© *International Regional Science Review,* Vol. 15, No. 2, pp. 199–222, 1993

The Utility of Impulse Response Functions in Regional Analysis: Some Critical Issues

Jeff B. Cromwell
Institute for Labor Studies
West Virginia University
Morgantown, West Virginia 26506-6031 USA

Michael J. Hannan
Department of Business Administration and Economics
Edinboro University of Pennsylvania
Edinboro, Pennsylvania 16444 USA

ABSTRACT Regional scientists have long been interested in measuring the effects of various external and internal stimuli on a regional economy. Measuring the actual size and timing of exogenous and endogenous impacts has been of special interest, as numerical or estimation techniques allow regional actors (governments, business, and others) to make policy-type probability statements and actions in response to changes to these stimuli. Recently, the use of vector autoregressive (VAR) models and, consequently, impulse response functions has become increasingly popular. This paper will closely examine the VAR methodology and its assumptions and will address the types of empirical issues that arise from actual regional implementation. The issues of stationarity, model specification and selection, order determination, and impulse responses are discussed.

1. Introduction

While early applications of the vector autoregressive (VAR) methodology at the regional level were mainly concerned with forecasting (e.g., Hoehn, Gruben, and Fomby 1984; Kinal and Ratner 1986; Cargill and Morus 1988; Cromwell and Hannan 1988; Gruben and Long 1988; Lesage and Reed 1989a; Lesage 1990b), recent emphasis has shifted to impact analysis and theory testing (e.g., Corman, Joyce, and Lovitch 1987; Raynauld 1988; Sherwood-Call 1988; Koray and Lastrapes 1989; Mjelde and Paggi 1989; Lesage and Magura 1986; Lesage and Reed 1989a, 1989b; Lesage 1990a, 1990b). Lesage's papers were the first to address regional theory with VAR modeling. Lesage and Magura (1986) employed VAR modeling to determine the causal structure of interindustry-intercity employment relationships in three cities and among five industries. Lesage

Received 1 March 1991; in final revised form 22 May 1992.

and Reed (1989a) demonstrated the applicability of VAR modeling for examining the interregional wage transmission hypothesis and suggested that VAR modeling holds great promise in many areas of regional science. Lesage and Reed (1989b) examined the relationship between changes in export and local employment. They obtained the impulse response functions to analyze local employment impacts as well as to test the ability of the location-quotient approach of economic-base modeling to measure regional exports. Building on the results of the previous studies, Lesage (1990a) examined the economic-base model and the issue of cointegration, which resulted in the error correction model (ECM) specification along with the Bayesian VAR in an excercise of pure forecasting. The final paper in this series, Lesage (1990b), used different forecasting models (e.g., VAR, VAR Moving Average, Bayesian VAR, ECM, Bayesian ECM) to examine the dynamics of a regional labor market disaggregated by standard industrial classification (SIC). Lesage's papers, along with Mjelde and Paggi (1989), are the primary articles that couple regional economic issues with the VAR methodology and will be referred to throughout this study.

This article first reviews the theoretical specification of the VAR methodology by dealing with the issues of stationarity, model specification and selection, order determination, and impulse responses. Next it examines these theoretical issues in an illustrative empirical exercise through a randomly selected regional data set, and then concludes with some thoughts on how to deal with the issues raised.

2. The Vector Autoregressive Methodology

While the more restrictive VAR approach is used most often in the literature, a look at the more general vector autoregressive moving average (VARMA) case will help put the VAR methodology into perspective.

Given that a data vector $X_1, \ldots, X_t$, where $X_t = (x_{1t}, \ldots, x_{kt})'$, is a mean-zero Gaussian jointly covariant stationary data vector, it can be represented by a k-dimensional VARMA(p,q) process such that

$$\Phi_p(L)X_t = \Theta_q(L)u_t, \tag{1}$$

where $\Phi_p(L) = (I - A_1L^1 - \ldots - A_pL^p)$ and $\Theta_q(L) = (I + B_1L^1 + \ldots + B_qL^q)$.[1] L^i is the backshift operator such that $L^iX_t = X_{t-i}$ for $i \neq 0$, $\Phi_p(L)$ and $\Theta_q(L)$ are matrix polynomials in the backshift operator L, I is a $k \times k$ identity matrix, and p and q are the orders of the

[1] The data vector is assumed to be mean corrected, thus eliminating the constant term.

autoregressive and moving average, respectively. A_i and B_i are $k \times k$ matrices of parameters, and $u_t = (u_{1t}, \ldots, u_{kt})'$ is a k-dimensional vector of strict white noise that results in $E(u_t) = 0$ and $E(u_t u_s')$ as a diagonal, positive definite, variance-covariance matrix Σ_u.[2]

Since this process is assumed to be jointly covariant stationary (i.e., $\det[\Phi_p(z)] \neq 0$ for $|z| \leq 1$), then the matrix polynomial $\Phi_p(L)$ is invertible and there exists a moving-average representation

$$X_t = \Gamma(L)u_t, \tag{2}$$

where $\Gamma(L) = [\Phi_p(L)]^{-1}\, \Theta_q(L)$.

Because of the numerical difficulty involved with empirical estimation and identification of the VARMA approach, most researchers assume q to be zero in equation (1) and exploit the diagonal assumptions of the Σ_u matrix, which enables them to eliminate any contemporaneous correlation problems. The resulting VAR specification is not as parsimonious as the VARMA approach. The analogy can be made to the univariate case where a finite order autoregressive moving average can be represented by an infinite order autoregressive model (Priestley 1980). Estimation problems are reduced, however, because all right-hand side variables are predetermined and are the same in each equation, thus allowing consistent estimation of each equation separately with ordinary least squares techniques.

This article will center on the VAR approach but still maintain the generality of the results to the VARMA model specification (Judge et al. 1985). Even though, in practice, the VAR approach is used most often, the researcher should recognize the potential model misspecification as well as such problems as multicollinearity that may result from this specification. In addition, it should be realized that if p = 0 in equation (1), then estimation of the model will yield the impulse response functions directly through a vector moving-average process. This is similar to the univariate case of an MA(q) model that is estimated efficiently by nonlinear least squares. One could hypothesize that multivariate moving-average models could be estimated by univariate methods even though there may be losses with respect to efficiency.

The impulse responses are forthcoming from the moving-average representation of the VAR model (i.e., q = 0 in equation [1]), which can be written as

$$X_t = \sum_{j=1}^{\infty} \Gamma_j u_{t-j}, \tag{3}$$

[2] The term "strict white noise" is used instead of the familiar "white noise" to ensure that the residual vector will be independent and identically distributed. The assumption of Gaussianeity ensures this and also restricts the data vector to a linear model.

where Γ_j represents the series of impulses in X_t as a result of shocks imposed to the error term (Fuller 1976, Judge et al. 1985). The temporal impulse coefficient matrices are derived from the following numerical expression:

$$\Gamma_j = \sum_{i=1}^{j} \Gamma_{j-i} A_i, \tag{4}$$

where j is defined as in equation (3). Also, $\Gamma_0 = I_k$ is a $k \times k$ identity matrix, and $A_i = 0$ for $i > p$, where p is the order of the model. Equation (4) illustrates the recursive nature of the impulse response calculation.[3] Thus, the selection of p becomes extremely important in determining the magnitude and time path of the impulses.

