Courage to Capital?
A Model of the Effects of Rating Agencies on Sovereign Debt Roll–over

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Abstract

With the rise of international bond markets in the 1990s, the role of sovereign credit ratings has become increasingly important. In the aftermath of Asian Crises a series of empirical studies on the effects of sovereign ratings appeared. The theoretical literature on the topic, however, remains rather scarce. We propose a model of rating agencies that is an application of global game theory in which heterogeneous investors act strategically. The model is consistent with the main findings by the empirical literature. In particular, it is able to explain the independent effect of sovereign ratings on the cost of debt and the failure of rating agencies to predict crises. Our model also predicts that, in addition to affecting the level of debt roll–over, the mere existence of the rating agency's announcement can increase the magnitude of the response of capital flows to changes in fundamentals. The model also allows us to explore the reasons why agencies may over–react to crises and how they can spread financial contagion.

JEL classification: F34, G14, G15

Key words: credit rating, rating agency, sovereign debt, global game

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1 Introduction

> "By attaining a sovereign credit rating, your country will help reduce risk and encourage investment. A sovereign credit rating gives courage to capital." Secretary Colin L. Powell. Washington, DC. April 23, 2002.

Credit rating agencies have played an increasingly important role in international capital markets by gathering information that would not be available to investors and helping these investors assess the economic strength and likelihood of default of international borrowers. This is particularly true for sovereign borrowers: the number of countries that have received a credit rating increased from about a dozen in 1980 to about a hundred in 2002.[1] Wider use of credit ratings has likely increased sovereign governments' access to capital markets and improved their ability to raise funds.[2]

Investors appear to have found the information provided by rating agencies to be useful and have incorporated it into their investment decisions as indicated by movements in yields on sovereign bonds in response to changes in credit ratings (Cantor and Packer 1996, Clark and Lakshmi 2003, Larrain, Reisen, and von Maltzan 1997). The Basel II capital accords may further increase the importance that markets attach to the assessments of credit rating agencies.

The increased use of credit ratings, however, is not without controversy. On occasion, countries have argued that credit rating agencies have provided incorrect assessments of their likelihood of default.[3] Since the interest rates that countries must pay on their bonds depend on the rating that they receive, countries may not provide an unbiased assessment of the accuracy of their own credit rating. However, some academic studies have also raised concerns about the accuracy of sovereign credit ratings. Reinhart (2002) and Reisen and von Maltzan (1998) find that financial crises are generally not predicted by rating downgrades in contrast to what might be expected if rating agencies provide timely information about credit quality.

While empirical studies of sovereign ratings and their impacts are abundant, theoretical analysis (to

[1]As of today, Moody's and Standard and Poors provide ratings for over 100 countries each, while twenty years ago only 11 sovereigns were rated by Standard and Poors (Ferri, Liu, and Majnoni 2000).

[2]See International Monetary Fund (1999) *International Capital Markets: Developments, Prospects and Key Policy Issues.*

[3]See "Japan Rebukes Ratings Agencies For Falling Grades," Wall Street Journal. May 13, 2002, page C1.

the best of our knowledge) is rather scarce. In their foundational work, Millon and Thakor (1985) show the potential Pareto improvement due to information sharing that rating agencies provide. However, they do not allow for imperfect knowledge of the rating agency itself.

Our paper proposes a tractable framework that is able to explain a number of stylized facts regarding the impact of sovereign credit ratings on the investors behavior and gives a potential explanation for the failure of rating agencies to predict sovereign debt crises. The model also provides a useful framework for exploring the reasons the rating agencies may have "excessively downgraded" Asian economies in the aftermath of the 1997 financial crisis, as Ferri, Liu, and Stiglitz (1999) find to be the case. Our model also allows us to discuss the ways in which credit ratings could be a vector of international financial contagion.

The theoretical model presented in this paper focuses on the interactions between a credit rating agency and international investors.[4] It is an application of a global game, reviewed and discussed in Morris and Shin (2003) and is characterized by heterogeneous private signals, an announced public signal, and strategic agents that form beliefs about other agents' signals. We show that the global game structure of the model is crucial to understanding the stylized facts.

In our model, investors receive imperfect information regarding the ability of a sovereign to repay its debt as well as the credit rating agency's assessment of the sovereign's creditworthiness. Investors incorporate the agency's assessment of the economy into their own forecast; thus, the agency provides a focal point toward which the investors' beliefs gravitate. As a result, the agency affects the amount of debt the sovereign is able to roll–over and the probability of default.

We show that even if credit ratings are influenced by the same fundamentals as investors' private information, ratings will still affect investors' decisions. This is consistent with the findings of several empirical studies (Cantor and Packer 1996, Larrain, Reisen, and von Maltzan 1997). Credit ratings have this effect in part because the revision of the rating agency's signal in response to changes in fundamentals causes investors to revise their expectation regarding what other investors will do. Markets focus on ratings because they provide useful and public information.[5] In that respect, the

[4]While this paper is written with sovereign debt ratings in mind and some issues may be specific to these markets, the general structure of the model is more widely applicable.

