to immediately after the occurrence of an arrival, conditioned on the arrival being either tagged or background. They maintain the internal state information of the background (tagged) arrival processes. Hence, we can effectively transfer the arrival components of the system from one arrival to another without incorporating their inter-event time.

The Markov state space for the heavy traffic model with a single server can thus be reduced to a single macro state per service class.

Each state represents the type of packet at the server (background or tagged). The $\boldsymbol{H}$-operators are used to transition from the state representing the service of a departing class to the state representing the beginning of service for the class "next in line". We can directly construct the MEP representation of the departure stream for the tagged packets denoted as $\boldsymbol{B}_d$ and $\boldsymbol{L}_d$:

$$\boldsymbol{B}_d = \begin{bmatrix} \widehat{\boldsymbol{B}}_s^t & 0 & \cdots & \cdots & 0 \\ -\widehat{\boldsymbol{L}}_s^{b1}\widehat{\boldsymbol{H}}_t & (\widehat{\boldsymbol{B}}_s^{b1} - \widehat{\boldsymbol{L}}_s^{b1}\widehat{\boldsymbol{H}}_{b1}) & \cdots & \cdots & -\widehat{\boldsymbol{L}}_s^{b1}\widehat{\boldsymbol{H}}_{bN} \\ \vdots & -\widehat{\boldsymbol{L}}_s^{b2}\widehat{\boldsymbol{H}}_{b1} & \ddots & & -\widehat{\boldsymbol{L}}_s^{b2}\widehat{\boldsymbol{H}}_{bN} \\ \vdots & \vdots & & \ddots & \vdots \\ -\widehat{\boldsymbol{L}}_s^{bN}\widehat{\boldsymbol{H}}_t & -\widehat{\boldsymbol{L}}_s^{bN}\widehat{\boldsymbol{H}}_{b1} & \cdots & \cdots & (\widehat{\boldsymbol{B}}_s^{bN} - \widehat{\boldsymbol{L}}_s^{bN}\widehat{\boldsymbol{H}}_{bN}) \end{bmatrix} \tag{7}$$

and

$$\boldsymbol{L}_d = \begin{bmatrix} \widehat{\boldsymbol{L}}_t^s \widehat{\boldsymbol{H}}_t & \widehat{\boldsymbol{L}}_t^s \widehat{\boldsymbol{H}}_{b1} & \cdots & \cdots & \widehat{\boldsymbol{L}}_t^s \widehat{\boldsymbol{H}}_{bN} \\ 0 & 0 & \cdots & \cdots & 0 \\ \vdots & 0 & \ddots & & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix}. \tag{8}$$

Notice, that each class can have different service requirements and these service requirements can be correlated. Such a model allows us to compute first- and second-order characteristics of the process under scrutiny. Let us define $m_t^a = \dim(\widehat{\boldsymbol{B}}_t)$, $m_{bi}^a = \dim(\widehat{\boldsymbol{B}}_{bi})$, $m_t^s = \dim(\widehat{\boldsymbol{B}}_s^t)$ and $m_{bi}^s = \dim(\widehat{\boldsymbol{B}}_s^{bi})$. The dimension of the $\boldsymbol{B}_d$ and $\boldsymbol{L}_d$ matrices is $(N+1)m_t^a m_t^s \prod_{i=1}^N m_{bi}^a m_{bi}^s$. Fortunately, the number of classes in access networks is quite small and future research will attempt to incorporate further state space reduction techniques.

3. Numerical Results and Discussion

The model developed in section (2) allows us to study first- and second-order characteristics of an isolated stream sharing a single server with N background service classes. In this section, we present numerical results for packets of a tagged service class multiplexed with packets of a background service class having correlated service times. In order to quantify the impact caused by the background classes of traffic, we also present results for delays encountered by the departing tagged class when passing through a queue. We perform extensive numerical experiments to explore various combinations and present the interesting and insightful ones, while discussing trends and special cases.