For example, assume that the accepted model sets $p = 1$, while the true model is of order two. For the accepted model, the impulse responses for the first three periods ($j = 1,2,3$) are found from equation (4) and are

$$\begin{aligned} \Gamma_1 &= \Gamma_0 A_1 = A_1 \\ \Gamma_2 &= \Gamma_1 A_1 + \Gamma_0 A_2 = A_1^2 \\ \Gamma_3 &= \Gamma_2 A_1 + \Gamma_1 A_2 + \Gamma_0 A_3 = A_1^3. \end{aligned} \tag{5}$$

These results follow because of the recursive substitution through the equations and the fact that in this model, A_i is zero for $i > 1$. If the true model is of order two, the true impulse responses would be

$$\begin{aligned} \Gamma_1 &= \Gamma_0 A_1 = A_1 \\ \Gamma_2 &= \Gamma_1 A_1 + \Gamma_0 A_2 = A_1^2 + A_2 \\ \Gamma_3 &= \Gamma_2 A_1 + \Gamma_1 A_2 + \Gamma_0 A_3 = A_1^3 + A_2 A_1 + A_1 A_2. \end{aligned} \tag{6}$$

This case differs from the one above in that A_i is now zero for $i > 2$.

These results illustrate the importance of choosing the appropriate order in specifying a model. The impulse responses will only be the same for the first period, after which they will differ as long as A_i for $i > 1$ is nonzero. That is, the impulse responses are not unique. This problem is related to the non-uniqueness of an autoregressive moving-average representation for a stochastic process and is also similar to the identification problem in simultaneous equation

[3] Typically, the coefficients are referred to as "dynamic multipliers" or impulse responses of the system. The matrix Γ_j represents the response of X_k ($t = k$) to a unit shock in the error process over j periods. Thus, the response of X_k to a unit shock in equation i at $t = 0$ is simply the kth column of the Γ_j matrix. (See Doan 1988, Sargent and Sims 1977, Sims 1980.)

models. Identification requires the imposition of a particular structure on the economic model or the use of various data fitting exercises. Quenouille (1957) and Hannan (1969, 1970) discuss the uniqueness issue in more detail.

In addition to the uniqueness problem, the properties of the residual covariance matrix, Σ_u, also hinder interpretation of the impulse responses. This problem stems from the difference between a one-time standard error shock and a one-time unit shock. The former does not exploit the contemporaneous correlations between the residuals in equation (1) and enables the size of the shock to be related to the fit of the equation. For example, if the effect of a shock to national industrial production on state employment were to be examined, the size of the one standard error shock would be equal to the standard error from the equation for industrial production in the VAR system. Alternatively, the unit shock introduces an orthogonalization that eliminates the contemporaneous residual correlations. Specifically, a lower triangular matrix P is chosen such that $\Sigma_u = PP'$, and v_t is set equal to $P^{-1}u_t$. This enables Σ_v to equal I_k, which is the identity variance-covariance matrix. The selection of P can be accomplished by various decomposition methods that try to find the square root of a matrix (e.g., Cholesky, Singular value, QR). The Cholesky factorization, currently the dominant approach in the regional literature, suffers from the numerical problem that a matrix square root is not unique (except in the case of a two-variable VAR), which suggests that the different ordering of the residuals will imply a different matrix P.

The orthogonalized impulse responses, $\phi_i = \Gamma_i P$, suggest that the lower triangular matrix P is a weight matrix. This weight matrix takes into account the contemporaneous correlations and forms linear combinations of the original (untransformed) impulses into their orthogonal representations. Thus, the orthogonal impulses, ϕ_i, are a function of the original impulses and the decomposition method. The Cholesky decomposition and Sim's orthogonalization procedures are examples of methods commonly employed (Doan 1988). The coefficients from these procedures comprise the weights in the lower triangular matrix P.

Stationarity in VAR Modeling

Given the suggestion by Nelson and Plosser (1982) that most national economic time series are nonstationary, it is plausible that regional economic time series would also possess this trait. Equations (1)–(4) have no theoretical support, however, unless the processes that are being modeled are Gaussian jointly covariant stationary. Although a model could possibly forecast well even in the presence of nonstationarity, the precise calculation of impulse response functions requires a necessary and sufficient condition of stationarity.

In the literature, transformations of the data commonly are performed to alleviate the problem of nonstationarity. One transformation method employed is that of differencing. First-order differencing implies that the variable in question follows a random walk model, denoted by $(1 - L)X_t = e_t$. Besides the traditional technique of visual inspection (i.e., eye-balling) to confirm that the series is stationary, some empirical tests for the univariate case have been developed to examine the random walk, or unit root, hypothesis (e.g., Dickey and Fuller 1979, 1981; Bhargava 1986). Univariate differencing, however, even if placed in the hypothesis testing framework, may not be sufficient to ensure that the model is jointly covariant stationary. It is possible that the model could have polynomial trends or an increasing variance, in which cases the unit root hypothesis may be rejected. Alternatively, the individual series may be stationary, but when placed in a vector are not jointly covariant stationary.

Lutkepohl (1982, 238) states that "differencing nonstationary univariate component series of a multiple time series to induce stationarity prior to building an [autoregressive] model for the multivariate generation process is in general inadequate." In other words, merely testing for joint covariance stationarity at the univariate level is not sufficient. It must be supplemented with a multivariate test. One such test, recently proposed by Fountis and Dickey (1989), is a multivariate version of the univariate tests proposed by Dickey and Fuller (1979, 1981). It involves examining the eigenvalues of the coefficient matrices, A_i. Fountis and Dickey demonstrate that this method is the vector analog to differencing in the univariate case. An interesting feature of this testing methodology is that the data are recast in their original form of levels even though a unit root is found. While this method is potentially very attractive, it may be computationally expensive for regional VAR models since they typically involve dimensions of high order. This dimensionality comes from having both many variables and longer lag lengths than typical national counterparts, especially true when monthly data is used. For example, in a typical regional application a five-variable VAR with 12 lags could be specified, which would result in 60 characteristic roots. Not only could there be an unwieldy number of characteristic roots, but also the possibility of multiple unit roots. The one limitation of the method proposed by Fountis and Dickey is that it tests for only one unit root.

In the regional literature, transformations for stationarity have frequently been performed but rarely tested for success. Table 1 summarizes the transformation methods employed in some selected regional applications. With the exception of LeSage (1990a), there is a uniform lack of testing the series for a unit root. This is surprising given the existence of univariate testing procedures during the time period covered by the selected literature. Neither is multivariate

TABLE 1
Stationarity in Regional VARs

Author(s)	Method of transformation[a]			Hypothesis testing[b]	
	DO	TR	PT	Univariate	Multivariate[c]
Hoehn/Gruber/Fomby (1984)	Y	N	Y	N	N
Kinal/Ratner (1986)	Y	N	Y	N	N
LeSage/Magura (1986)	N	N	N	N	N
Corman/Joyce/Lovitch (1987)	N	Y	Y	N	N
Cargill/Morus (1988)	N	N	N	N	N
Cromwell/Hannan (1988)	Y	N	Y	N	N
Gruben/Long (1988)	N	N	N	N	N
Raynauld (1988)[d]	N	N	Y	N	N
Sherwood-Call (1988)	Y	N	Y	N	N
Koray/Lastrapes (1989)	N	Y	Y	N	N
LeSage/Reed (1989a)	N	Y	N	N	N
LeSage/Reed (1989b)[e]	N	N	N	N	N
Mjelde/Paggi (1989)	N	N	Y	N	N
LeSage (1990a)	Y	N	N	Y	N

[a]The methods of data transformation used to attempt stationarity are: DO = use of a difference operator; TR = trend removal by regression or inclusion in the VAR; PT = power transformation for stabilizing variances (e.g., log transformation).