[5]Boot and Milbourn (2002) suggest that rating agencies might act as market coordination mechanisms as a way to separate and discipline firms that have access to both a "good" and a "bad" production technology in an environment

level of the agency's announcement is very important and the focus of empirical literature on the accuracy of the ratings is well placed.

We also find that the addition of the rating agency increases the magnitude of the response of the capital flows to changes in fundamentals, thus providing a rationale for the complaints about the volatility of "hot" money flows that have accompanied the rising use of sovereign credit ratings. It is also consistent with the Calvo (1998) argument that a market coordination mechanism may lead to a "sudden stop" in the flow of funds to sovereign borrowers and precipitate a crisis. Our model also suggests that a rating agency can exacerbate the spread of financial contagion.

We see our paper as a contribution to a growing body of research that involves higher order beliefs, that seems to be gaining ground in various fields of economics (Corsetti, Dasgupta, Morris, and Shin 2004, Goldstein and Pauzner 2003, Angeletos, Hellwig, and Pavan 2003). We believe that this way of thinking about investors' behavior could be also fruitful in analyzing other issues of international finance literature.

The paper is organized as follows. In part two, we summarize up–to–date empirical findings on the impacts of sovereign credit ratings and discuss potential explanations. Part three develops a model of debt roll–over with a rating agency. In part four, we show how the introduction of a rating agency into a global game framework can help explain stylized facts and discuss further implications and extension of the model. Part five concludes.

2 Stylized facts

In this section, we summarize recent research on the effects of sovereign credit ratings on the markets. Rather then reviewing the literature chronologically, we group the papers by their findings to formulate stylized facts.

with multiple equilibria. We ignore this function of the ratings in our model.

Markets react to changes in sovereign ratings

The response of investment to fundamentals has been well documented (Corbo and Hernandez 2001, Montiel and Reinhart 1999). Cantor and Packer (1996) show that while over 90% of variation in the sovereign ratings can be explained by publicly observed fundamentals, credit ratings have an independent influence on yields *over and above* their correlation with other publicly available information. This finding is confirmed by Clark and Lakshmi (2003), who find that ratings influence debt yields even after accounting for economic fundamentals.

Rating agencies fail to predict crises

A number of studies have shown that credit ratings fail to predict financial crises. In particular, Cantor and Packer (1996), Larrain, Reisen, and von Maltzan (1997), and Kaminsky and Schmukler (2002) find that rating changes tend to occur in the midst of or after movements in financial markets. Confirming this findings, Reinhart (2002) finds that rating downgrades tend to follow financial crisis rather than precede them.

Rating agencies may have over-reacted in the aftermath of the Asian crisis

Ferri, Liu, and Stiglitz (1999) find evidence that rating agencies were excessively optimistic preceding the Asian crisis in 1998 and excessively pessimistic following them. They note that there were sudden large drops in ratings for several countries following the 1998 crisis in East Asia. They speculate that this behavior could be due to agencies' attempt to rebuild their reputation. Reisen and von Maltzan (1998) also conclude that ratings agencies overreact following financial crises.

Errors or overreaction by rating agencies may be of particular concern because the effects of changes in the ratings on sovereign bonds tend to spill over to other markets and other countries. Kaminsky and Schmukler (2002) find that changes in the credit rating on sovereign bonds also affect the country's stock price indices. Gande and Parsley (2002) find that negative rating events increase interest rate spreads on sovereign bonds in nearby countries. Kraussl (2003) finds that sovereign credit rating changes have effects on both bond yield spreads and the short term international liquidity positions.

We now turn to presenting a model and return to these stylized facts in part four to see if we can explain them in a framework of our model.

3 Model

In this section, we develop a model to show how credit rating agencies affect markets. We start by developing a benchmark model without a credit rating agency in which imperfectly informed investors decide whether to roll over the debt of a sovereign government based on their own assessments of the sovereign's ability to repay and on their expectations of the actions of the other investors. We then introduce the credit rating agency which provides a public assessment of the sovereign's creditworthiness. With this model, we are able to assess the effect of the rating agency on the equilibrium, both in terms of the level of debt and the change in the equilibrium in response to changes in the sovereign's ability to pay.

We assume that the sovereign government has an outstanding amount of one period debt that it wishes to rollover. The government debt is held by N risk–neutral investors where we suppose that each investor holds one unit of this debt. Thus, the total stock of debt is equal to N. The government can and is willing to repay an exogenous share θ of this debt while the remaining amount of debt $(1-\theta)N$ needs to be rolled over.[6] Each investor decides individually whether or not to roll over her unit of debt and there is no cooperation among investors. Denote investor i's decision d_i: $d_i = 1$ if investor i decides to roll over her unit of debt and $d_i = 0$ otherwise. The total amount of debt that will be rolled over is then

$$D = \sum_{i=1}^{N} d_i,$$

which implies that $D \in [0, N]$.

