Advanced Materials Research *Online: 2011-05-31*
ISSN: 1662-8985, Vols. 255-260, pp 797-800
doi:10.4028/www.scientific.net/AMR.255-260.797

Vertical Load-Carrying Natural Frequency of Railway Double-Track Pre-stressed Concrete Continuous Bridge

Jianying Ren[1,a], Wenping Li[2,b] ,Mubiao Su[3,a] and Qingyuan Zeng[1,a]

[1] College of Civil and Architectural Engineering, Central South University China.

[2] College of Civil Engineering, Shijiazhuang Tiedao University China.

[3] Structural Health Monitoring and Control Institute, Shijiazhuang Tiedao University China.

[a]aren1977@sina.com, [b]liwenping77@163.com

Keywords: railway double-track bridge, vehicle-bridge system, vertical load-carrying natural frequency, pre-stressed concrete continuous bridge

Abstract. The railway double-track continuous bridge vertical load-carrying frequency is theoretical derived with the vehicle-bridge system mode1. The vertical load-carrying frequencies of the 40 m+4×72 m+40 m party pre-stressed concrete continuous box-girder bridge are calculated, when two trains with 20 same high-speed passenger vehicles are traveling on the bridge from both ends. The change regularity of this double-track continuous bridge vertical load-carrying natural frequency with two trains on it is similar with that of the single-track simply supported bridge and this bridge as one train on it. This bridge vertical load-carrying frequency when two trains on it is smaller than that only train on it. The maximum deviation percentage between the vertical load-carrying natural frequency and the vertical natural frequency is 1.17837%.This bridge vertical load-carrying frequency can be replaced by its natural frequency, and there isn't obvious error. If the vertical load-carrying frequency should be calculated, it can be replaced by the average value.

Introduction

When the train is on the bridge, the structural natural frequency, which is defined by the bridge dead load and the live load on the bridge, is called *load-carrying natural frequency*. When the forced vibration frequency on the bridge is equal the bridge load-carrying natural frequency, there will be resonance [1]. And When the successive vehicles travelling on the bridge at speed of v, the vehicles (which length is L_v) wheel loads are equal to a dynamical periodical load with fundamental frequency v/L_v act on the beam. When the integer times of the frequency due to weight loads of a series of successive vehicles acting periodically on the bridge agree with the fundamental vertical load-carrying natural frequency, that is $jv/L_v=f_{1y}$ (j=1,2,3…), the sum of free-vibration response components generated by each moving load acting successively on the bridge will result in the resonance [2]. And the vehicle speed is higher, the resonant maximum amplitude is bigger too.

Based on the reference [3], the railway double-track continuous bridge vertical load-carrying frequency will be theoretical derived with the vehicle-bridge system mode1. The vertical load-carrying frequencies of the 40 m+4×72 m+40 m party pre-stressed concrete continuous box-girder bridge will be calculated, when two trains with 20 same high-speed passenger vehicles are traveling on the bridge from both ends.

Vertical Motion Equation of the Vehicle-Bridge System

The Vehicle-bridge System Model. When the train travels on the bridge from the left end, let the first and the last wheel-axle set on the bridge be represented by KL and ML at time t. And when the train travels on the bridge from the right end, let the first and the last wheel-axle set on the bridge be represented by KR and MR at time t. Vehicle and bridge interact at the same vertical plane (Fig. 1-2). Each vehicle is considered as a model of a rigid body with four wheel-axle sets, which has two degrees of freedom only in the vertical plane. The two degrees of freedom of the vehicle correspond to bounce and pitch motion (Fig. 3). The vehicle vertical two series suspension spring is considered as a linear spring, and the stiffness is substituted as an equivalent stiffness [4]. The mass of the redirector truss is averaged to the rigid body and wheel-axle sets. It is hinged between the vehicles. The vertical displacements of the wheel and the wheel/rail contact point on the rail are always considered uniform.

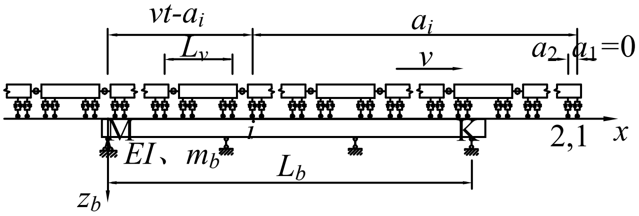

Fig.1 The vehicle-bridge model when the train travels on bridge from the left end

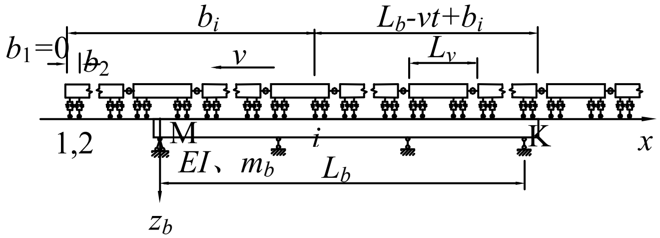

Fig.2 The vehicle-bridge model when the train travels on bridge from the right end