3.1. Construction of Distributions

We characterize distributions using their first- and second-order characteristics. We specify the mean and squared coefficient of variation, c^2, for marginals and the correlation, $r[k]$, as the second-order parameter in the construction of the arrival and service processes. We construct the processes in such a manner that we still have independent control over c^2 and r. When $c^2 = 1$, we use the Poisson process with $\boldsymbol{B} = [\lambda]$ and $\boldsymbol{L} = [\lambda]$. For non-exponential renewal processes, we use Marie's construction [17] for distributions (valid for $c^2 > 1/2$) which is derived by fitting the parameters of a Coxian distribution with third moment equal $\frac{3}{2}(2c^4 + c^2 + 1)(E[X])^3$. Such a distribution can be represented in LAQT representation [17] as

$$\boldsymbol{p} = [\,1 \quad 0\,], \qquad \boldsymbol{B} = \frac{2\lambda}{c^2}\begin{bmatrix} (c^2+1/2) & -1 \\ c^2/2 & 0 \end{bmatrix}, \qquad \boldsymbol{L} = \boldsymbol{B}\boldsymbol{e}'\boldsymbol{p} \tag{9}$$

The process we have just described leads to uncorrelated random variables. In order to construct correlated processes that share the same marginals, we use the approach presented in [18] to construct processes with geometrically decaying covariance. Define $\boldsymbol{L}^{(\gamma)}$ for $-1 < \gamma < 1$ as

$$\boldsymbol{L}^{(\gamma)} = (1-\gamma)(\boldsymbol{B}\boldsymbol{e}'\boldsymbol{p} - \boldsymbol{B}) + \boldsymbol{B}. \tag{10}$$

The $\boldsymbol{L}^{(\gamma)}$ constructed introduces geometrically decaying correlations in the process, while leaving the marginals invariant.

$$r[k] = \frac{(\boldsymbol{p}\boldsymbol{V}^2\boldsymbol{e}' - (\boldsymbol{p}\boldsymbol{V}\boldsymbol{e}')^2)}{(2\boldsymbol{p}\boldsymbol{V}^2\boldsymbol{e}' - (\boldsymbol{p}\boldsymbol{V}\boldsymbol{e}')^2)}\ \gamma^k. \tag{11}$$

Not all possible combinations of c^2 and γ result in valid processes. In particular, for some $\gamma < 0$ and $c^2 < 1.0$, the resulting expression (2) is no longer a joint probability density. Numerical experimentation has shown that values $c^2 > 1.0$ and $\gamma \geq 0$ generally result in proper joint densities. This formulation enables us to vary either the covariance decay or the c^2 of a process (or both). In order to better evaluate the distortions in the tagged service class departing from a multiplexer, we also pass the departing stream into an exponential server with an infinite queue (see Figure 2). We used TELPACK [20] to solve the resulting $G/M/1$ system for queue length distribution and waiting times.

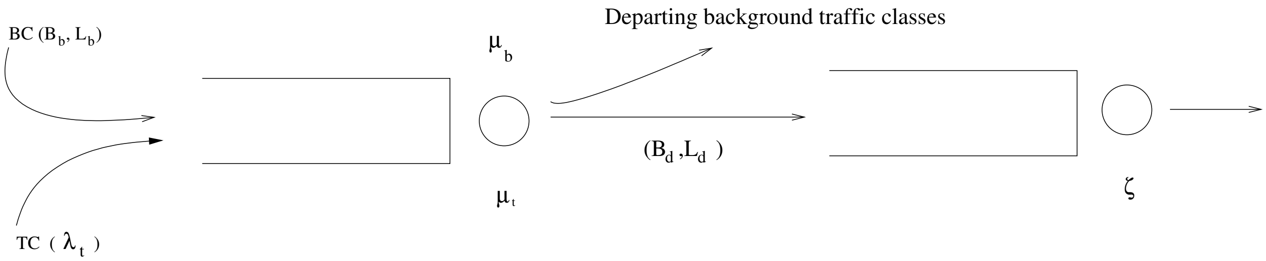

Figure 2. Diagram of the multiplexer under consideration

3.2. Discussion

In order to perform parametric studies and clearly understand the distortions that the tagged traffic experiences, we use Poisson distributed arrivals and exponential service times for the tagged class. The arrival and service processes for the background class were constructed using equations ((9) and (10)). The mean service rate ratio, defined as α, is $\alpha = \frac{\mu_t}{\mu_b}$.