[6] One can think of θ as a measure of fundamentals. Stronger fundamentals lead to more economic growth providing the government with a larger revenue base to use to repay the debt.

If the investor decides not to roll over her unit of debt, she can withdraw it without any premium or punishment and invest it in a risk–free asset with gross return normalized to 1.[7] If she decides to roll over her debt, she receives a gross return of $R > 1$ if there is no default.[8] This gross rate of return implies that risk premium is $R - 1$. If an insufficient number of investors roll over their debt holdings, i.e. $D < (1-\theta)N$, the country is forced to default on its debt and none of the investors that rolled over their holdings of debt are paid.[9] Thus, the payoff structure for investors is as follows:

$$u_i(d_i, d_{-i}, \theta) = \begin{cases} R & \text{if } d_i = 1 \ \& \ D \geq (1-\theta)N \\ 0 & \text{if } d_i = 1 \ \& \ D < (1-\theta)N \\ 1 & \text{if } d_i = 0 \end{cases} \tag{1}$$

This payoff structure implies that investors will choose to roll over the debt if and only if they believe that

$$\text{Prob}(D \geq (1-\theta)N)R \geq 1. \tag{2}$$

Suppose that at the time of the investors' roll–over decisions θ is not known. Assume however, that it is public knowledge that the realization will be drawn from normal distribution with mean $\bar{\theta}$ and variance normalized to 1. The *apriori* probability $1-p$ of being repayed in (3) is

$$1 - p = \text{Prob}(D \geq (1-\theta)N) = \Phi(\frac{D}{N} - 1 - \bar{\theta}), \tag{3}$$

where Φ is standard normal CDF, and p is the probability of default.

Our assumptions imply that if $\theta < 0$, there will be default unless "new money" is injected. In our model, we do not allow for this possibility, therefore there will always be default if $\theta < 0$, and therefore not rolling over the debt $d = 0$ is a dominant strategy for every investor. Likewise, if

[7] In this model we assume that those who want to withdraw their money can always do it before the draw of θ occurs. We will not focus on how this is financed.

[8] Again, we do not specify how the premium is financed. Presumably economic growth, as reflected by θ, would allow the country to repay part of the debt while the remainder is financed by future debt rollovers. This is sustainable as long as the growth of revenue is greater than or equal to the growth of interest payments. It would probably be more realistic to assume that θ falls as R rises, however this complicates the model without adding any insight.

[9] This implicitly assumes that a country bears costs of default that are fixed and independent of the amount of debt defaulted, and that the country does not build long–run relationships with investors. Under these conditions it will be optimal for the country to always default on the entire debt stock.

$\theta > 1$, the default will not occur even if $D = 0$, because the government can repay all the debt from its own resources, thus rolling over the debt $d = 1$ is a dominant strategy in this case. We focus our attention on the equilibria with $\theta \in (0; 1)$, which will depend on the information structure regarding θ, however, as shown in Morris and Shin (2003), the dominance regions described are necessary for the existence of the unique equilibrium.

For simplicity, we will restrict our attention to pure strategy equilibria, although the general results of the paper still hold if mixed strategies are allowed. We also normalize $N = 1$ to simplify notation. In our case, since θ is a share of total debt, the important variable is the share of investors that will choose to roll over the debt D/N. Thus, N has no effect on equilibrium. However, in an alternative formulation, Morris and Shin (2004) show that the coordination problem grows more severe with more disperse debt (higher mass of investors).

3.1 Full information

If the share of debt that the government can repay, θ, is common knowledge to all investors, then there are two symmetric equilibria in which either $D = 0$ or $D = 1$. Both are self–fulfilling and can occur at any level of $\theta \in (0; 1)$. There are also infinitely many non–symmetric equilibria (Morris and Shin 2004). This multiplicity will disappear as we introduce imperfect heterogeneous signals and strategic behavior.[10]

This perfect information set–up is frequently a benchmark model for thinking about the investor coordination. It is common to think that with perfect information the role of credit rating agencies would be to help investors coordinate on one of the two equilibria, which then implies that credit rating agencies could be responsible for swings in international capital flows. We will see from the model below that some of this coordination effect is present when we allow for imperfect information and heterogeneous beliefs. However, this multiple equilibria model does not help us understand the stylized facts described in the previous section. We now turn to the model with a unique equilibrium.

[10] The same logic applies if all investors get the same noisy signal about θ, and this signal is common knowledge.

