D. PAUL MEHTA

Department of Mechanical Engineering
Bradley University
Peoria, Illinois 61625
U.S.A.

EFFECTS OF THERMAL INSULATION ON THE DYNAMICS OF AN OCCUPIED SPACE IN A BUILDING SYSTEM

Manuscript received September 19, 1981

ABSTRACT: A mathematical model which describes the dynamic interactions of thermal insulation with the other components in a building system is presented in this paper.

INTRODUCTION

IN THE PAST eight years much international activity has been directed towards the energy problem. In the United States, a large proportion of this activity has been focused on energy conservation in buildings. Buildings were identified as prime candidates for saving energy as early as the oil embargo of 1973. Of the various energy saving strategies developed for buildings, thermal insulation has found greatest acceptance mainly because it is readily available at a low cost.

Unfortunately, the developments and applications of thermal insulation have occurred without much consideration to other factors such as its effects on the building, HVAC (heating, ventilating, and air-conditioning) system, control strategy, weather, and thermal comfort. In reality, the insulated envelope interacts dynamically with all the components in a building system affecting the dynamic characteristics of the total building system. It is very important to understand the mechanisms of these interactions because this knowledge can be used to produce a desirable influence on other energy saving strategies.

The intent of this paper is to report on an analytical model which describes the dynamic interactions of thermal insulation with the other components in a building system. It is shown that the model can be applied to predict the effects of thermal insulation on the dynamics of the building system, and to study their influence on other energy saving strategies.

Journal of THERMAL INSULATION, Vol. 5 (October 1981), p. 78
00148-8287/81/02 0078—15 $04.50/0

BACKGROUND

Interest in dynamic thermal performance of materials can be traced back to 1822 when Fourier explained the basic mathematics for transient heat transfer in solids [1]. Applications of analytical techniques to predict transient thermal response of building envelopes began in 1944 [2] and for the next 25 years research efforts in this area were of academic interest only [3—6]. With the advent of high speed digital computers, efforts were initiated to analyze actual building structures for dynamic performance. In 1967, Mitalas and Stephenson initiated the response factor method [7—8]. In 1969, Kusuda extended the calculations of response factors to multilayer structures of various curvatures [9]. Stephenson and Mitalas further developed the response factor by using Z-transforms for the inversion [10].

A few people have directed their research efforts to establishing dynamic responses of the heating, ventilating and air-conditioning components and their control systems. Zelenski, Lund, Harrison, and Sowls investigated a closed-loop system for a duct air temperature control loop [11]. Room and envelope dynamics were excluded from this loop. Dynamics of the heat exchanger, valve, actuator, controller and transducer were lumped together in the forward block, and it was concluded that this block could be approximated by a dead time and a time constant. It was further reported that this system which contained one slow component exhibited a first-order transient response corresponding to the dominant time constant. The fast elements simply introduced a delay or a dead time. Stoecker et al have also studied the dynamic characteristics of an air temperature control loop to find the limiting value of gain of the loop for stability [12]. This effort also excluded the room and envelope dynamics. All efforts of this kind regarding dynamic performances of HVAC components and control systems have been concerned mainly with finding the limiting gain of the loop to keep the system stable while achieving minimum steady-state deviation or offset.

Following the oil embargo of 1973, studies of the dynamics of air temperature control loops included energy consumption considerations. Hamilton, Leonard and Pearson investigated the dynamic responses near full and part heating loads of a discharge air temperature control system [13]. It was emphasized that design criteria based on the dynamics of the components of the loop and envelope should be developed to aid the designers in selecting components for HVAC systems.

Escalating fuel costs within the past 3—5 years have resulted in a greater emphasis on insulating materials which has led to an increased interest in their thermal performance. Many factors have been identified which have an effect upon thermal performance of insulations: compaction, thickness, humidity, temperature, and moisture/air barriers [14]. In general, the properties of generic insulating materials have been sufficiently established under static (steady state) conditions. Additional work is necessary to evaluate these materials under dynamic (transient) conditions.

Properties of insulating materials have usually been presented without consideration for the remainder of the building and there is lack of sufficient experimental verification of the predicted performance of the building systems as they are affected by the insulating materials [15]. Very few research efforts, if any, have been devoted to developing models that describe dynamic interactions of insulating materials with the rest of the building system and which can predict the effects of insulation interactions on other energy saving strategies. The purpose of this paper is to develop a mathematical model which describes the interactions and predicts the effects of thermal insulation on the dynamics of the building system.