Our experimental setup is such that the utilization of the second queue is kept fixed at (ρ_d=0.8) and the mean arrival rate of the tagged and background classes is the same (λ_t=λ_b=5.0). For the first queue, we determine the tagged class service rate based on the value of α. We then choose a background service rate such that the multiplexed queue utilization is 1.0, so that our heavy traffic model can be used. Using the mean value analysis, we find that the average rate of departing tagged stream is

$$\lambda_d = \frac{\mu_t\ \mu_b\ \lambda_t}{(\lambda_t\ \mu_b + \lambda_b\ \mu_t)}. \tag{12}$$

We have exercised the model extensively and present here the numerical results only for interesting and typical cases to validate or invalidate common expectations and to provide insights into the system behavior. In particular, we want to study the impact of the service time distributions of the background traffic at a multiplexer. First, we look at the squared coefficient of variation of the departing tagged class.

3.2.1. Characterization of the departing tagged stream

The characteristics of this tagged stream is described in terms of its marginal distribution and its correlation structure. In Figures 3, 4, 5 and 6, we show how the background class causes an increase in the squared coefficient of variation in the tagged stream, which had $c^2 = 1$ before it entered the multiplexer. The c^2 of the tagged packet stream is shown as a function of both the

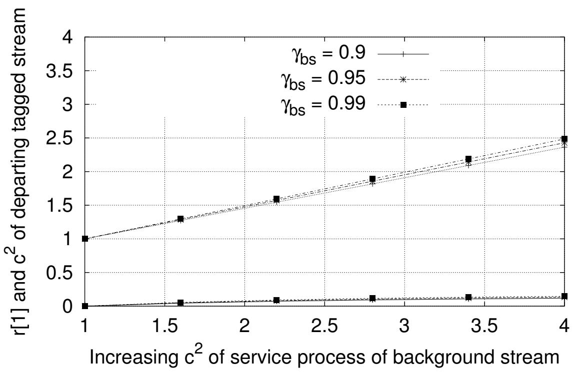

Figure 3. r[1] and c^2: c_b^2=1, α=1

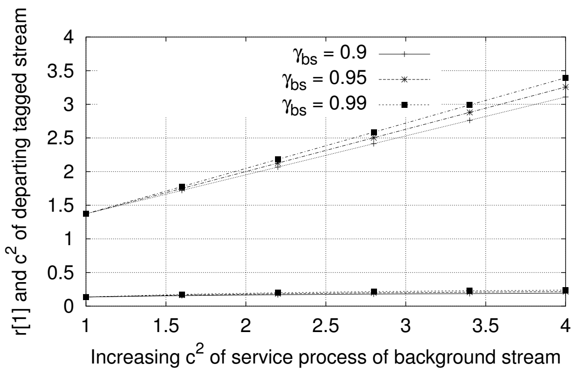

Figure 4. r[1] and c^2: c_b^2=4, γ_b=0.99, α=1

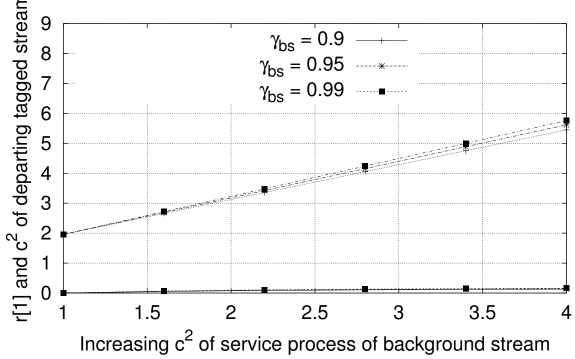

Figure 5. r[1] and c^2: c_b^2=1, α=4

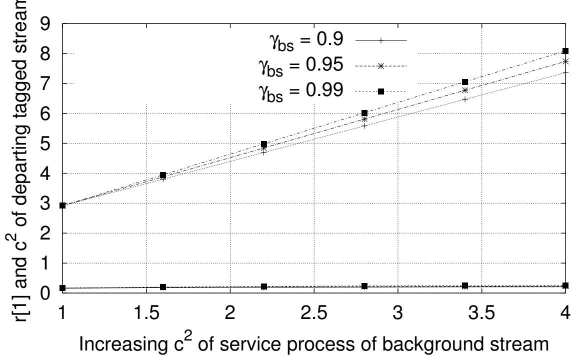

Figure 6. r[1] and c^2: c_b^2=4, γ_b=0.99, α=4

covariance decay and the c^2 in the service process of the background stream, where either the multiplexer is equally shared between the tagged and background class ($\alpha = 1$) or there are on an average 4 times more packets of background stream as compared to tagged packets (α=4).

Observe that the higher c^2 of the service process of the background stream invariably (for cases observed) increases the c^2 and the covariance of the departing tagged stream. It should also be noted that the presence of more background traffic distorts the tagged traffic even further, even though the traffic intensity for the tagged class becomes relatively smaller.

We now study the correlation structure of the departing tagged class. Note that if no multiplexing takes place there would not be any correlation. In Figure 7, we consider a background class with a Poisson arrival process to the multiplexer. However, the service time at the server is not exponential (the squared coefficient of variance is elevated, $c_{bs}^2 = 4$), but the covariance for the service times for the background class is only slowly decreasing, $\gamma_{bs} = 0.99$. Remember that α represents the ratio of background traffic classes to tagged traffic: if α = 4, then there are on average 4 background packet arrivals for each tagged packet. In other words: if $\alpha > 1$ then the background class dominates the multiplexer. We observe that correlations in the tagged class is introduced by sharing the multiplexer with the background traffic, even though both classes of traffic were originally Poisson. Also, note that the amount of correlation can be fairly high. Hence, we stress the importance of modeling the service time distribution correctly in these shared situations.