3.2 Private signals

Suppose now that in addition to their common prior about θ, investors get private noisy signals about θ. Denote investors i's signal $\tilde{\theta}_i$:

$$\tilde{\theta}_i = \theta + \varepsilon_i, \qquad \varepsilon_i \ \sim \ N\left(0; \frac{1}{\beta}\right),$$

where β is the "precision" of private signal. We will assume that only investor i observes her signal $\tilde{\theta}_i$, while β is common knowledge. This gives investor i's posterior belief about θ, θ_i:

$$\theta_i \ \sim \ N\left(\frac{\bar{\theta} + \beta\tilde{\theta}_i}{1+\beta}, \frac{1}{1+\beta}\right).$$

This posterior distribution of beliefs about θ gives us a unique equilibrium if private signals are sufficiently informative, $\beta \geq \frac{1}{2\pi}$.[11] In what follows, we assume that this condition holds.

The unique equilibrium can then be described by the threshold level of θ below which there will be a default in equilibrium, because fewer than $1 - \theta$ investors will choose to roll over their debt. Above that level, a sufficient number of investors will choose to roll over the debt and there will be no default. This unique level of θ should then be consistent with the belief of a pivotal investor who is indifferent between rolling over debt and not doing so. We denote this equilibrium threshold of θ as θ^*.

The equilibrium in this model, θ^*, is a switching point such that the fundamental $\theta \geq \theta^*$ results in a successful roll–over, while $\theta < \theta^*$ results in default. Since for any $0 < \theta < \theta^*$ there will be default in equilibrium, and this default would not occur if all investors could coordinate on rolling over the debt, θ^* could be interpreted as a measure of inefficiency due to coordination failure.

Proposition 1 *(Morris and Shin 2004) Given the information structure above, and if* $\beta \geq \frac{1}{2\pi} \simeq 0.16$*, the equilibrium is unique and* θ^* *is implicitly determined by*

$$\theta^* = \Phi\left(\frac{1}{\sqrt{\beta}}\ \left[\theta^* - \bar{\theta} + \sqrt{1+\beta}\ \Phi^{-1}\left(\frac{1}{R}\right)\right]\right) \tag{4}$$

[11] See, Morris and Shin (2004) for the proof in a similar setting.

and is $\in (0;1)$*. In addition,* $\partial\theta^*/\partial\bar{\theta} < 0$*,* $\partial\theta^*/\partial R < 0$*.*

Proof. Follows from Morris and Shin (2004) with α replaced by 1.

For convenience, we will denote $\Phi^{-1}\left(\frac{1}{R}\right)$ as ρ in what follows. It will be useful to keep in mind that $\rho > 0 \ \Leftrightarrow \ R \in (1;2)$ and $\rho < 0 \ \Leftrightarrow \ R > 2$. We can also establish the following necessary and sufficient condition:

Proposition 2 $\partial\theta^*/\partial\beta < 0$ *if and only if*

$$\bar{\theta} < \frac{\rho}{\sqrt{1+\beta}} + \Phi\left(\sqrt{\frac{\beta}{1+\beta}}\ \rho\right), \tag{5}$$

Which implies the following sufficient conditions:

1. If $\rho > 0$ *and* $\bar{\theta} < \Phi\left(\sqrt{\frac{\beta}{1+\beta}}\ \rho\right)$*,* $\partial\theta^*/\partial\beta < 0$*.*

2. If $\rho < 0$ *and* $\bar{\theta} > \Phi\left(\sqrt{\frac{\beta}{1+\beta}}\ \rho\right)$*,* $\partial\theta^*/\partial\beta > 0$*.*

Proof. See Appendix.

We are most interested in the actual probability of default, p^*. This is equal to the probability that actual θ will be below the threshold θ^*.

$$p^* = \text{Prob}\,(\theta < \theta^*) = F_\theta(\theta^*) = \Phi(\theta^* - \bar{\theta}),$$

which is monotonically increasing in θ^*. Therefore for any variable $\bullet$, $\text{sign}\,(\partial\theta^*/\partial\bullet) = \text{sign}\,(\partial p^*/\partial\bullet)$.

We can see from Proposition 1 that the probability of default is lower if $\bar{\theta}$ is higher and if R is higher. Both better fundamentals and a higher risk premium on sovereign debt increase the incentives to rolling over the debt and therefore more investors choose to do so, which in turn lowers the probability of default.

Proposition 2 establishes the effects of precision of private signals on probability of default. The sufficient conditions present two interesting cases. In Case 1, interest rates are low while the fundamentals are relatively poor, which could happen if the interest rates were preset and then the (expected) fundamentals have worsened. In this case, higher precision of private information lowers the probability of default. In the absence of the coordination problem, the investors will be hesitant to roll over the debt. However, with more precise private information, investors would

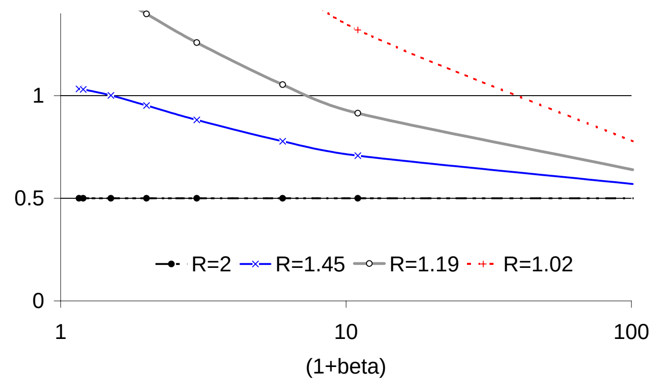


Figure 1: Right–hand side of equation (5)

put less weight on public information ($\bar{\theta}$, which is low and is therefore bad news) and are therefore more likely to invest.