ANALYTICAL MODEL

The thermal processes which take place in an occupied space of a building are shown in Figure 1. The following assumptions are made.

1. The properties of the thermal process involved in heating the occupied space are those at the location at which the occupants experience the controlled conditions. This assumption permits us to describe the process with ordinary linear differential equations instead of partial differential equations.
2. Changes in the stored energy of space air are due only to changes in dry-bulb temperature; those due to humidity changes are considered negligible.
3. Heat transfer by radiation is absent.
4. The air in the occupied space is uniformly mixed.
5. Infiltration losses are considered to be a function of supply air flow rate, occupant acivity, internal heat sources, room temperature, and construction details in a given occupied space.

From an energy balance on the occupied space (Figure 1): Rate of energy in = Rate of energy out + Rate of change in stored energy, or

$$\dot{Q}_f + \dot{m}_a c_{pa} T_i + \dot{Q}_{occ} + \dot{Q}_{int} = \dot{m}_a c_{pa} T + \dot{Q}_{cond} + \dot{Q}_{inf} + M_a c_{pa} \frac{dT^*}{dt} \quad (1)$$

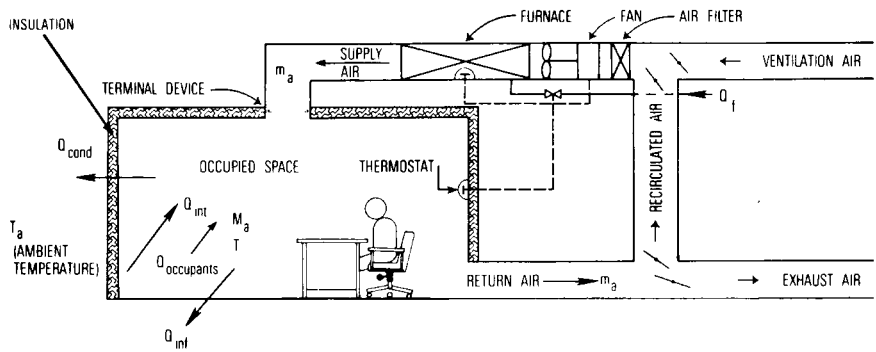

Figure 1. Schematic representation of a building system.

If we define,

$$\dot{Q} = \dot{Q}_f + \dot{Q}_{occ} + \dot{Q}_{int} \quad (2)$$

and

$$\dot{Q}_{cond} = \frac{1}{(1/hi + \sum y/k_y + \frac{x_{insu}}{k_{insu}} + 1/ho)} \cdot A\,(T - T_a) = U\,(T - T_a) \quad (3)$$

then we may rewrite Eq 1 as:

$$\frac{dT}{dt} = \frac{\dot{Q}}{M_a c_{pa}} + \frac{\dot{m}_a c_{pa}}{M_a c_{pa}} (T_i - T) - \frac{UA(T - T_a)}{M_a c_{pa}} - \frac{\dot{Q}_{inf}}{M_a c_{pa}} \quad (4)$$

For most applications, relatively small fluctuations in net heat gains and space temperature will be experienced within the occupied space. For this reason, only small perturbations about the operating or set point will be considered. The operating point may be found by setting the time derivative in Eq. 4 to zero:

*See list of nomenclature at the end of this article.

$$\frac{\bar{\dot{Q}}^*}{M_a c_{pa}} + \frac{\dot{m}_a c_{pa}}{M_a c_{pa}}(\bar{T}_i - \bar{T}) - \frac{UA(\bar{T}-\bar{T}_a)}{M_a c_{pa}} - \frac{\bar{\dot{Q}}_{inf}}{M_a c_{pa}} = 0 \tag{5}$$

Eqs. 4 and 5 may be combined to yield an equation in terms of perturbations about the operating point:

$$\frac{d(T - \bar{T})}{dt} = \frac{\dot{Q} - \bar{\dot{Q}}}{M_a c_{pa}} + \frac{\dot{m}_a c_{pa}}{M_a c_{pa}}[(T_i - \bar{T}_i) - (T - \bar{T})] - \frac{UA}{M_a c_{pa}}[(T - \bar{T}) - (T_a - \bar{T}_a)] - \frac{(\dot{Q}_{inf} - \bar{\dot{Q}}_{inf})}{M_a c_{pa}} \tag{6}$$

