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Non-iterative eigenstructure assignment based finite element model updating of a Mindlin–Reissner plate in Duncan form of state space using ambient vibration respnose

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ABSTRACT

Eigenstructure assignment (ESA) based model updating is a control based technique for systematic calibration of finite element models using measured response from real structure. Application of this technique in physical space restricts simultaneous updating of stiffness and damping matrices of any mechanical system. On the other hand ESA when used in state space domain demands state space eigenstructure to be identified which is a challenging job. It is not certain that the identified state space eigenstructure will be in the same order and orientation as desired by the ESA algorithm. In this paper we used Duncan form of state space model of the mechanical system so that assignable eigenstructure in this form can be easily constructed using modal properties of the system in its physical space and thus problems regarding orientation is avoided. To achieve compatibility between assignable state space eigenstructure and state space model the later has been reduced using structural equivalent reduction expansion program (SEREP). Assignable eigenstructure is then used along with ESA algorithm given by B.C. Moore to update the reduced primary model of the mechanical system to simultaneously update the stiffness and damping matrices. Proposed algorithm is tested on a Plate modeled using Mindlin-Reissner plate element and updated model demonstrated a good agreement with the desired result.

Keywords: Finite element model updating, Subspace identification, Eigenstructure assignment.

1. INTRODUCTION

Finite element models of real life structures fail to replicate the reality owing to Improper modeling approach and assumptions towards boundary condition, parameter values and model order. Systematic calibration of the primitive model is therefore required before using it as a reliable predictor model. This can be achieved by combined use of system identification and model updating. System identification which mostly considers the real system as a black box, tries to identify important characteristic features (modal properties, nonlinearity etc.) of the real system through which the system can be interpreted. This is done by constructing parametric or nonparametric models using little analytical sense. Thus resulting model may or may not be physically understandable. Model updating take this effort one step further by using these characteristic features to alter a primary model constructed taking physics of the system into consideration. Updated model therefore retains the physical

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significance while its response conforms to that of the real system. Finite element model updating therefore can be found as an interesting field of research especially in the fields of structural health monitoring for the past few decades. Existing methods for finite element model updating are mostly vibration based where modal properties which are basically physical space eigenstructure are used to update a model. Different optimization algorithms ranging from gradient or hessian based, sensitivity based, perturbation based to nature mimicking types (GA, PSO, Hybrid techniques) are tried by different researchers in this endeavor. Apart from the regular requirement of matching the modal properties of the FE model with the identified structure, conditions to qualify as a good FE model updating technique include ability to retain the exploitable properties (positive, symmetric, banded, and sparsity) of stiffness and mass matrix. Unfortunately most of the existing techniques suffer from problems regarding fulfillment of the second condition. Besides some follow sequential updating technique (first stiffness then mass or vice versa) and most don’t give much attention to update damping matrix. Besides these optimization algorithm has their own drawbacks including improper convergence, multiple possible solutions and computational expense.

Control theory based eigenstructure assignment is on the other hand is a good approach to update any finite element model updating. In this method desired eigenstructures (modal parameters) are embedded in to the system model so that updated model has the same eigenstructure as desired by the designer. Eigenvalue assignment or pole placement has always been an interesting field of research for control engineers. Generally pole placement techniques are used to control a system with minimum control effort possible. There are several pole placement techniques exist in literature. Arbitrary assignment of eigenvalues for a closed loop system has been discussed by Wonham [2]. B. C. Moore [3] was the first person to identify the flexibility offered by state feedback in multivariable systems beyond closed loop eigenvalue assignment. He further demonstrated in his paper that a specific number of elements of each eigenvector of a closed loop MIMO system can be freely assigned. Kautsky, et. al. [4], Srinathkumar [5] discussed robust pole assignment technique in linear time invariant system. Several other researchers (Sobel et. al. [6]) also developed algorithm to place eigenstructure for closed loop system.