In Case 2, interest rates are extremely high (gross interest rate $R > 2$, which implies risk premium of over 100 percentage points), however, expected fundamentals are decent. In this case, each investor has a strong incentive to roll over the debt. More precise private information makes investors pay less attention to the public signal (which is good in this case) and therefore makes them less likely to invest, thus raising the probability of default.[12]

For the rest of the paper we assume that $\rho > 0$ ($R < 2$). Figure 1 shows the sufficient condition for $\partial\theta^*/\partial\beta < 0$ for several values of ρ. The region below each line indicates the values of $\bar{\theta}$ for which the condition holds. We can see that equation (5) is more likely to hold for smaller β, since there is no coordination problem if on average $\bar{\theta} > 1$. Thus, we see that an increase in the precision of the private signals typically lowers the probability of default unless the private signals are already quite precise. It is worth keeping in mind that β represents the *relative* precision of private information with respect to the precision of the public signal, which we normalized to 1.

[12] For further discussion of the intuition behind the effects of precision of public and private information, see Morris and Shin (2002) and Metz (2002).

3.3 Truthful rating agency

Suppose now that there is a rating agency that has the same prior as all the investors and receives a signal $\tilde{\theta}^A$.[13]

$$\tilde{\theta}^A = \theta + \nu, \qquad \nu \sim N\left(0, \frac{1}{\alpha}\right).$$

We will assume that only the rating agency observes $\tilde{\theta}^A$ but α is common knowledge. Suppose the agency announces directly $\tilde{\theta}^A$ and investors then update their prior accordingly. New prior has mean

$$\theta^A \equiv \frac{\bar{\theta} + \alpha\,\tilde{\theta}^A}{1+\alpha}$$

and variance $1/(1+\alpha)$. We will assume that investors get the same private signals as before, and therefore the new posterior beliefs θ_i^A are

$$\theta_i^A \sim N\left(\frac{\bar{\theta} + \alpha\,\tilde{\theta}^A + \beta\,\tilde{\theta}_i}{1+\alpha+\beta}, \frac{1}{1+\alpha+\beta}\right). \tag{6}$$

Now, for the equilibrium to be unique, we have to impose stricter condition on β. While before it was necessary and sufficient that $\beta \geq \frac{1}{2\pi}$, now it is necessary that $\beta \geq \frac{(1+\alpha)^2}{2\pi} > \frac{1}{2\pi}$ if $\alpha > 0$. This gives us the first effect that introducing rating agency can have:

Proposition 3 *For* $\beta \in \left[\frac{1}{2\pi}, \frac{(1+\alpha)^2}{2\pi}\right)$*, there is a unique equilibrium in the absence of a rating agency and multiple equilibria if a rating agency is introduced.*

Proof. Follows from Morris and Shin (2004) with α replaced by $1+\alpha$.

In other words, if private signals are precise enough to ensure uniqueness of equilibrium in the absence of a rating agency, but not precise enough relative to the precision of the agency's signal, introducing a rating agency will lead to a multiplicity of equilibria. In particular both $D = 0$ (default) and $D = 1$ (no default) symmetric equilibria will exist, which is similar to the full information case. In this case the argument that rating agencies can help investors coordinate on no–default equilibrium could be valid. We will not engage here in the discussion of equilibrium selection and will focus instead on the cases when a unique equilibrium exists with a rating agency,

[13] The three rating agencies report that they base their rating largely on economic fundamentals (Fitch 1998, Moody's 1999, Standard and Poor's 2002).

i.e. $\beta \geq \frac{(1+\alpha)^2}{2\pi}$. We discuss the issue of multiplicity in the next section.

Proposition 4 *(Morris and Shin 2004) Given the information structure above and* $\beta \geq \frac{(1+\alpha)^2}{2\pi}$*, the equilibrium is unique and* θ^{A*} *is implicitly determined by*

$$\theta^{A*} = \Phi\left(\frac{1}{\sqrt{\beta}}\ \left((1+\alpha)\theta^{A*} - \bar{\theta} - \alpha\,\tilde{\theta}^{A} + \sqrt{1+\alpha+\beta}\ \rho\right)\right) \tag{7}$$

and is $\in (0;1)$. *In addition,* $\partial\theta^{A*}/\partial\tilde{\theta}^{A} < 0$, $\partial\theta^{A*}/\partial\bar{\theta} < 0$, $\partial\theta^{A*}/\partial R < 0$.