Eq. 6 may be rewritten as:

$$\frac{d\Delta T}{dt} = \frac{\Delta \dot{Q}}{M_a c_{pa}} + \frac{\dot{m}_a c_{pa}}{M_a c_{pa}}(\Delta T_i - \Delta T) \frac{UA}{M_a c_{pa}}(\Delta T - \Delta T_a) - \frac{\Delta \dot{Q}_{inf}}{M_a c_{pa}} \tag{7}$$

From assumption 5, we may write

$$\dot{Q}_{inf} = F(\dot{Q}, T) \tag{8}$$

Linearizing Eq. 8 about the operating point yields:

$$\Delta \dot{Q}_{inf} = K_1 \Delta \dot{Q} + K_2 \Delta T \tag{9}$$

The constants, K_1 and K_2, are defined as follows and are evaluated at the operating point:

$$K_1 = \left.\frac{\partial \dot{Q}_{inf}}{\partial \dot{Q}}\right|_T \tag{10}$$

$$K_2 = \left.\frac{\partial \dot{Q}_{inf}}{\partial T}\right|_{\dot{Q}} \tag{11}$$

*Bar (—) notations indicate values at the operating point.

Substituting Eq. 9 into Eq. 7, taking the Laplace transform, and rearranging, we obtain for step inputs:

$$\Delta T(s) = \frac{K_{\dot{Q}}\,\Delta\dot{Q}(s)}{S(1 + \tau_{os}S)} + \frac{K_{Ti}\,\Delta T_i(s)}{S(1 + \tau_{os}S)} + \frac{K_{Ta}\,\Delta T_a(s)}{S(1 + \tau_{os}S)} \qquad (12)$$

where:

$$K_{\dot{Q}} = \frac{1 - K_1}{\dot{m}_a c_{pa} + UA + K_2} \qquad (13)$$

$$K_{Ti} = \frac{\dot{m}_a c_{pa}}{\dot{m}_a c_{pa} + UA + K_2} \qquad (14)$$

$$K_{Ta} = \frac{UA}{\dot{m}_a c_{pa} + UA + K_2} \qquad (15)$$

$$\tau_{os} = \frac{M_a c_{pa}}{m_a c_{pa} + UA + K_2} \qquad (16)$$

DYNAMIC CHARACTERISTICS

The dynamics of systems are best described in terms of two essential traits: stability and response. Stability is the measure of the system's inherent tendency to be self-restoring. This means that a stable system, when acted upon by some input, eventually assumes a finite output. The output of an unstable system eventually becomes infinite in magnitude. The response feature of dynamic performance indicates how quickly a finite output is achieved.

Eq. 12 describes the dynamic performance of the occupied space of the building system shown in Figure 1. This equation predicts the dynamic changes in the ouput (space temperature) of the system when different inputs, viz, heat transfer from the furnace, temperature of the adjacent space, and weather (ambient temperature) are changed. $K_{\dot{Q}}$, K_{Ti} and K_{Ta} are respectively the gains of the system relative to the

above input variables and they govern the stability characteristics of the system. The time constant τ_{os} determines the response feature of the occupied space.

It can be seen from Eqs. 13—16 that the stability characteristics (gains) and the response feature of the occupied space to different inputs are dependent upon the overall heat transfer coefficient, U, of the envelope which in turn is a function of the type and amount of applied thermal insulation (see Eq. 3). The dynamic parameters of the occupied space, described by Eqs. 13—16, include characteristics of the remainder of the building system such as mass of enclosed air, supply air flow rate in the HVAC system, and infiltration. Thus the response of the building system predicted from Eqs. 12—16 will include interactions of the thermal insulation with the rest of the system.

EFFECTS OF THERMAL INSULATION OF DYNAMIC CHARACTERISTICS

Table 1 shows the calculated values of the dynamic characteristics of an occupied space when one inch equivalent of different types of insulating materials are used. Figure 2 is a graphical presentation of the data given in Table 1.