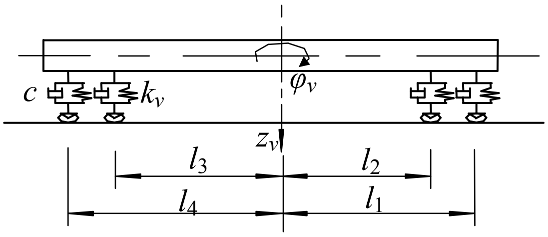

Fig. 3 The vehicle model

Motion Equations. Referring to Fig. 3, the vertical motion equations for each vehicle can be expressed as [2]:

$$\begin{cases} m_v \ddot{z}_v(t) + c_v \sum_{j=1}^{4} \dot{u}_j + k_v \sum_{j=1}^{4} u_j = 0 \\ J_v \ddot{\varphi}_v(t) + c_v \sum_{j=1}^{4} \dot{u}_j(\pm l_j) + k_v \sum_{j=1}^{4} u_j(\pm l_j) = 0 \end{cases} \tag{1}$$

Where u_j is a deformation of the vehicle jth axle suspension spring, z_v and φ_v are vertical and pitch displacements of the vehicle from the equilibrium position, c_v and k_v represent the spring damping and stiffness of the vehicle's suspension spring for each wheel-axle set, m_v and J_v are, respectively, the mass and pitch moment of inertia for each vehicle, l_j is the longitudinal distance between the centroid of the vehicle and its jth wheel-axle set (Fig. 3).

The motion equation for the bridge can be expressed as:

$$\begin{aligned} &\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 z_b}{\partial x^2}\right) + m_b\frac{\partial^2 z_b}{\partial t^2} + c_b\frac{\partial z_b}{\partial t} = \\ &\sum_{i=KL}^{ML}\{\delta[x-(vt-a_i)](P_i - m_s\frac{\partial^2 z_b}{\partial t^2} + c_v\dot{u}_i + k_v u_i)\} + \sum_{i=KR}^{MR}\{\delta[x-(L_b\text{-}vt+b_i)](P_i - m_s\frac{\partial^2 z_b}{\partial t^2} + c_v\dot{u}_i + k_v u_i)\} \end{aligned} \tag{2}$$

Where, EI, m_b, and c_b are the vertical flexural rigidity, unit length mass, and damping coefficient of the bridge, respectively, v is the train running speed, a_i and b_i are the longitudinal distance from the ith wheel-axle set to the first wheel-axle set of the train which travel on the bridge from left end and right end (Fig. 1-2), P_i is the static weight of the ith wheel-axle, m_s is the unsprung mass of the ith wheel-axle set, c_v and k_v are the damping and stiffness of the vehicle's suspension spring for the ith wheel-axle set, $\delta(x-\eta)$ is Dirac function, u_i is deformation of the vehicle ith wheel-axle set suspension spring.

The vehicle-bridge system motion equation can be obtained by coupling Eq.1 and Eq.2. Using orthogonality of the vibration mode, the characteristic equation can be obtained:

$$\left|K - \omega^2 M\right| = 0 \tag{3}$$

Vertical Load-carrying Natural Frequency of Railway Double-track Pre-stressed Concrete Continuous Bridge

The Results. This paper calculated the continuous bridge vertical load-carrying natural frequency of the 40 m+4×72 m+40 m party pre-stressed concrete continuous box-girder bridge. This continuous bridge is a double-track 40 m+4×72 m+40 m party pre-stressed concrete continuous box-girder bridge in the Shimen to Changsha railway line. The maingirder is non-uniform continuous beam, the fixed support locate at the middle pier. The beam depth is 2.6 m at the end support point and the midspan, and the beam depth is 4.8 m at the middle support point. The beam cross section is single box and single room.Because it is only study the bridge vertical vibration but the bridge internal force in this paper, the bridge model uses the beam element to analyze. The non-uniform continuous beam is regarded as a echelon variational beam, the beam is divided into 128 beam elements along the bridge longitudinal direction. The semi-bridge model is shown in Fig. 4, the other part is symmetrical with this part.

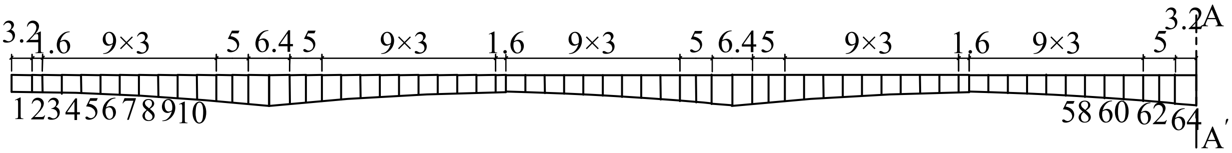

Fig. 4 The mode of 40 m+4×72 m+40 m party pre-stressed concrete box-girder bridge

There are two trains with 20 same high-speed passenger vehicles at the speed of 160 km/h travelling on the bridge from both ends of the bridge at same time. When the first wheel-axle set is on the bridge, it is start time. The variation curves with time of five (from 1 to 3) load-carrying natural frequencies are obtained. For the page limit, there only list two variation curves of the first and the second load-carrying natural frequencies (Fig.5-6).