Proof. Follows from Morris and Shin (2004) with α replaced by $1+\alpha$.

We can also establish the following necessary and sufficient condition:

Proposition 5 $\partial\theta^{A*}/\partial\beta < 0$ *if and only if*

$$\theta^{A} = \frac{\bar{\theta} + \alpha\tilde{\theta}^{A}}{1+\alpha} < \frac{\rho}{\sqrt{1+\alpha+\beta}} + \Phi\left(\frac{\sqrt{\beta}\ \rho}{\sqrt{1+\alpha+\beta}}\right), \tag{8}$$

given that on average $\tilde{\theta}^{A} = \bar{\theta} = \theta^{A}$, $\partial\theta^{A*}/\partial\alpha < 0$ *on average if and only if*

$$\theta^{A} = \tilde{\theta}^{A} = \bar{\theta} > \frac{\rho}{2\sqrt{1+\alpha+\beta}} + \Phi\left(\frac{(1+\alpha+2\beta)\rho}{2\sqrt{1+\alpha+\beta}\sqrt{\beta}}\right), \tag{9}$$

Proof. See Appendix.

Note that the condition is now on θ^{A}, not $\bar{\theta}$. In most cases, equations (8) and (9) hold as can be seen from Figure 2. This means that more precise private information most likely lowers the probability of default, while more precise public information increases it. The intuition for both results is that relatively more precise public information makes investors disregard their private information thus worsening the inefficiency — the result shown in a different (more general) setting in Morris and Shin (2002).

It is important to remember that both the precision of private signals and of the agency's signal are relative to the precision of pure public information (which we normalized to 1). Therefore, an increase in the precision of pure public information (if combined with an improvement of the quality of private signals) is equivalent a fall in α and therefore will lower the probability of default.

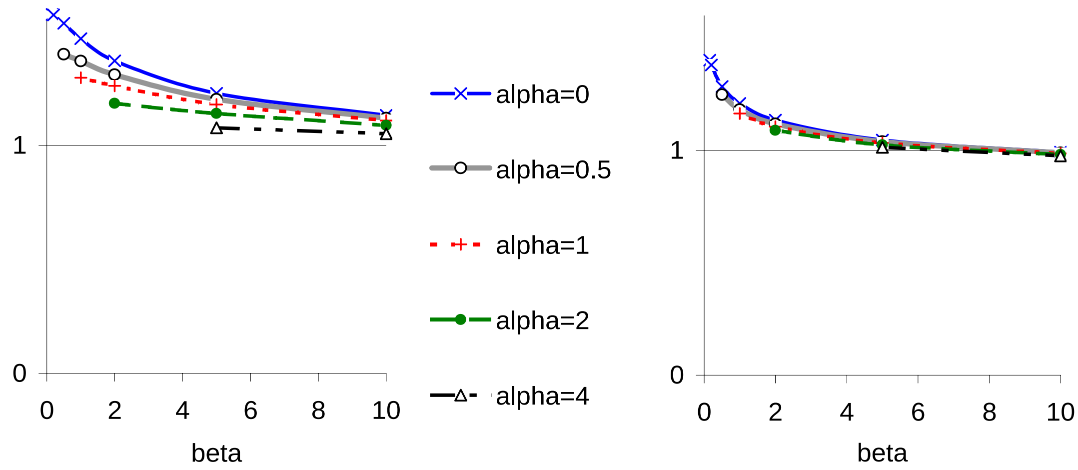


Figure 2: Right–hand side of equations (8) (left) and (9) (right) with $\rho = 1$

As before, the probability of default is lower if the prior mean of θ, $\bar{\theta}$, is higher or the risk premium R is higher. In addition, a better announcement by the agency lowers the probability of default, as expected.

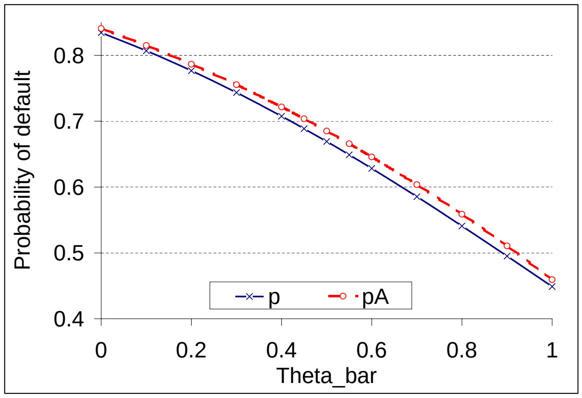
We are also interested in the effects of a general increase in quality of information. What if the precision of both public and private signal improves with respect to the precision of the *ex ante* common prior? To evaluate this, we set $1 + \alpha = \beta \equiv \gamma$, so that the total precision of the public signal is equal to the total precision of the private signal. This implies

$$\theta^{A*} = \Phi\left(\frac{1}{\sqrt{\gamma}}\left(\gamma\,\theta^{A*} - \bar{\theta} - (\gamma - 1)\,\tilde{\theta}^A + \sqrt{2\gamma}\;\rho\right)\right)$$

and

$$\frac{\partial\theta^{A*}}{\partial\gamma} = \frac{\phi}{2\gamma\sqrt{\gamma}}\frac{\gamma\theta^{A*} + \bar{\theta} - (1+\gamma)\tilde{\theta}^A}{1 - \sqrt{\gamma}\,\phi}.$$