[b]Hypothesis testing indicates tests for stationarity.

[c]The multivariate test is not that of Fountis and Dickey (1989), but a test to see if the VAR's characteristic roots are less than unity.

[d]This study had transformed the data logarithmically, though this was done for interpretational purposes of the impulse responses and not for stationarity.

[e]The authors addressed stationarity by regressing the raw data on national wage variables and employing the residuals from these regressions in the VAR model.

testing attempted by any of the authors. Although the Fountis and Dickey (1989) multivariate test is very recent, eigenvalues can be obtained for any application, which could lend support for an assumption of stationarity. (Some authors may be excused from this criticism if the purpose of their study was empirical forecasting.) Although impulse response functions can always be calculated from any coefficient matrix, it is the time path and magnitude of the impulse responses that are affected by stationarity.[4] Attempts to address the stationarity issue are demonstrated by performing the

[4] In the first-order model, $X_t = AX_{t-1}$, the ith impulse is simply A raised to the ith power. Since A is squared, a spectral decomposition can be performed to show that the dynamics of the impulses are a function of the characteristic roots of A.

transformations, but the transformations have no value unless they are tested for success.

Engle and Granger's (1987) concept of cointegration also relates to stationarity. In general, if two variables are integrated of order one, denoted by $I_a(1)$ and $I_b(1)$, then they are cointegrated if there exists a linear combination of the two variables that is stationary. An economic interpretation of this linear combination can be described as a long-run or equilibrium relationship among the variables, or more intuitively, a tendency of the economic system to move toward some particular point in time.

The idea of cointegration introduces an important constraint in VAR modeling that can be specified in the form of an error correction model. Engle and Granger (1987, 259) point out that "vector autoregressions estimated with cointegrated data will be misspecified if the data are differenced, and will have omitted important constraints if the data are used in levels." Thus, to overcome this problem, the Granger Representation theorem allows a VAR model in first differences to be specified with a constraint of an error correction variable included. The premise of this theorem, however, is that the components of the vector X_t are integrated of order one, I(1), which implies a random walk vector. This in turn implies that each variable of X_t is a random walk. Cointegration can exhibit higher orders than unity, however. Thus, error correction models can be estimated given those higher orders (Engle and Granger 1987). Some applied researchers may argue that higher levels of integration of variables may preclude easy interpretation of the results. The formal classical approach would imply that each variable in the vector first be tested for order of integration. This goes back to the transformation techniques mentioned earlier. Before the cointegration tests can be performed, the degree of differencing that is required to make the series stationary must be determined.

Cointegration can be tested at both the univariate level and the multivariate level (multi-cointegration) (e.g., Dickey and Fuller 1979, 1981). As shown in equation (4), the impulse response functions can be calculated from the first order model, $X_t = AX_{t-1} + U_t$,[5] which can be written as

$$\begin{bmatrix} (1 - Lx_{1,t} \\ (1 - L)x_{2,t} \\ Z_{t-1} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \rho_1 \\ \alpha_{21} & \alpha_{22} & \rho_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (1 - Lx_{1,t-1} \\ (1 - L)x_{2,t-1} \\ Z_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ 0 \end{bmatrix}.$$

[5] One could generalize that the vector Y_t is jointly covariant stationary since the data series are integrated of order one and the error correction variable is integrated of order zero; however, as discussed previously, one must be cautious about this generalization.

If the error correction variable is not included, then one returns to the two-variable VAR model. Even if the model is jointly covariant stationary, it is conceivable that it will still be misspecified by omitting the important error correction constraint, that is, the third column of the matrix A. Finding the eigenvalues associated with the A matrix will provide information concerning these cointegrating relationships.

One caveat should be mentioned regarding the use of location quotients for measuring exports and cointegration. Lesage (1990a) developed an export-base model where export-related employment was defined as a piece-wise linear function of local employment.

$$XE_{it} = \begin{cases} 0 & \text{if } NE_{it}/NE_t \geq AE_{it}/AE_t \\ AE_{it} - LE_{it} & \text{if } NE_{it}/NE_t < AE_{it}/AE_t, \end{cases}$$

where i is a given industry, and LE_{it} is the location quotient $AE_t(NE_{it}/NE_t)$. Thus, if the region's employment share in a given industry is greater than what is true at the national level, then export and local employment are negatively related. (See Isserman [1980] for a discussion of location quotient limitations.) Therefore, if the relationship holds true for the whole time period (or at least a large subset), XE and LE should be cointegrated. If this is the case, it is further known that Granger causality must occur in at least one direction. This suggests that an error correction model would be the most appropriate for calculating impulse response functions.

Model Selection and Order Determination

Suppose there is a cointegrating relationship among the variables. An appropriate model specification would then be the error correction model, or if multicollinearity in a model of high dimension is a concern, a Bayesian error correction model could be specified. Alternatively, if no cointegration exists, a VAR or Bayesian VAR model could be employed. In either case the impulse response function can be calculated from any of these models. With respect to the Bayesian VAR model in particular, however, the methodology is difficult to manage from an empirical perspective. For example, to minimize forecast error from a Bayesian VAR model, the researcher must (1) know the number of periods ahead planned to be forecast,[6] (2) find the appropriate Bayesian priors for the model (e.g., harmonic vs. geometric vs. another decay), which becomes a multidimensional minimization problem,[7] and (3) determine the appropriate forecasting

[6] Lesage (1990b) showed how various VAR and Bayesian VAR models minimize the root mean squared error (RMSE) criterion depending on the number of periods ahead to forecast.

[7] Magura (1990) has shown that the dimensionality problem may be reduced in regional applications based on information obtainable from input-output relationships.

environment for evaluation (i.e., static vs. dynamic, ex post vs. ex ante). Some have suggested that guidance for specifying priors is provided in the literature; however, these broad principles are not necessarily appropriate in every empirical application. Given these limitations, the remainder of this discussion will be restricted to the non-Bayesian VAR approach.

Model selection is based on the context of the problem, that is, the choice of decision criteria. Such criteria could include ex ante forecast performance, goodness-of-fit measures, or rules gained from prior experience. Within each of these criteria, a number of additional decisions would need to be made. For example, if ex ante forecasts were used as the decision criterion, one would also need to decide whether to use static or dynamic forecasts, how long to extend the forecast period, and how large of a sample period to use for estimation of the forecast model. The dynamic nature of the linear model discussed earlier requires the determination of the model order. Next to the issue of stationarity, order determination is the most complex issue facing the regional modeler.