Eigenstructure assignment for model updating is however relatively new field. Generally vibration data and modal properties have been used for model updating in most of the

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literature [7]. Quadratic Partial Eigenvalue Assignment and Partial Eigenstructure Assignment technique to update models are discussed by Datta [8]. J. Carvalho [9] showed how state estimates can be used to update an FE model using optimization techniques. However, symmetry and other exploitable properties of updated stiffness and damping matrix have always been a major concern. Several optimization techniques are used with different metaheuristics in different literature to maintain these properties

ESA for model updating thus has been extensively used in both physical and state space domain for aerospace and vehicular motion control where the objective has been to control the path of a moving body with minimum control effort possible. However these types of problems do not have any special kinds of structure while state space model owns a very specific structure which can be exploited to gain lots of other information about the health of the system. Thus use of ESA in motion control problems is characteristically different from the use in case of FEM updating. Use of ESA based FEM updating is although exists in the literature. However most of the work that has been performed in this effort is cast in physical space domain while state space domain offers a greater flexibility of simultaneous updating of stiffness and damping matrices rendering the updating method to be more practical. But updating a state space model of a mechanical system maintaining its basic exploitable structure has its own challenges. First of all this demands that the eigenstructure in state space domain needs to be identified from the real structure in proper order and orientation compatible with the primary model which is supposed to be updated. However it may happen that identified state matrix can be rotated by a transposition matrix and it is not certain that the identified state matrix will be in the same orientation as the system model. Being coordinate independent eigenvalues can be identified easily, but problem arises while identifying eigenvectors in assignable orientation. In order to avoid this problem a different form of state space modeling namely Duncan form, has been adopted in this paper. Eigenstructure of state matrix in Duncan form of state space model has clear relation with modal properties of the system in its physical space. Therefore instead of identifying state space eigenstructure, modal properties of the system have been identified. Using these modal properties desired eigenstructure for Duncan form of state space model has been reconstructed. This eigenstructure is then used as the desired eigenstructure to update the system state matrix using ESA method.

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2. THORY

2.1 Discrete time stochastic subspace identification

As proposed damage identification technique is finite element model updating based it starts with an identification step to extract the modal parameters of the system from its response. State space modeling is good approach to identify the system in this regard. Any n order differential equation of a system can be defined as 2n number of coupled first order equation, termed as state space form of the system. Considering a mechanical system in continuous time of mass M, stiffness K and damping D, state transposition matrix A_c, vector x_t and output matrix C_c can be defined as:

$$A_c = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}; \quad x_t = \begin{Bmatrix} q_t \\ \dot{q}_t \end{Bmatrix}; \quad C_c = \begin{bmatrix} C_d - C_a M^{-1} K \\ C_v - C_a M^{-1} D \end{bmatrix}^T \qquad 1$$

Where $y_t = C_a \ddot{q}_t + C_v \dot{q}_t + C_d q_t$ is the output vector and C_a, C_v, C_d are the output matrix for acceleration $\ddot{q}_t$, velocity $\dot{q}_t$ and displacement q_t in continuous time respectively. Using these terms system model in continuous time can be expressed as:

$$\begin{aligned} \dot{x}_t &= A_c x_t + w_t \\ y_t &= C_c x_t + v_t \end{aligned} \qquad 2$$

The model structure is considered here to be a stochastic with unknown input. w_t and v_t are process noise and measurement noise respectively. Discrete time stochastic subspace identification algorithm given by Vanoverschee & Demoor [10] has been used to identify this continuous system using sampling. This is a non-iterative approach of state space modeling. The discrete time state space model of the system can be written in state space form as:

$$\begin{aligned} x(k+1) &= Ax(k) + w(k) \\ y(k+1) &= Cx(k) + v(k) \end{aligned} \qquad 3$$

Where *A* is state transposition matrix, *C* is output matrix relating state vector to output, *x(k)* is the discrete time state vector at k^{th} time instant, *y(k)* is output vector or measurement terms. Using Kalman's [11] forward innovation technique the same system is described as:

$$x(k+1) = Ax(k) + K_g e(k)$$
$$y(k+1) = Cx(k) + e(k) \quad 4$$

Where $e(k)$ is the innovation vector and K_g is called Kalman gain matrix. Using stochastic subspace identification algorithm given by Vanoverschee and Demoore state transposition matrix *A*, output matrix *C* and gain matrix K_g can be easily identified using output signal. Here we used output or measurement vector *Y(k)* which is the time history of acceleration response obtained from sensors placed at appropriate location of the structure. Identified system is then transformed in to continuous system using zero-order-hold technique yielding new set of state and output matrices in continuous domain. Post multiplying eigenvector of this new state matrix in with output matrix yields array of mode shape coordinates of the system in physical space in the predefined sensor locations. Eigenvalues however, being insensitive towards orientation of the state matrix, can be identified easily from the state matrix. These identified physical space eigenstructure is then used to construct eigenstructure for the Duncan form of state space model described in the following section.