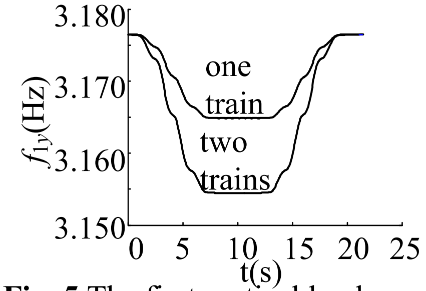

Fig. 5 The first vertical load-carrying natural frequency.

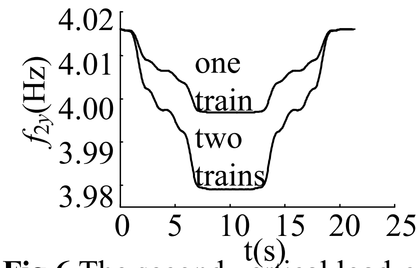

Fig.6 The second vertical load-carrying natural frequency .

Results Analysis. From the results, the conclusion is similar with the conclusions of the reference [3] and [5]. There are three stages in the time-varying curve of the vertical load-carrying natural frequency during the train traveling on the bridge. Firstly, as soon as the trains run on the bridge from the two ends, the bridge vertical load-carrying natural frequency begins to gradually decline. Secondly, when the vehicles are distributed on the whole bridge, the bridge vertical load-carrying natural frequency is periodically varied. Thirdly, when the vehicles begin to off the bridge, the bridge vertical load-carrying natural frequency begins to gradually increase, until the train completely leaves the bridge, the bridge vertical load-carrying natural frequency gradually returns to the natural frequency (Fig. 5-6).

When the vehicles distribute on the bridge, the variation range of the vertical load-carrying natural frequency of the bridge is small, the maximum D-value is 0.00420 Hz<0.1 Hz. And then, this bridge vertical load-carrying natural frequency can be replaced by the average value.

Table 1 The vertical load-carrying natural frequency of this railway double-track bridge

Frequency number	Natural frequency/Hz	Variation range /Hz	Maximum D-value /Hz	Medium value /Hz	δ /%
1	3.17646	3.15443~3.15446	0.00003	3.15444	-0.69367
2	4.01599	3.97913~3.97916	0.00004	3.97915	-0.91786
3	5.12380	5.06936~5.06949	0.00013	5.06942	-1.06253

where: $\delta = \frac{\text{the minimum vertical load - carrying natural frequency - the vertical natural frequency}}{\text{the vertical natural frequency}} \times 100\%$

This double-track continuous bridge five vertical load-carrying natural frequencies are shown in Table 1. From Table 1 and Fig. 5-6, the D-value between the vertical load-carrying natural frequency and the vertical natural frequency, when two trains on bridge, is larger than that when one train on bridge, the maximum deviation percentage as two train on bridge is 1.17837%, and the maximum deviation percentage as one train on bridge is 0.60174% [3].

Conclusions

1 The variation regular of this double-track pre-stressed concrete continuous bridge vertical load-carrying natural frequency as two trains on it is similar with that of the simply supported bridge[5] and this bridge as one train on it [3].

2 The minimum deviation percentage of the five vertical load-carrying natural frequencies as two trains on this bridge is 0.69367%, the maximum deviation percentage is 1.17837%. They are all less than 5%. In practical use, this bridge vertical load-carrying frequency can be replaced by its natural frequency, and there isn't obvious error.

3 When two trains are distributed on the bridge, the vertical load-carrying natural frequencies periodically vary, but the variation amplitude is small, the maximum is only 0.00420Hz. Consequently, if the vertical load-carrying natural frequencies of this pre-stressed concrete continuous bridge should be calculated, it can be replaced by the average value.

4 The analysis results may provide reference for multiple track railway bridge vertical load-carrying frequency and vibration response.

Acknowledgements

This work was financially supported by Technological Research and Development Programs of the Ministry of Railways (2007G028).

References

[1] Ministry of Railways of the People's Republic of China: Code for Rating Existing Railway Bridges(China railway publishing house, Beijing 2004) (in Chinese)

[2] Jianzhong Li and Mubiao Su: Journal of Sound and Vibration, Vol. 224(5)(1999), p.897

[3] Jianying Ren, Mubiao Su, Wenping Li and Binbo Liang: Journal of the China Railway Society, Vol. 27(5)(2005), p.111 (in Chinese)

[4] Futian Wang: Vehicle dynamics (China railway publishing house, Beijing 1983) (in Chinese)

[5] Mubiao Su, Jianzhong Li and Zhiguang Liang: Journal of the China Railway Society,Vol. 23(2)(2001), p.76 (in Chinese)