Table 2 summarizes the methods used for order determination in the selected regional VAR literature. In many cases, model order was determined by minimizing the static forecast error, based on a sample. These methods are predominantly minimization criteria involving probabilistic foundations and are related to typical forecast performance measures such as mean absolute percent error (MAPE) and root mean squared error (RMSE). Some of these methods include the Akaike Information Criterion (AIC) (Akaike 1973, 1974); the Schwarz Criterion (SC) (Schwarz 1978); Criterion for Autoregressive Transfer function (CAT) (Parzen 1974); Final Prediction Error (FPE) (Akaike 1969); Hannan-Quinn (HQ) (Hannan and Quinn 1979) and (HQ1) (Quinn 1980); and the Shibata criterion (Shibata 1980). A classical statistical testing procedure for order determination, the Likelihood Ratio (LR) (Hannan 1970; Anderson 1971), has also begun to surface as an employed technique (table 2). In other cases, the order was not determined by using any of the above measures but through other conditions, such as finding the maximum lag lengths reasonable given degrees of freedom limitations, or using a large number of lags to fully capture the dynamics of the economic relationships.[8]

Given the many measures for determining order, the question arises is one measure better than the others? One simulation study compared the relative effectiveness of the methods above and found the Schwarz and Hannan-Quinn criteria to be most successful in identifying the true order of the model (Lutkepohl 1985). The

[8] The typical VAR model is constructed with symmetric lags, though this is not a requirement. It is possible to employ asymmetric lag lengths, which necessitates the use of generalized least squares estimation techniques for efficiency.

TABLE 2
Order Determination in Regional VAR Studies

Author(s)	Test Statistic
Hoehn/Gruber/Fomby (1984)	None
Kinal/Ratner (1986)	None
LeSage/Magura (1986)	Step-wise Final Prediction Error
Corman/Joyce/Lovitch (1987)	None
Cargill/Morus (1988)	None
Cromwell/Hannan (1988)	None
Gruben/Long (1988)	None
Raynauld (1988)	None
Sherwood-Call (1988)	None
Koray/Lastrapes (1989)	None
LeSage/Reed (1989a)	Liklihood Ratio
LeSage/Reed (1989b)	Likelihood Ratio
Mjelde/Paggi (1989)	Likelihood Ratio (Tiao-Box U-statistic)
LeSage (1990a)	Likelihood Ratio

applicability of Lutkepohl's results in regional analysis is limited, however, because he only considered simulation models of low order (i.e., one and two lags) and low dimensionality (i.e., two and three variables). It is common to find models of a much higher order and dimensionality in the regional literature due to the typical use of quarterly and monthly data and the large number of variables employed in regional applications. Simulation studies demonstrate only what is feasible within a given experimental context. Therefore, it cannot be concluded that the Hannan-Quinn and Schwarz methods will dominate all other measures in applications with different conditions. The issue of a dominant order criterion remains unresolved.

Calculation of Impulse Response Functions

Empirical researchers must consider the question of orthogonalization when calculating impulse response functions. In the case of one-time standard error shocks, economic interpretation of the resulting impulses is straightforward, provided there is no contemporaneous correlation in the residuals (Raynauld 1988; Backus 1982). If contemporaneous correlation does exist (often the case in practice), then the impact of one variable on another cannot be isolated unless the variance-covariance matrix, Σ_u, is diagonalized. This has the effect of making some variables act exogenously in order to remove the contemporaneous correlation of the residuals between equations.

Orthogonalization is also another way of identifying the model by restricting covariance instead of restricting lag lengths.

Identification of the model coupled with its recursive structure potentially limits the use of impulse responses in regional studies. For example, to examine the timing and magnitude of impacts on a regional labor market, a typical regional VAR specification may include nominal average hourly earnings as a proxy for wages, manhours as a proxy for regional employment, and various variables that represent nonwage determinants of demand and supply. For the sake of simplicity, let one determinant be the national consumer price index. This three-variable VAR can then be used to examine the response of nominal wages and employment to a shock in the consumer price index (e.g., Lesage 1990b). To isolate these responses, the Cholesky decomposition method may be employed. For the responses to be unique, an a priori ordering based on economic theory has to be chosen to carry out the orthogonalization procedure. The resulting recursive model will place certain constraints on the elasticities of the regional labor demand and supply functions. For example, if the employment variable is placed before the wage variable in the ordering, then either labor demand or supply is assumed to be perfectly inelastic. After this ordering, the system defined above can be represented by

$$\begin{aligned} \text{CPI} &= f(\text{CPI}) \\ \text{Employ} &= f(\text{Employ, CPI}) \\ \text{Wage} &= f(\text{Wage, Employ, CPI}) \end{aligned}$$

where Employ or Wage can represent demand or supply. With this ordering, changes in wages will have no effect on employment. Alternatively, the reverse ordering implies that labor demand or supply is perfectly elastic. Although both demand and supply shocks can be identified, the recursive structure allows each shock to be associated with the equation for a specific variable. Simultaneous models, however, would allow labor demand and supply to have their more traditional elasticities (i.e., nonperfect elasticity). Thus, in regional applications of the VAR methodology, the economic consequences of the recursiveness assumption must be realized as well as the effects of identification procedures on regional economic policy making.

Both standard error and unit impulse responses have been used in the regional literature (table 3). Confusion generally exists as to interpretation, however, because many authors refer to standard deviation and unit shocks interchangeably, but the term "unit" shock should be used only when an orthogonalization procedure has been used. Some may argue that there is no difference between standard

TABLE 3
Impulse Response Functions in Regional VAR Studies

Author(s)	Shock-type	Confidence interval	Hypothesis test
Corman/Joyce/Lovitch (1987)	Unit	No	No
Cargill/Morus (1988)[a]	--	No	No
Raynauld (1988)	Standard error	Monte Carlo	No
Sherwood-Call (1988)[a]	--	No	No
Koray/Lastrapes (1989)	Unit	No	No
LeSage/Reed (1989a)	Unit	No	No
LeSage/Reed (1989b)	Standard error	Monte Carlo	No
Mjelde/Paggi (1989)	Unit	No	No

[a]Only variance decompositions are presented; however, the impulse response functions are needed to obtain these decompositions (Lutkepohl 1990).

error and unit shocks as long as the data are similarly scaled. For the two types of shocks to be the same, however, the variance-covariance matrix, Σ_u in equation (1), must be diagonal. Having similarly scaled data does not in itself generate a diagonal variance-covariance matrix. Off-diagonal elements may still be of considerable magnitude (i.e., there may be contemporaneous correlation). Given this possibility, one cannot generalize, even through standardization of the data, that the two shocks will be the same.

Testing for significance of coefficients is commonplace in any estimation procedure, since without it, parameter values have no statistical meaning. Unfortunately, the literature lacks a formalized hypothesis testing procedure for impulse responses. At most, there have been attempts to formulate confidence intervals around the impulse responses, though nearly all of these confidence intervals rely on an assumption of normality for the error process in equation (1), either through direct implementation (i.e., assumption) or the Central Limit Theorem.

There are three techniques commonly used to construct confidence intervals. One uses the Monte Carlo integration method based on normality of the residuals (Kloek and Van Dijk 1978). A second method deals with bootstrapping a mean and variance from the empirical distribution of the residuals in equation (1) (Efron 1982). The third method is based on asymptotic normal approximations of the error process (Lutkepohl 1990, Runkle 1987). Until recently, most confidence intervals were constructed using the Monte Carlo procedure outlined by Doan (1988). Also, the Monte Carlo method has been the only one employed in regional applications, and only in cases where standard-error shocks have been calculated (table 3).