This implies, on average ($\tilde{\theta}^A = \bar{\theta} = \theta^A$), that $\partial\theta^{A*}/\partial\gamma < 0$ if and only if $\theta^A > \Phi(\sqrt{2}\,\rho)$. Note that for values of ρ discussed before, this implies that θ^A needs to be more than 0.75 in order for information precision to lower the probability of default. In most cases, more precise information will exacerbate the coordination problem. This result is due to the fact that the effect of an increase



Note: $\alpha = 1$, $\beta = 2$, $R = 1.2$, $\tilde{\theta}^A = \bar{\theta}$.

Figure 3: Effect of a change in $\bar{\theta}$ with and without the agency.

in α dominates that of the increase in β due to a feed–back mechanism of higher order expectations. However, the effect is numerically small.

We can also see that the rating agency affects the equilibrium in two ways — through its effect on the mean of the new common prior and through its effect on the precision of public information. Clearly, higher rating leads to a "better" equilibrium, however the following is true:

Proposition 6 *$\theta^{A*} \leq \theta^*$ if and only if $\tilde{\theta}^A \geq \theta^* + W$, where $W = \frac{\sqrt{1+\alpha+\beta}-\sqrt{1+\beta}}{\alpha}\rho$ and is positive for $\rho > 0$ and $\alpha > 0$.*

Proof. See Appendix.

In other words, rating agency equilibrium (weakly) dominates equilibrium without the agency if and only if the signal that agency gets is sufficiently better than the threshold level of θ in equilibrium without the agency. The wedge W is positive as long as rating agency's signal is somewhat informative ($\alpha > 0$) and the interest rate is not too high $R < 2$. See illustration on Figure 3.

It has been argued that rating agencies affect the volatility of capital flows. One way to examine this in our model is to see how introducing a rating agency affects the response of the share of investors who choose to roll over the debt to changes in fundamentals: $\partial D/\partial\bar{\theta}$. The equilibrium share of investors who choose to roll over their debt is $D^{A*} = 1 - \theta^{A*}$ with agency and $D^* = 1 - \theta^*$

without.[14] Thus, $\partial D^*/\partial\bar{\theta} = -\partial\theta^*/\partial\bar{\theta}$ and $\partial D^{A*}/\partial\bar{\theta} = \partial\theta^{A*}/\partial\bar{\theta}$.

Proposition 7 *If on average $\partial\tilde{\theta}^A/\partial\bar{\theta} = 1$, $\partial D^{A*}/\partial\bar{\theta} > \partial D^*/\partial\bar{\theta}$ at a point where $\theta^* = \theta^{A^*}$.*

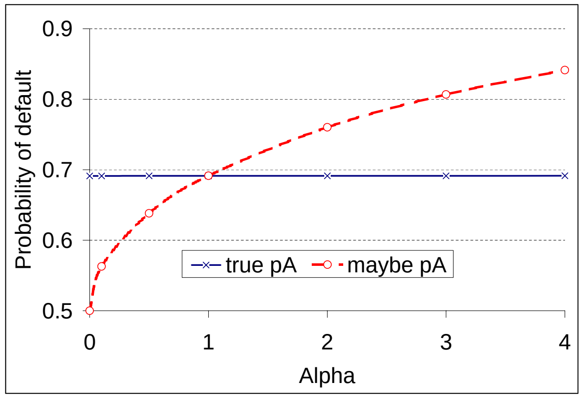
Proof. See Appendix.

In other words, the introduction of a rating agency increases the magnitude of the response of capital flows to changes in fundamentals. This suggests that the rating agency may increase the volatility of capital flows if fundamentals are volatile. This occurs because investors put some weight on the agency's announcement, which is public information and is the same for everyone, and put lower weight on their own private signals. As a result, the posterior distribution of beliefs tightens and the same change in fundamentals will induce a larger number of investors to change their minds regarding rolling over their holdings of the debt.

One might ask then, if rating agencies increase the probability of default and the volatility of capital flows, why should they exist? We must emphasize that our model is not a full model of the contribution of the rating agency to financial markets. In fact, we omit a very important feature, the cost of gathering information. In the presence of costly information acquisition, the public signal provided by rating agencies may provide information to investors for whom private information is prohibitively expensive and thus increase the number of investors willing to participate in a particular market. Further, since such costs affect the payoffs to investors directly, as well as through their effect on α and β, they will have an effect on overall welfare that could be positive or negative. We leave aside the question of the costs of obtaining information in this paper, for we analyze it in another project.