2.2 Duncan form

Duncan form was first given by Duncan in his paper [12] In this state space form the dynamics of the continuous system is described by the following equations:

$$R\,\dot{x}(t) + \overline{K}\,x(t) = F(t) \quad 5$$

$$\text{where } R = \begin{bmatrix} M & D \\ 0 & M \end{bmatrix};\ \overline{K} = \begin{bmatrix} 0 & K \\ -M & 0 \end{bmatrix};\ F(t) = \begin{bmatrix} 0 \\ f(t) \end{bmatrix} \quad 6$$

M, K, D are the system mass, stiffness and damping matrices. To obtain the homogeneous solution of this first order system we assume a solution of this form:

$$x(t) = e^{\alpha t}\{\phi\} \quad 7$$

Which gives a solution in the form: $\alpha R\{\phi\} + \overline{K}\{\phi\} = [0]$. This equation is manipulated as an eigenvalue problem of a matrix term *U* as:

$$U\{\phi\} = \frac{1}{\alpha}\{\phi\} \quad 8$$

Where $U=-\overline{K}^{-1}R=\begin{bmatrix} -K^{-1}M & K^{-1}D \\ 0 & I \end{bmatrix}$ and eigenvectors of this problem can be described using eigenvector of the system in its physical space as:

$$\left\{\Phi^n\right\}=\begin{bmatrix} \left\{\phi^n\right\} \\ \alpha_n\left\{\phi^n\right\} \end{bmatrix} \qquad 9$$

Where $\left\{\phi^n\right\}$ is the eigenvector in the physical space i.e. eigenvector of the quadratic pencil:

$$M\lambda^2+D\lambda+K=0 \qquad 10$$

In connection to the previous section it can be shown that eigenvectors of system described in Duncan form of state space i.e. Φ^n can easily be constructed in its desired orientation using eigenstructure of the system in its physical space i.e. ϕ^n and α_n which are actually mode shapes and natural frequencies. This approach has been tried in this paper. The eigenstructure in physical space identified using subspace identification algorithm described in previous section is used to construct desired eigenstructure for Duncan form. After updating is performed stiffness, mass and damping matrices are again extracted from the updated state matrix using following equation. However, one has to consider one of these matrices to be standard and unchanged even after updating. Mass matrix being most reliable in this regard has been considered to be standard in most other literatures dealing with these kind of simultaneous updating situation. We here adopt that same strategy to extract the other two system matrices using following equations:

$$\begin{aligned} A_{identified} &= \begin{bmatrix} A_{ul} & A_{ur} \\ A_{dl} & A_{dr} \end{bmatrix} \\ K_{identified} &= -MA_{dl}; \quad C_{identified} = -MA_{dr}; \end{aligned} \qquad 11$$

2.3 Eigenstructure assignment

Eigenstructure assignment is a control based technique to properly place desired eigenstructure in to a system. Transient response of a system is a function of its eigenstructure and to alter the system's transient response eigenstructure has to be altered. ESA uses feedback to calculate a controller or gain matrix to update the eigenstructure of the system. Consider dynamics of a system has been described as:

$$\begin{aligned} x(k+1) &= Ax(k)+Bu(k) \\ y(k) &= Cx(k) \end{aligned} \qquad 12$$

Where *x(k)* is the state vector at k^{th} instant, *A* is state transposition matrix, *B* is input matrix, *u(k)* is input vector, *C* is output matrix and *y(k)* is measured output vector. If a input sequence is selected in a such a way that $u(k) = -K_c x(k)$; then equation can be rewritten as:

$$x(k+1) = Ax(k) - BK_c x(k)$$

Which then can be manipulated as:

$$x(k+1) = (A - BK_c)x(k) = A'x(k)$$

This yields altogether a new system with transposition matrix A' which has the same eigenstructure as desired by the designer. This is termed as full state feedback where every state has been used. We here used algorithm given by B.C Moore [3] to assign desired eigenstructure in to the primary state matrix to update it in such a way that eigenstructure of the updated state matrix coincides with desired eigenstructure. The algorithm is described below.