Confidence intervals generated with these methods may be limited. Because the process is stationary, the impulses will taper off to zero as the number of impulses, i, becomes large. This is reasonable, since one would expect that the effect of a shock on an exogenous variable should logically move toward zero over time. In other words, if a system is shocked from equilibrium at $t = 0$, then over time the system should return to equilibrium at some future period. If the error vector is assumed to be zero at $t = 0$, then it should return to zero at some point also. Further, Lutkepohl (1990) states that if the data generation process is stationary in a finite order VAR model, the probability is close to one that impulse response values will be close to zero as i increases. Intuitively, a test of the null hypothesis that the impulse is zero for large i should be accepted. A review of the literature suggests, however, that acceptance might not occur due to the large confidence intervals as i becomes large. Standard practice is to exploit the assumption of a symmetric distribution and generate confidence intervals using the Empirical Rule, for example, ± two standard deviations for a 95 percent confidence interval. A more useful procedure than constructing 95 percent confidence intervals would be to develop hypothesis tests concerning the significance of the impacts. Recent work by Lutkepohl (1989, 1990) now allows the construction of such tests through the use of normal approximations.[9]

3. An Illustrative Application

To illustrate the theoretical issues discussed above, a two-variable model for Erie, Pennsylvania, has been chosen. It includes Erie 2-digit SIC manufacturing employment (EMFG) as the regional variable and the Index of Industrial Production–All Industries (INDPROD) as a national measure of economic activity. Monthly data for EMFG were obtained from the Bureau of Labor Statistics (1991) and for INDPROD from Citibank (1991). The model is estimated for the period from May 1970 to April 1991.

Model Identification

The first step is to determine the appropriateness of the model selection by performing the McLeod-Li (1983) diagnostic test for linearity (table 4). The Box-Pierce and Ljung-Box tests were calculated for the two variables in levels, differences, and differences squared. The null hypothesis that the series is white noise was rejected in all cases except for the squared difference of EMFG. The results provide evidence of nonlinear dependence for EMFG and suggest that the series is linearly uncorrelated. Thus, a linear model for this variable is inappropriate, whereas a linear model for INDPROD would be appropriate. Consequently, a VAR model of these variables could be misspecified.

[9] See Cromwell (1991) for a more technical discussion of some of the caveats mentioned in the preceding theoretical sections.

TABLE 4
Diagnostic Test for Linearity

	Box-Pierce	Ljung-Box
Levels		
EMFG	4418.82**	4662.72**
INDPROD	4193.10**	4420.07**
First Difference		
(1–L) EMFG	51.53**	54.48**
(1–L) INDPROD	121.26**	126.10**
First Difference Squared		
$[(1-L)\ EMFG]^2$	15.43	16.18
$[(1-L)\ INDPROD]^2$	47.21**	48.55**

EMFG = Erie manufacturing employment
INDPROD = Index of industrial production
* = significant at the 5% level
** = significant at the 1% level
} Likewise for subsequent tables

The null hypothesis is that serial correlation does not exist.

Stationarity and Cointegration Tests

Univariate and multivariate tests for stationarity were also performed for the series in levels and differenced form (see Dickey and Fuller 1979). These results are shown in table 5. In the univariate case, both series are integrated of order one. In the multivariate case, the Fountis and Dickey (1989) tests were performed for a 1-lag model. The null hypothesis of a unit root for the single-order model could not be rejected. The characteristic roots examined for the differenced model suggest that for these estimated coefficients, the model is jointly covariant stationary.

Since both variables are integrated of order one, it is appropriate to test whether both variables are cointegrated. Engle-Granger (1987) cointegration tests were performed, and the results conclusively showed that the variables are not cointegrated. Had they been cointegrated, a pure VAR in differences would have been a misspecification (Engle and Granger 1987).

Order Determination

Several criteria were used to determine the optimal lag length for the Granger-causality test and to find the order that provided the true impulse responses. Table 6 illustrates that different order criteria provide different estimates of the optimal lag length. From the Granger-causality perspective (looking at differenced data), the

TABLE 5
Tests for Stationarity

	Levels		First differenced	
	EMFG	INDPROD	(1–L)EMFG	(1–L)INDPROD
UNIVARIATE				
Dickey-Fuller Test				
Trend Included	–1.73	–1.45	–19.44**	–10.15**
No trend included	–1.21	3.71**	–19.49**	–9.79**
Constant included	–0.69	–0.61	–19.52**	–10.17**
Augmented Dickey-Fuller Test				
p = 1				
Trend included	–1.74	–2.37	–13.34**	–7.56**
No trend included	–1.20	2.24*	–13.40**	–7.23**
Constant included	–0.73	–0.77	–13.42**	–7.58**
p = 4				
Trend included	–1.92	–3.07	–6.73**	–5.71**
No trend included	–1.18	1.67	–6.74**	–5.35**
Constant included	–0.96	–0.98	–6.78**	–5.73**
MULTIVARIATE[a]				
Roots of $\mid I_t - A_1 \mid$				
Differenced one-lag model	–0.01336			
	0.42446			

[a]Instead of looking at the roots of the matrix polynomial $\mid \Phi_1(z) \mid$, the eigenvalues are examined with the new requirement of $\mid t \mid < 1$ for stationarity. The Fountis and Dickey (1989) test is not performed since it is desirable to keep the data in differenced form.

Significance levels are based on MacKinnon (1990). The null hypothesis is that a unit root exists.

AIC, SC, H-Q, and FPE measures all indicate that national industrial production is exogenous. That is, their optimal lag lengths fall in the range where the Granger-causality tests indicated exogeneity. The remaining measures, H-Q1, Shibata, and LR, indicate industrial production to be endogenous. Therefore, researchers can obtain greatly different results depending on which order criterion they happen to be using. The endogeneity vs. exogeneity issue also raises the question of whether a VAR model or a structural econometric model is more appropriate.

Impulse Response Functions

To provide some information on the recursive nature of the model, Granger-causality tests were performed for the two variables in levels and differences for model orders 1–15 (table 7).[10] For Erie

[10] The possible presence of serial correlation in the results is recognized. This

TABLE 6
Order Determination Tests Performed on Differenced Data

	Test Statistics						
Lags	AIC	SC	H-Q	H-QI	FPE	Shibata	LR
1	−2.4796	−2.4239	−2.4571	−2.4976	0.0851	0.0851	104.656
2	−2.4959	−2.3845	−2.4511	−2.5319	0.0837	0.0836	92.507
3	−2.4820	−2.3149	−2.4148	−2.5360	0.0849	0.0847	88.039
4	−2.4817	−2.2589	−2.3921	−2.5538	0.0849	0.0845	80.103
5	−2.4766	−2.1981	−2.3645	−2.5667	0.0854	0.0848	73.413
6	−2.4711	−2.1369	−2.3366	−2.5792	0.0858	0.0850	66.803
7	−2.4535	−2.0635	−2.2966	−2.5796	0.0874	0.0862	63.285
8	−2.4639	−2.0182	−2.2846	−2.6080	0.0865	0.0850	52.637
9	−2.4568	−1.9554	−2.2551	−2.6190	0.0871	0.0853	46.438
10	−2.4771	−1.9200	−2.2530	−2.6573	0.0854	0.0832	33.286
11	−2.4618	−1.8490	−2.2153	−2.6600	0.0867	0.0841	29.169
12	−2.4640	−1.7955	−2.1951	−2.6802	0.0866	0.0835	20.615
13	−2.4748	−1.7506	−2.1834	−2.7090	0.0857	0.0822	9.872
14	−2.4673	−1.6852	−2.1526	−2.7206	0.0863	0.0823	3.334
15	−2.4469	−1.6066	−2.1088	−2.7195	0.0882	0.0835	na
	Order Suggested by Test						
	2	1	1	14	2	13	9, 10[a]

[a]An order of 9 is indicated at the 5% significance level; an order of 10 at the 1% level.