Table 1. Dynamic Parameters of a Building System with One Inch of Different Insulating Materials at Supply Air Rate of 7104 Pounds/Hr

Insulating Material	Approx. R/Inch	Average R/Inch	U (Btu/hr°F ft²)	$K_Q^{\cdot} \times 10^{-4}$	K_{Ti}	K_{Ta}	τ_{os} (hr)
LOOSE FILL							
Fiberglass Mineral Wool	3.0-3.2	3.10	0.32	2.95	0.5527	0.4073	0.6469
Cellulose	3.1-3.7	3.40	0.29	3.07	0.5747	0.3838	0.6725
Vermiculite	2.2	2.20	0.45	2.53	0.4743	0.4914	0.5550
BATTS OR BLANKETS							
Fiberglass Mineral Wool	3.2	3.20	0.31	2.98	0.5599	0.3996	0.6552
RIGID BOARD							
Polystyrene beadboard	3.6	3.60	0.28	3.10	0.5824	0.3755	0.6816
Extruded polystyrene	5.3	5.30	0.19	3.53	0.6623	0.2896	0.7751
Urethane	6.2	6.20	0.16	3.70	0.6941	0.2557	0.8123
Fiberglass	4.0	4.00	0.25	3.23	0.6068	0.3493	0.7101
Polyisocyanurate	8.0	8.00	0.13	3.89	0.7290	0.2182	0.8532
FOAM							
Urea formaldehyde	4.8	4.80	0.21	3.43	0.6427	0.3108	0.7522
Polyurethane	6.2	6.20	0.16	3.70	0.6149	0.2557	0.8123

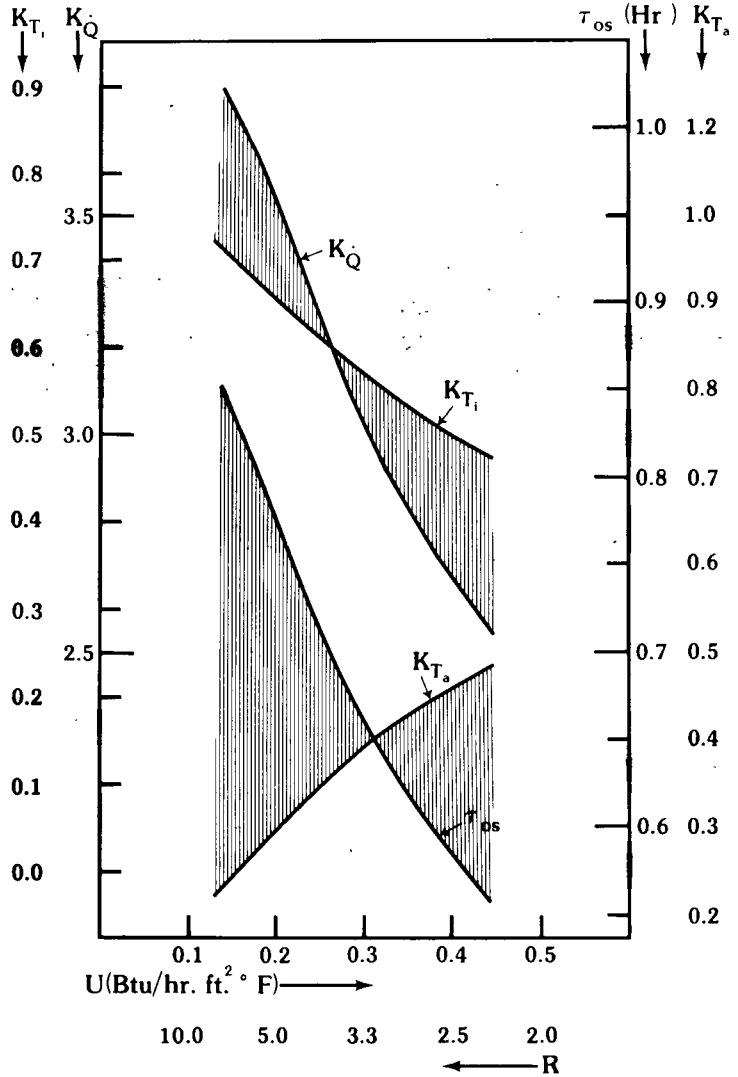

Figure 2. Effects of insulation on dynamic characteristics.