Algorithm:

1. *Define* $S_\lambda = [\lambda I - A \quad B]$ *and partition its basis vector as:* $R_\lambda = [N_\lambda \quad M_\lambda]$ *so that* N_λ *has the same order as A, where* λ *is desired eigenvalue.*
2. *Define dynamics with desired eigenstructure* $[\lambda_i, v_i]$ *with an gain matrix* K_c *as:*

$$(A - BK_c)v_i = \lambda_i I v_i$$

3. *Compare these two equations:*

$$[\lambda_i I - A \quad B]\begin{Bmatrix} N_{\lambda_i} \\ M_{\lambda_i} \end{Bmatrix} = 0 \text{ and } [\lambda_i I - A \quad B]\begin{Bmatrix} v_i \\ K_c v_i \end{Bmatrix} = 0$$

This comparison signifies that N_{λ_i} *and* v_i *spans the same vector space whereas* M_{λ_i} *and* $K_c v_i$ *spans another vector space.*

3. Calculate z_i *that relates two vector spaces of* N_{λ_i} *and* v_i*; and also vector spaces of* M_{λ_i} *and* $K_c v_i$ *using this equation as:* $z_i = N_{\lambda_i}{}^{\dagger} v_i$ *where* $\dagger$ *symbolizes Moore-Penrose pseudo inverse.*

4. Calculate gain matrix as: $K_c = -M_\lambda z_i v_i^{-1}$

5. Update the state matrix as: $A' = (A - BK_c)$

In this method we virtually use an array of input vector which stabilizes the system (in this case match systems eigenstructure to the desired one). We here used an arbitrary B matrix which only ensures that $Rank([\lambda_i I - A \quad B]) \geq n$. Using this B matrix we follow the algorithm given by B. Moore to obtain a gain matrix K_c which updates the state matrix A to match its eigenstructure to the desired values.

3. FINITE ELEMENT MODEL UPDATING

To update the primary model firstly assignable eigenstructure needs to be identified from the response history of the real structure. This is done by state space identification of the real system using subspace identification algorithm as described in the previous section. Identified system state matrix and output matrix (*A* and *C*) is the converted into continuous time domain by using Zero order hold technique. Modal parameters are the extracted from these two matrices using following equations:

$$\begin{aligned} &Eigenstructue\ of\ \ A\ in\ state\ space = \{\alpha_n;\Phi_n\} \\ &Eigenvector\ \ in\ physical\ space\ \ \ \phi_n = C\Phi_n \end{aligned} \qquad 13$$

Thereafter using equation (9) assignable eigenstructure for the Duncan form of state space model is reconstructed. Primary FE model of the system is then reduced down to only measured degrees of freedom using Structural equivalent reduction expansion program (SEREP) algorithm to fulfill the order criteria of the assignable eigenstructure. ESA is then applied on the Duncan form of state space model constructed using reduced order system matrices. Updated stiffness and damping matrices are the identified using equation (11).

4. NUMERICAL VALIDATION

To validate the proposed method a numerical experiments have been performed on an aluminum plate. A finite element model of the plate is prepared with assumed parameter value listed in Table 1. This has been considered as the real system for which acceleration response history is simulated using Newmark-beta method with a sampling frequency 500Hz. To demonstrate the noise sensitivity of the proposed method numerically obtained time signal has been contaminated using 10% noise. The plate is excited with a white noise sequence of zero mean and unit standard deviation to replicate ambient vibration excitation condition which is obvious in large size structural system identification. Simulated time history signal is then put through the stochastic subspace algorithm to obtain eigenstructure of identified state matrix. In this process we developed state space model of different order and then using some human intervention based on practical constraints on the identified eigenvalues (e.g. practical value of damping i.e. negative and less than 20%, removing unstable eigenvalues) we collected only feasible eigenstructure of the identified state matrix. In the next step the eigenvalues of the identified state matrix and eigenvector of the system in physical space is extracted using equation (13). Desired eigenstructure for the Duncan form of state space model is then constructed from outcome of the last step using equation (9). A primary FE model is also prepared with a set of assumed elasticity values different from the original one and this model is considered as primary model (listed in table 1). The identified physical space eigenvector is having the same order as of the number of sensor points and its coordinate corresponds only to vertical degrees of freedom (DOF) (considering structure has been instrumented with accelerometer for vertical movement only). For that reason undamaged FE model of the system is reduced to the specified degrees of freedom by using SEREP algorithm so that identified eigenstructure can be used to update the undamaged model. Using this reduced stiffness, mass and damping matrices Duncan form representation for undamaged state of the system is constructed and updated using ESA method. Using equation (11) updated stiffness matrix are then extracted and compared with the initial stiffness matrix of the system. Figure 1lists the initial, target and updated frequencies of the finite element model which demonstrates very close conformation with the desired result. Figure 2 shows the MAC values for first five modes is very close to the desired value of 1. Thus we can conclude that the updated model is representing the real system better than the initial model.