AIC = Akaike Information Criterion (Akaike 1973, 1974)
SC = Schwartz Criterion (Scwarz 1978)
H-Q = Hannan-Quinn (Hannan and Quinn 1979)
H-QI = Hannan-Quinn-I (Quinn 1980)
FPE = Final Prediction Error (Akaike 1969)
Shibata = Shibata Criterion (Shibata 1980)
LR = Likelihood Ratio (Hannan 1970; Anderson 1971)

Fifteen was used as an upper bound for all tests, therefore the LR is not applicable for 15 lags.

manufacturing employment (EMFG) in levels, national industrial production causes local employment in lags 2–15, whereas in differences, all lags of the national variable cause local employment at the 1 percent significance level. An anomaly results in the causal effect of local employment on national industrial production. When measured in levels, Erie manufacturing employment causes national industrial production in its first six lags, and the causal effect then

presence would indicate that the changes in the causality results are not solely dependent on model order. However, modeling the contemporaneous correlations of the residual matrices from these VARs to eliminate possible serial correlation was beyond the scope of this example.

TABLE 7
Granger-Causality Test

Model Order	Levels[a] EMFG	Levels[a] INDPROD	First Differenced (1–L) EMFG	First Differenced (1–L) INDPROD
1	0.32	22.13**	17.50**	0.10
2	9.19**	7.01**	12.52**	0.21
3	8.58**	4.34**	8.24**	0.20
4	6.24**	3.27*	8.10**	0.15
5	6.45**	2.66*	6.97**	0.20
6	5.66**	2.23*	6.28**	0.27
7	5.15**	1.90	5.72**	0.31
8	4.78**	1.68	4.98**	1.31
9	4.18**	2.09*	4.64**	1.70
10	3.92**	2.20*	3.73**	2.23*
11	3.23**	2.89**	3.59**	2.10*
12	3.11**	2.74**	3.45**	1.79*
13	2.99**	2.36**	3.38**	2.07*
14	3.00**	2.44**	3.59**	1.80*
15	3.20**	2.17**	3.24**	1.95*

[a]Since the data are I(1), VAR estimation in levels is misspecified and not covariant stationary. This portion of the table is included to illustrate the problems that occur in ignoring stationarity.

Values are F-statistics. The null hypothesis for EMFG states that EMFG is not Granger-caused by INDPROD, while the null for INDPROD is that it is not Granger-caused by EMFG; likewise for the differenced cases.

reoccurs in lags 9–15. This result is theoretically unsupportable when considering the size of Erie manufacturing compared to national levels of economic activity. Because of the stationarity results above, however, the implications from the differenced data are more appropriate.

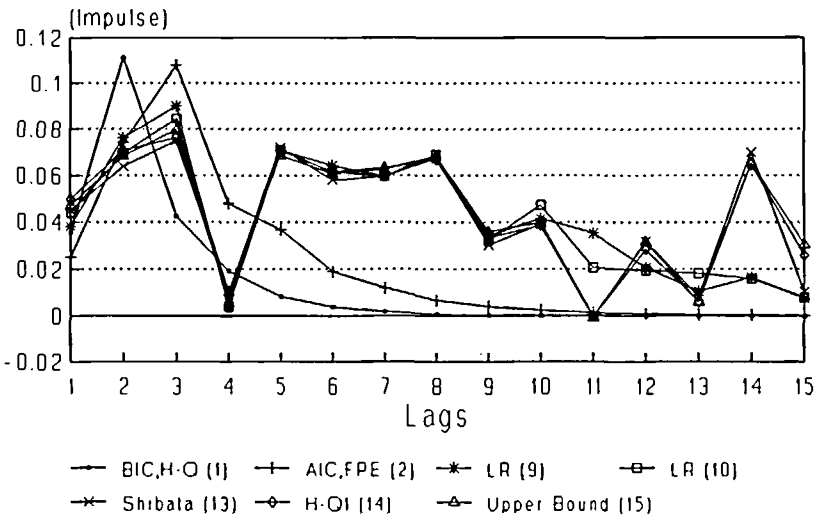
In the case of first differenced data, Erie employment does not cause national industrial production. Yet, a causal effect at the 5 percent significance level is still noted at lags 10–15. This result suggests that industrial production is exogenous to the model in lags 1–9 and endogenous from lags 10–15. To determine whether industrial production is exogenous or endogenous, an appropriate order of the model must be determined. Since this is a bivariate model the ordering is unique; however, the causality tests are still instructive.

The orthogonal impulse responses were calculated with the weights of the P matrix (obtained by the Cholesky decomposition method) for the response of Erie employment to a shock in national production, where $\phi_{21} = 0.63114\Gamma_{21} + 0.22585\Gamma_{22}$. This calculation states that 63 percent of the orthogonal response ϕ_{21} is the response of employment to a shock in national production and 22.6 percent is the response of employment to a shock in employment. Since the values of these weights remain constant for the entire time path of the impulse responses, the constant variance assumption of stationarity is needed. Once again, the stationarity issue surfaces.

For each order criterion from table 6, a set of orthogonal impulse responses was calculated for a fifteen-month period. From equations (5) and (6) it would be expected that the time paths as well as the magnitudes of the impulse responses would differ across different model orders. Figure 1 shows that impulses from the first and second order models track closely, reaching peaks in either month 2 or 3, respectively. The higher order models also track closely, except that the order 10 model tends toward zero impact after month 10.