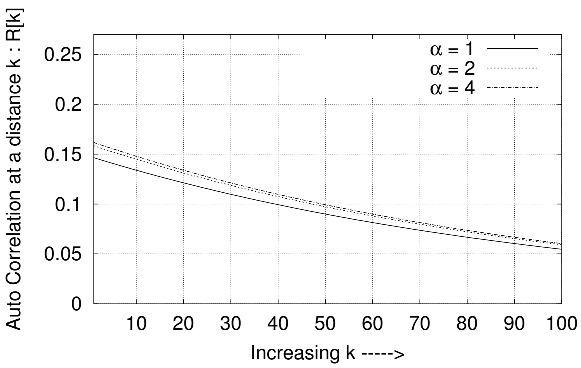

Figure 7. r[k]: c_b^2=1, c_{bs}^2=4, γ_{bs}=0.99

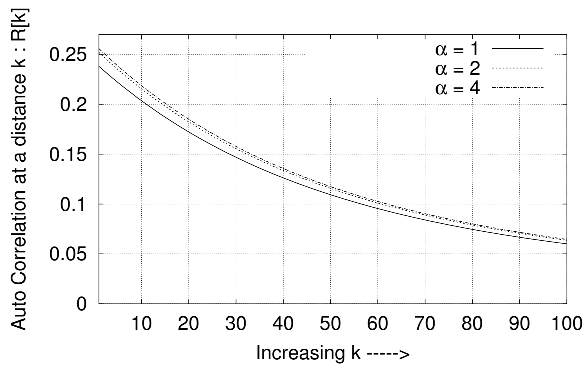

Figure 8. r[k]: c_b^2=4, γ_b=0.99, c_{bs}^2=4, γ_{bs}=0.99

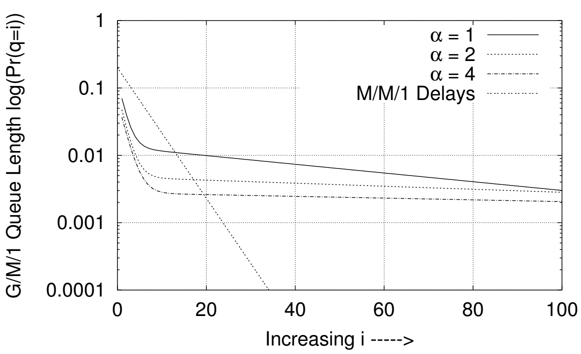

Figure 9. Pr[q=k]: c_b^2=1, c_{bs}^2=4, γ_{bs}=0.99

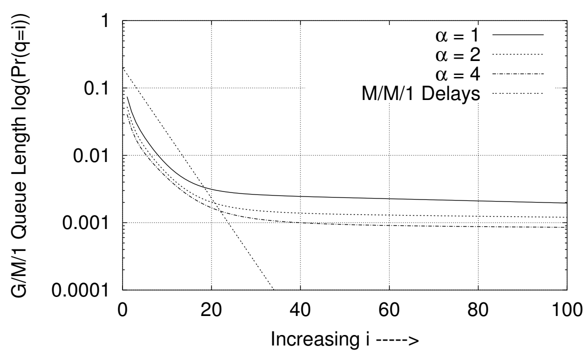

Figure 10. Pr[q=k]: c_b^2=4, γ_b=0.99, c_{bs}^2=4, γ_{bs}=0.99

3.2.2. Impact on an exponential server

In order to study the impact that heavy traffic multiplexing might have on the rest of the network, we study the behavior of an exponential server fed with the output of the multiplexer, compared to the behavior of the server being fed by the original tagged stream. We compare the queue lengths to get insights into the price paid by the tagged stream at an exponential server (in terms of queue length distribution) for passing through a heavy traffic multiplexer. In Figure 9, we present the results for the situation where the arrival process for tagged class is still Poisson, but where now both the arrival process and the service time distributions for the background class are correlated non-exponential (both with $c_b^2 = c_{bs}^2 = 4$ and both with slowly decreasing covariances, $\gamma = 0.99$). The correlations in the departure process of the tagged class has a similar pattern as when the background class had Poisson arrivals (Figure 7), but the induced correlations are now much higher (see Figure 8). More importantly, these higher correlations have a significant impact on the queue length distribution at the exponential server. The queue length now has a heavier tail as compared to the behavior encountered by the original tagged stream at the exponential server. This experiment is repeated, again with both the arrival process and the service time distributions for the background class being correlated non-exponential (but now both with $c^2 = 4$ and both with slower decrease of covariances, $\gamma = 0.99$). The tagged class, before being multiplexed, is still Poisson with exponential service times. Now we see that the