Table 1: Assumed material properties for FE model

Structure	Aluminum plate
Elasticity	63 GPa
Poisson’s ratio	0.334
Dimension	0.25 m x 0.45m x 0.006 m
Density	2796.9 Kg/m^3
Element	Mindlin-Reissner Plate element
Boundary condition	Cantilever (Clamped-free-free-free)
Total elements	5x5=25
Assumed elasticity	70% of original i.e. 44.1 GPa

5. CONCLUSION

In this paper we tried using ESA based finite element model updating technique to update a primary model of a Mindlin-Reissner plate. We also envisaged robustness of the proposed method under presence of noise which enhances its applicability in the real field scenario. While existing methods uses gradient or hessian based or evolutionary algorithm based optimization technique to update FE models which are iterative in nature and most often leads to wrong result, proposed method is non-iterative in nature which updates the primary model in state space domain to embed desired state space eigenstructure in the updated FEM.

Table 2: Comparison of natural frequencies between initial, target and updated FE models for first five modes

Modes	Target FE	Initial FE	Updated FE
First Mode	23.67	19.80	23.43
Second Mode	90.52	75.73	89.52
Third mode	161.43	135.07	159.72
Fourth mode	319.43	267.26	316.03
Fifth mode	516.99	432.54	518.76

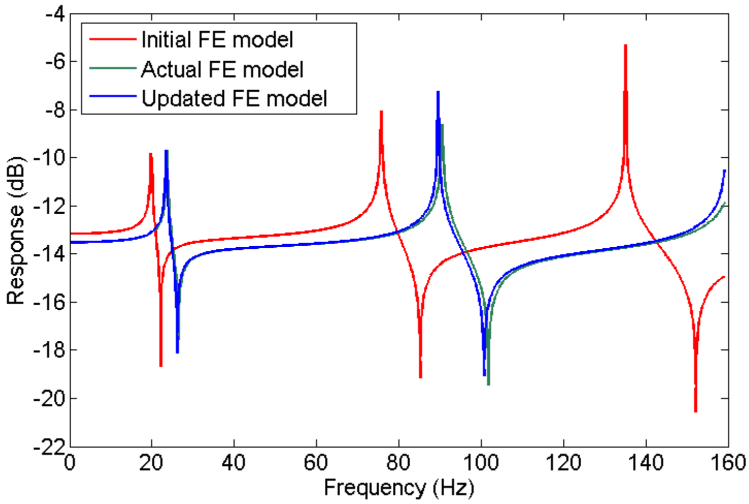

Figure 1: Frequency response function of initial, actual or target and updated FE models

Unlike other optimization based FEM updating technique this method uses linear algebra application to have a computationally inexpensive solution of the said problem. Because of its non-iterative nature and less computation requirement this method is also justified to be taken up as an online damage identification technique. In this paper we employed the proposed method in a Mindlin-Reissner Plate and for practicality white noise excitation has been used to simulate ambient vibration condition. Numerical experiments show that proposed method is capable of capturing damage features even in the presence of up to 10%

noise sufficiently. This same is also testified from numerical experimentation and results are satisfactorily close to actual scenario. The goodness of the method lies in the fact that it uses human intervention during state space identification of the damaged structure to separate real physical roots from the non-physical roots through the use of stabilization plots. These non-physical roots are attributed to either computational outcome or presence of noise in the signal. Removing these non-physical roots by human intervention leads to removal of noise influence as well as making the job of working with noisy signals less complicated. Thus this method combines linear algebra application to have a non-iterative solution along with human decision which makes the method more practical to be used as damage identification for real life structure.

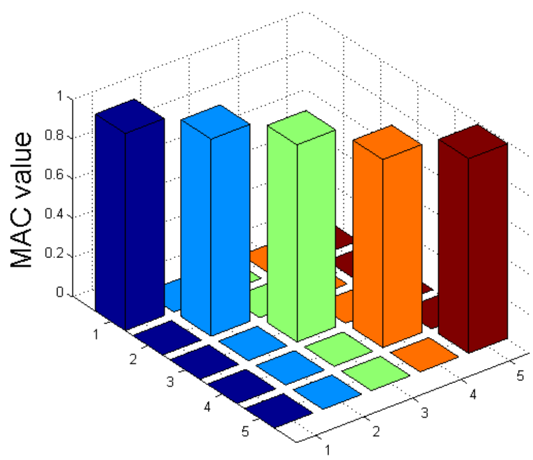

Figure 2: MAC value for first five modes

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