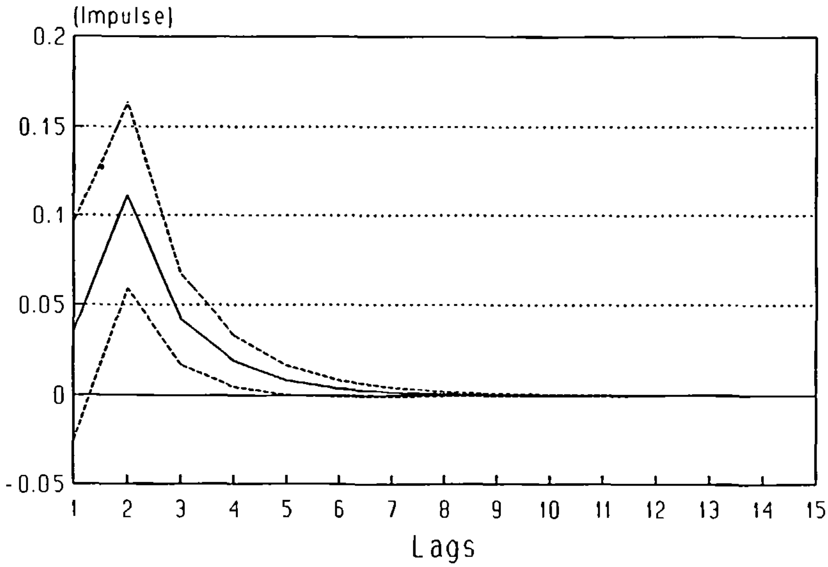
Also important are the different implications that the various models provide. If a low order model is employed, the results indicate that changes in industrial production at the national level have a large impact on local employment within two or three months, but the impact quickly falls to zero thereafter. On the other hand, if a higher order model is employed, the results imply that a change in industrial production has a large effect after three months, which remains strong a number of months beyond this period. These extreme differences in time paths between the first and second order models and the upper bound (15-lag) model are significant given the literature's reliance on high order models for generating impulse responses (under the assumption that they allow sufficient time to capture all dynamics in the system).

FIGURE 1
Impulse Responses of EMFG to a Unit Shock in INDPROD



Model order numbers are in parentheses

FIGURE 2
Impulse Response on Manufacturing Employment, First Order Model



Figures 2 and 3 show the impulse responses for the 1-lag and 15-lag models, respectively, with 95 percent confidence intervals generated using the Monte Carlo technique.[11] As expected, after an initial impact, the confidence interval narrows to the zero level in the low order model (figure 2). For the 15-lag model, the mean impact continues to be positive at 15 lags, while the confidence interval remains large. These results demonstrate the divergence of outcomes that are possible simply by choosing different model orders. The results for these two models are overlaid in figure 4 for comparison. For most of the lags (3, 5, 6, 7, 8, 9, 10, 12, 14, and 15), the mean impulses do not fall within both models' confidence intervals. This suggests that appropriate order determination in VAR model construction is important since impulse response results can vary significantly. This is consistent with expectations, given the manner in which impulses are calculated, as was shown in equations (5) and (6).

[11] Confidence interval values were also generated for the remaining model orders and are available from the authors. The results for the 1-lag and 15-lag models are presented to compare the two model-order extremes estimated in this exercise.

FIGURE 3
Impulse Response on Manufacturing Employment, 15 Order Model

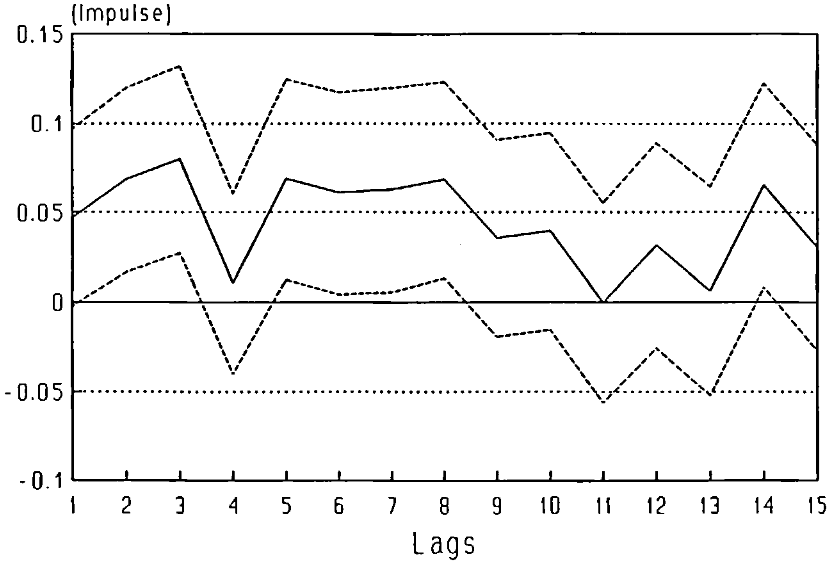
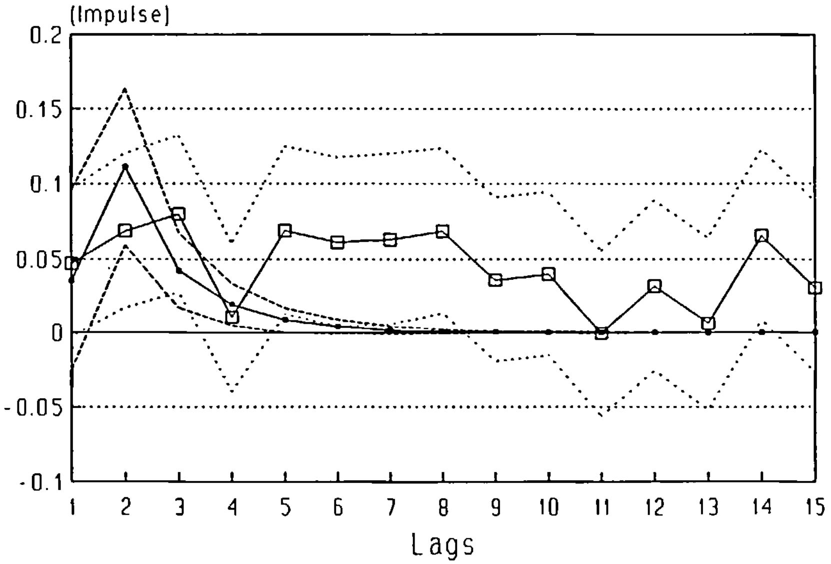


FIGURE 4
Impulse Response on Manufacturing Employment, 1 and 15 Order Models



4. Conclusions

Impulse responses have an important contribution to make to regional analysis given their inexpensive and dynamic nature, but only if a formal statistical testing framework can be implemented that would resolve the various issues raised in this paper. Criticisms here regarding the plausibility of results concern the power of the test statistics, not the methodologies. In most of the regional literature these crucial testing issues have been left unaddressed. Virtually all testing procedures are fraught with difficulty, but this does not necessitate their abandonment in practice. Given what is currently available in the literature, a researcher is able to employ adequately powerful test statistics at each stage of the analysis so that defensible results could be forthcoming. While this approach may be considered dogmatic by some, formalism at least provides a vehicle for determining the plausibility of results, whereas other less formal approaches provide results that are conditioned on the plausibility of the researcher's assumptions.

References

Akaike, H. 1969. Fitting autoregressive models for prediction. *Annals of the Institute of Statistical Mathematics* 21: 243–47.

Akaike, H. 1973. Information theory and an extension of the maximum likelihood principle. In *2nd International symposium on information theory*, eds., B. N. Petrov and F. Csaki. Budapest: Akademiai Kiadi

Akaike, H. 1974. A new look at statistical model identification. *IEEE Transactions on Automatic Control* 19: 716–23.

Anderson, T. W. 1971. *The statistical analysis of time series.* New York: John Wiley and Sons.

Backus, D. 1982. The Canadian-U.S. exchange rate: Evidence from a vector autoregression. Kingston, Ontario: Queen's University, Department of Economics Working Paper No. 515.

Bhargava, A. 1986. On the theory of testing for unit roots in observed time series. *Review of Economic Studies* 53: 369–84.