correlations induced have about the same absolute values, but decay slower. This has a significant impact on the G/M/1 queue, where the queue length is very slow to decay (see Figure 10) compared to the decay observed by the tagged stream had it avoided the multiplexer and reached directly to the exponential server.

In table 1, we present the mean queue lengths, $E[q]$, for the various scenarios observed in

Table 1
Mean queue length for c_{bs}^2=4 and γ_{bs}=0.99

$E[q]$		$c_b^2 = 4$		
	c_b^2=1	γ_b=0.9	$\gamma_b = 0.95$	$\gamma_b = 0.99$
$\alpha = 1$	48.3348	65.9326	79.8815	167.382
$\alpha = 2$	141.097	170.596	196.489	377.0
$\alpha = 4$	247.965	289.675	327.749	605.494

the experiments. The expected queue lengths also agree with the assertion that the degree of distortion encountered by the tagged stream as it passes through the multiplexer, can be made higher by increasing the proportion of background cells, increasing the c^2, or increasing the correlation decay of the background arrival and the service processes. Higher distortions in the tagged departing streams are evident in the high value of expected queue length at the exponential server.

4. Summary

This paper investigates the effect on a tagged class at a multiplexer by several background classes. First we build a model based on Linear Algebraic Queueing principles assuming heavy traffic. The model is developed in its full generality where all processes involved can be correlated and non-exponential. Using this model, we show that significant correlations can be introduced by the background traffic when the arrival or service time distribution for the background class deviates significantly from those of the tagged class. These distortions can be particularly severe at access nodes under heavy traffic, where the tagged class is being forwarded with distorted second-order statistics. If a single class with low traffic intensity is multiplexed with similar streams of different classes, then every such single class can be severely effected.

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