Eqs. 13—16 have been used to calculate the values given in Table 1. Building System parameters (enclosed air mass, supply air flow rate, envelope area etc.) of the Iowa State University (ISU) Energy Research House were used. The constants K_1 and K_2, which are defined by Eqs. 10—11 and are used in Eqs. 13—16, were determined experimentally at

the ISU Energy Research House according to the following procedures:

Procedures for K_1

1. The ISU Energy Research House was instrumented and metered to record weather data, indoor temperatures, and energy consumed by the different components of the HVAC system. Data points were scanned and recorded by a programmable data logger.
2. The HVAC system was turned on and was allowed to cycle automatically. A record of the system's ON and OFF periods was maintained. From the total ON period, for which the space temperature was kept constant, energy transferred to the space by the system was measured.
3. Simultaneous measurements on infiltration rates were made using a nondispersive infrared (NDIR) gas analyzer to monitor the concentrations of methane which was charged into the house. The output of the gas analyzer was recorded on a strip chart recorder. The time constant of the indoor air leakage process due to infiltration was calculated from the slope of the methane concentration decay record (see Figure 3). The number of air changes per hour, and hence $\dot{m}_{ainf}$, were calculated from the values of the time constant for infiltration and volume of indoor air in the house.
4. Eq. 10 was used to calculate the value of K_1 using the energy transferred to the space and energy lost by infiltration at constant space temperature.

Procedures for K_2

1. Eq. 11 was modified by substituting:

$$\dot{Q}_{inf} = \dot{m}_{ainf}\, c_{pa}\, (T - T_a) \tag{17}$$

Therefore,

$$K_2 = \frac{d}{dT}\, [\dot{m}_{ainf}\, c_{pa}\, (T - T_a)] \quad \dot{Q} = \text{constant}$$

$$\text{or } K_2 = [\dot{m}_{ainf}\, c_{pa} - m_{ainf}\, c_{pa}\, \frac{dTa}{dT}\,] \quad \dot{Q} = \text{constant} \tag{18}$$

2. Recorded ambient temperatures and average space temperatures were plotted for a period when the HVAC system was off. The slope

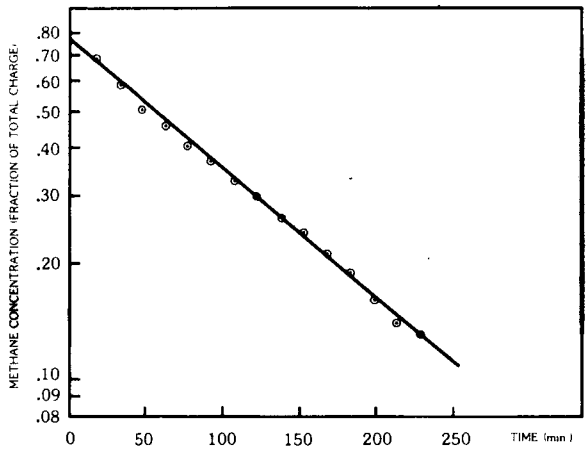

Figure 3. Methane concentration vs. time at ISU Energy Research House.

of the plot was used to determine dTa/dT.

3. Simultaneous measurements on infiltration rates were made using the tracer gas techniques described above.
4. Eq. 18 was used to calculate K_2.

EFFECTS OF INSULATION ON ENERGY SAVING STRATEGIES

For an energy efficient design of a building, it is desirable

- To minimize the value of K_{Ta}
- To maximize the value of $K_{\dot{Q}}$
- To optimize the value of K_{Ti}

It can be seen from Figure 2 that adding insulation to a building envelope decreases the value of K_{Ta} and increases the value of $K_{\dot{Q}}$. These two effects of thermal insulation reduce the energy consumption in operating the building. The effects of insulation on K_{Ti} and on energy consumption follow the same patterns as $K\dot{Q}$.

The occupant's thermal comfort considerations put a limit on the maximum value for the time constant τ_{os}. This maximum value depends upon the amplitude and frequency of the space temperature drifts which can be acceptable to the occupant without discomfort. Temperature drifts in built environments have been reported as a mode of energy conservation [16—18]. Although the effect of these drifts on the occupant's acceptability are not very clear at present [19,20], Griffith and McIntyre have recommended a maximum rate of temperature

change at 0.75°C/hr with a maximum deviation of 2.25°C from the mean comfort temperature [21]. Thus a maximum value of 1.9 hrs for τ_{os} is suggested when such energy saving strategies are planned.

It can be seen from Figure 2 that addition of insulation to a building envelope increases its time constant. Thus it appears that thermal comfort considerations under certain conditions discussed above put a limit on the amount of thermal insulation that can be applied and which could be insufficient with respect to other criteria. Applications of the analytical model to study the interactions of thermal insulation with the other parameters of the building system show that design optimization can be accomplished. To illustrate this point, Figure 4 has been constructed using Eqs. 14—16, and it shows the effects of air supply rates on the dynamics of the building system. The following procedures are suggested to use Figures 2 and 4 for an optimal design which can have desirable effects of insulation on comfort and on other energy saving strategies.