Bureau of Labor Statistics. 1991. Erie MSA: Employment by industry benchmark updated. Unpublished data.

Cargill, T. F., and S. A. Morus. 1988. A vector autoregression model of the Nevada economy. *Economic Review (Federal Reserve Bank of San Francisco)* 1: 21–32.

Citibank. 1991. CITIBASE: Citibank economic database, 1946–91. New York: Citibank North America.

Corman, H., T. Joyce, and N. Lovitch. 1987. Crime, deterrence and the business cycle in New York City: A VAR approach. *Review of Economics and Statistics* 69: 695–700.

Cromwell, J. B. 1991. Some technical notes on stationarity and hypothesis testing with impulse responses. Unpublished manuscript.

Cromwell, J. B., and M. J. Hannan. 1988. Regional econometric modeling using Bayesian vector autoregressive techniques: The case of West Virginia. Unpublished manuscript.

Dickey, D. A., and W. A. Fuller. 1979. Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association* 74: 427–31.

Dickey, D. A., and W. A. Fuller. 1981. Likelihood ratio tests for autoregressive time series with a unit root. *Econometrica* 49: 1057–72.

Doan, T. A. 1988. *User's manual: RATS version 3.00.* Evanston, Ill.: VAR Econometrics, Inc.

Efron, B. 1982. *The jackknife, the bootstrap, and other resampling plans.* Philadelphia: Society for Industrial and Applied Mathematics.

Engle, R. F., and C. W. J. Granger. 1987. Co-integration and error-correction: Representation, estimation, and testing. *Econometrica* 55: 251–76.

Fountis, N. G., and D. A. Dickey. 1989. Testing for a unit root stationarity in multivariate autoregressive time series. *The Annals of Statistics* 17(1): 419–28.

Fuller, W. A. 1976. *Introduction to statistical time series.* New York: John Wiley and Sons.

Gruben, W. C., and W. T. Long. 1988. Forecasting the Texas economy: Applications and evaluation of a systematic multivariate time series model. *Economic Review (Federal Reserve Bank of Dallas)* January: 11–28.

Hannan, E. J. 1969. The identification of vector mixed autoregressive moving average systems. *Biometrika* 56: 223–25.

Hannan, E. J. 1970. *Multiple time series.* New York: John Wiley and Sons.

Hannan, E. J., and B. G. Quinn. 1979. The determination of the order of an autoregression. *Journal of the Royal Statistical Society, Series B* 41: 190–95.

Hoehn, J. G., W. C. Gruben, and T. R. Fomby. 1984. Time series models of the Texas economy. *Economic Review (Federal Reserve Bank of Dallas)* May: 11–23.

Isserman, A. 1980. Estimating export activity in a regional economy: A theoretical and empirical analysis of alternative methods. *International Regional Science Review* 5: 155–84.

Judge, G. G., W. E. Griffiths, R. Carter Hill, H. Lutkepohl, and T.-C. Lee. 1985. *The theory and practice of econometrics.* New York: John Wiley and Sons.

Kinal, T., and J. Ratner. 1986. A VAR forecasting model of a regional economy: Its construction and comparative accuracy. *International Regional Science Review* 10: 113–26.

Kloek, T., and H. K. Van Dijk. 1978. Bayesian estimates of equation system parameters: An application of integration by Monte Carlo. *Econometrica* 46: 1–20.

Koray, F., and W. D. Lastrapes. 1989. Real exchange rate volatility and U.S. bilateral trade: A VAR approach. *Review of Economics and Statistics* 71: 708–12.

LeSage, J. P. 1990a. Forecasting metropolitan employment using an export-base error-correction model. *Journal of Regional Science* 30: 307–23.

LeSage, J. P. 1990b. A comparison of forecasting ability of ECM and VAR models. *Review of Economics and Statistics* 72: 664–71.

LeSage, J. P., and M. Magura. 1986. Econometric modeling of interregional labor market linkages. *Journal of Regional Science* 26: 567–77.

LeSage, J. P., and J. D. Reed. 1989a. The dynamic relationship between export, local, and total area employment. *Regional Science and Urban Economics* 19: 615–36.

LeSage, J. P., and J. D. Reed. 1989b. Interregional wage transmission in an urban hierarchy: Tests using vector autoregressive models. *International Regional Science Review* 12: 305–18.

Lutkepohl, H. 1982. Differencing multiple time series: Another look at Canadian money and income data. *Journal of Time Series Analysis* 3: 235–43.

Lutkepohl, H. 1985. Comparison of criteria for estimating the order of a vector autoregressive process. *Journal of Time Series Analysis* 6: 35–52.

Lutkepohl, H. 1989. A note on the asymptotic distribution of impulse response functions of estimated VAR models with orthogonal residuals. *Journal of Econometrics* 42: 371–76.

Lutkepohl, H. 1990. Asymptotic distributions of impulse responses and forecast error variance decompositions of vector autoregressive models. *Review of Economics and Statistics* 72: 166–26.

MacKinnon, J. G. 1990. Critical values for cointegration tests. San Diego: University of California Working Paper.

McLeod, A. J., and W. K. Li. 1983. Diagnostic checking ARMA time series models using squared-residual autocorrelations. *Journal of Time Series Analysis* 4(4): 269–73.

Magura, M. 1990. Using input-output data in a Bayesian autoregressive forecasting model. In *New directions in regional analysis: Multi-regional approaches*, eds. L. Anselin and M. Madden. London: Belhaven Press.

Mjelde, J. W., and M. S. Paggi. 1989. An empirical analysis of interregional price linkages. *Journal of Regional Science* 29: 171–90.

Nelson, C. R., and C. I. Plosser. 1982. Trends and random walks in macroeconomic time series: Some evidence and implications. *Journal of Monetary Economics* 10: 139–62.

Parzen, E. 1974. Some recent advances in time series modeling. *IEEE Transactions on Automatic Control* 19: 723–30.

Priestley, M. B. 1980. State-dependent models: A general approach to nonlinear time series analysis. *Journal of Time Series Analysis* 1: 47–71.

Quenouille, M. H. 1957. *The analysis of multiple time-series*. London: Griffin.

Quinn, B. G. 1980. Order determination for a multivariate autoregression. *Journal of the Royal Statistical Society, Series B* 42: 182–85.

Raynauld, J. 1988. Canadian regional cycles: The Quebec-Ontario case revisited. *Canadian Journal of Economics* 21: 115–28.

Runkle, D. 1987. Vector autoregressions and reality. *Journal of Business and Economic Statistics* 5: 437–49.

Sargent, T. J., and C. A. Sims. 1977. Business cycle modeling without pretending to have too much a priori economic theory. In *New methods in business cycle research: Proceedings from a conference*. Minneapolis, Minn.: Federal Reserve Bank.

Schwarz, G. 1978. Estimating the dimension of a model. *The Annals of Statistics* 6: 461–64.

Sherwood-Call, C. 1988. Exploring the relationships between national and regional economic fluctuations. *Economic Review (Federal Reserve Bank of San Francisco)* Summer: 15–26.

Shibata, R. 1980. Selection of the order of the model for estimating parameters of a linear process. *The Annals of Statistics* 8: 147–64.

Sims, C. A. 1980. Macroeconomics and reality. *Econometrica* 48: 1–48.