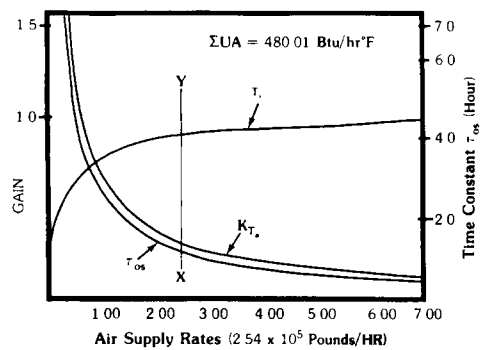

Figure 4. Calculated effects of air supply rates on the dynamics of occupied space.

1. Select the amount of thermal insulation so that the calculated value of τ_{os}, from Eq. 16 or from an extended version of Figure 2, lies to the left of the line XY in Figure 4.
2. Select supply air temperature, T_s, for a certain design load and a desired control temperature, T_c, so that $\dot{m}_a$ calculated from the equation

$$\dot{Q}_L = \dot{m}_a C_{pa} (T_s - T_c) \qquad (19)$$

lies to the right of line XY in Figure 4. This step will prevent the selection of a value of K_{Ta}, which is too high and will result in an optimum combination of ΣUA, m_a and K_2.

3. Determine the outlet air velocity for the proposed diffuser from the equation:

$$\dot{m}_a = A_e V_o \rho \tag{20}$$

4. Calculate V_x, the local velocity at the location of the occupants, for a specified occupied space temperature, T, using

$$\frac{X}{A_e} = K\frac{V_o}{V_x} = \frac{KQ_o}{V_x A_e} \tag{21}$$

where K is the constant of proportionality and can be obtained from manufacturer's data.

5. Calculate the effective draft temperature, θ, from

$$\theta = (T - T_c) - 7.66\,(V_x - 0.15) \tag{22}$$

6. Check that the values of θ lie between (-1.67) to $(+1.11)$ when velocity is in m/sec and temperatures are in °K.
7. If the calculated values in step 5 do not fall within the limits stated in step 6, choose a new effective area for the diffuser and repeat steps 2–5.

A design procedure based on steps 1–7 should coordinate the effects of thermal insulation, dynamic performance, whole body and local comfort conditions, and will allow the implementation of energy efficient strategies of the kind discussed above.

CONCLUSIONS

A mathematical model which describes the dynamic interactions of thermal insulation with the other components in a building system has been presented in this paper. The model has been applied to study the effects of thermal insulation on dynamic parameters. Procedures to produce desirable effects of insulation on some other energy saving strategies have been discussed.

ACKNOWLEDGMENTS

Some of the concepts reported in this paper are the result of studies

funded through the National Science Foundation. Special thanks are due to Dr. Max A. Wessler, Chairman, Mechanical Engineering Department at Bradley University, for his encouragement, support, and guidance. Appreciation is also extended to Mrs. Annelle Fletcher for her interest and efforts in typing this paper. Support from the staff at the Audio Visual Center of Bradley University in preparing the illustrations is gratefully acknowledged.

NOMENCLATURE

A	area: (m^2) or (ft^2)
K	throw constant, dimensionless; thermal conductivity: (J/hr.m.°C) or (Btu/hr.ft.°F)
L	length: (m) or (ft)
M	mass: (kg) or (lbm)
Q	energy: (J) or (Btu); volume flow rate (m^3/s)
T	air temperature: (°C) or (°F)
U	overall heat transfer coefficient: ($J/hr.m^2.°C$) or ($Btu/h.ft^2.°F$)
V	air velocity; (m/s) volume: (m^3) or (ft^3)
X	distance from outlet to occupant: (m) or (ft)
c	specific heat: (J/kg.°K) or (Btu/lbm.°F)
k	thermal conductivity: (J/h.m.°K) or (Btu/hr.ft.°F)
m	mass: (kg) or (lbm)
t	time: (hr) or (s)
x	thickness: (m) or (ft)
y	thickness: (m) or (ft)

Symbols and Subscripts

K	gain, proportionality constant
L	load
X	local, at the occupant's location
a	air, ambient
c	control
Cond.	Conduction
e	effective, outlet
i	inlet
inf	infiltration
insu	insulation
int	internal; integral

m	meter
o	outlet; outside
occ	occupant
os	occupied space
s	sensor, supply
x	local, at the occupant's position

Greek Symbols

Δ	perturbation of a variable
θ	effective draft temperature (°C) or (°F)
ρ	density: (kg/m³) or (lbm/ft³)
τ	time constant: (hr)

REFERENCES

1. J. B. Fourier, *Theorie Analytique de la Choleur.* Paris: Gauthier Villars, 1822.
2. C. O. Mackey and L. T. Wright, "Periodic Heat Flow-Homogeneous Walls or Roofs." *ASHVE Trans.* 50 (1944): 293.
3. C. O. Mackey and L. T. Wright, "Periodic Heat-Flow Composite Walls or Roofs." *ASHVE Trans.* 52 (1946): 283.
4. L. A. Pipes, "Matrix Analysis of Heat Transfer Problems." *Franklin Institute Journal* 263 (1957): 195.
5. I. R. Hill, *A Method of Computing the Transient Temperature of Thick Walls from Arbitrary Variation of Adiabatic-Wall Temperature and Heat Transfer Coefficient.* National Advisory Committee for Aeronautics, Tech. Note 4105, 1957.
6. R. W. Muncey, "Thermal Response of a Building to Sudden Change of Temperature or Heat Flow." *Australian Journal of Applied Science* 14 (1963): 123.
7. G. P. Mitalas and D. G. Stephenson, "Room Thermal Response Factors," *ASHRAE Trans.* 73 (1967): III-2.1.
8. D. G. Stephenson and G. P. Mitalas, "Cooling Load Calculations by Thermal Response Factor Method." *ASHRAE Trans.* 73 (1967): III-1.1.
9. T. Kusuda, "Thermal Response Factors for Multi-Layer Structures of Various Heat Conduction Systems." *ASHRAE Trans.* 75 (1969): 246.
10. D. G. Stephenson and G. P. Mitalas, "Calculation of Heat Conduction Transfer Functions for Multi-Layer Slabs." *ASHRAE Trans.* 77 (1971): 130.
11. R. E. Zelenski, R. A. Lund, H. L. Harrison and R. E. Sowls, "An Investigation of a Closed Loop System for Duct Air Temperature Control." *ASHRAE Trans.* 75 (1968): VI-1.1.

12. W. F. Stoecker, L. A. Rosario, M. E. Hadenreich and I. R. Phelan, "Stability of an Air-Temperature Control Loop." *ASHRAE Trans.* 84 (1978): 35.
13. D. C. Hamilton, R. G. Leonard and J. T. Pearson, "Dynamic Response Characteristics of a Discharge Air Temperature Control System at Near Full and Part Heating Load." *ASHRAE Trans.* 80 (1974): 181.
14. U.S. Department of Energy. *An Assessment of Thermal Insulation Materials and Systems for Building Applications.* BNL-50862. UC-95d. Prepared by Brookhaven National Laboratory. June, 1978.
15. D. Paul Mehta and J. E. Woods, "An Experimental Validation of a Rational Model for Dynamic Responses of Buildings," *ASHRAE Trans.* 86 (1980): 497.
16. D. Paul Mehta, "Dynamic Thermal Responses of Buildings and Systems." Ph.D. Dissertation. Iowa State University, Ames, Iowa, 1980.
17. L. G. Berglund and R. R. Gonzalez, "Application of Acceptable Temperature Drifts to Built Environments as a Mode of Energy Conservation." *ASHRAE Trans.* 84 (1978): 110.
18. R. R. Gonzalez and L. G. Berglund, "Efficacy of Temperature and Humidity Ramps in Energy Conservation." *ASHRAE Journal* 21 (1979): 34.
19. C. H. Sprague and P. E. McNall, "The Effects of Fluctuating Temperature and Relative Humidity on Thermal Sensation (Thermal Comfort) of Sedentary Subjects." *ASHRAE Trans.* 76 (1970): 34.
20. D. P. Wyon, I. Anderson, and G. R. Lundquist, "Spontaneous Magnitude Estimation of Thermal Discomfort During Changes in Ambient Temperatures." J. Hygiene (Cambridge) 70 (1972): 203.
21. I. D. Griffiths, and D. A. McIntyre, "Sensitivity to Temporal Conditions *Ergonomics* 17 (1974): 499.