In the above setup, rating agencies announce their beliefs about economic fundamentals. This is somewhat unrealistic, since rating agencies typically announce a rating that corresponds to the likelihood that the sovereign (or other borrower) will default. However, the results will be equivalent, as investors can always back out $\bar{\theta}^A$ on which the announcement of the probability is made. However, it is possible that rating agencies do not take the coordination problem into account when making their rating announcements. We now turn to the case of a "myopic" rating

[14]Default occurs if $D < 1 - \theta$. Thresholds θ^* and θ^{A*} are such that for any θ lower than threshold, there will be default, and there will be no default for θ above the threshold. Thus, by definition, default occurs if $1 - \theta$ is above $1 - \theta^* = D^*$ (or $1 - \theta^{A*} = D^{A*}$).



Note: $\beta = 1 + \alpha$, $R = 1.03$, $\tilde{\theta}^A = \bar{\theta} = 0.5$.

Figure 4: Effect of a change in $\bar{\theta}$ with and without the agency.

agency.

3.4 Myopic rating agency

We can imagine that the rating agency, instead of having in mind the coordination game that we just described, has in mind the multiple equilibria model like the one we described in section 3.1. Suppose the agency announces the probability that there might be default, i.e. the probability that $\theta < 1$ and government cannot repay without any debt roll-over. We denote this probability p^M. Agency gets signal $\tilde{\theta}^A = \theta + \nu$ as before, therefore

$$p^M = \text{Prob}(\theta < 1) = \text{Prob}(\nu > \tilde{\theta}^A - 1) = 1 - F_\nu(\tilde{\theta}^A - 1) = 1 - \Phi(\sqrt{\alpha}(\tilde{\theta}^A - 1)).$$

Note that this probability is higher if α is smaller, i.e. agency is more worried about the default if its information is more precise, which makes sense, since more precise information implies that distribution has thinner tails.

As Figure 4 illustrates, the myopic rating agency will be likely to underestimate the probability of default if its signal is not very precise. Notice also, that ratings will be more volatile as a result of changes in the quality of information if the rating agency is myopic than if the rating agency takes into account the coordination game. Since the rating agencies claim that the stability of the

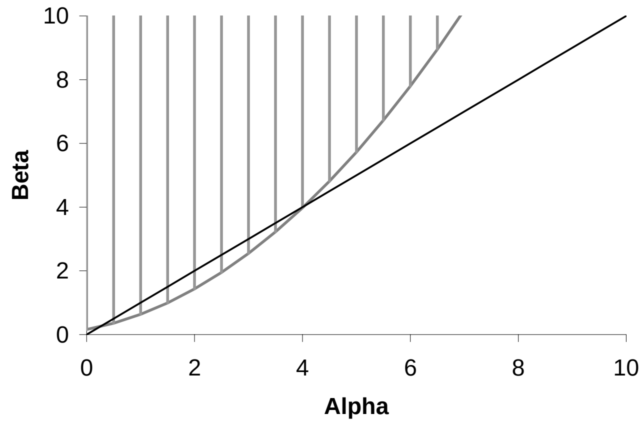


Figure 5: α, β and uniqueness condition (shaded area)

ratings is one of their goals (Mann and Cantor 2004), taking into account the coordination game (provided it is a correct model of the world) might help them achieve this goal.

4 Model discussion

We now discuss the implications of the model and reconcile its predictions with the stylized facts that we outlined in Part 2. In addition, we discuss how rating agencies might spread financial contagion.

4.1 Multiplicity versus uniqueness of equilibrium

In the previous section, we focused on the situation where the equilibrium was unique. In order to insure uniqueness, we had to assume that the condition on the precision of private signals relative to that of public information held, in particular, $\beta \geq (1+\alpha)^2/2\pi$. This condition requires that private information be rather precise relative to the rating agency's information. In fact, as shown on Figure 5, if $\alpha \geq 4$, uniqueness requires that $\beta > \alpha$, i.e. private signals are more precise than public information.

If we think of the original common prior (with precision normalized to 1) as easily available information, such as news media, it is not unrealistic to think that the information obtained by the analysts of rating agencies is much more precise. In this case, uniqueness will require that the private information that investors obtain is even more precise than that of the rating agency. This is not usually the way we think about the quality of information. The reason that investors make use of credit ratings is because they believe that rating agency have better access to information and therefore their information is more accurate. Indeed, it does not matter whether rating agency's information is more accurate in reality, what matters is investors' beliefs about the precision of the rating agency's information.

On the other hand, uniqueness without rating agency only requires that $\beta \geq 0.16$, which means that private signals only need to be 16% as precise as common prior. It is reasonable to believe that this condition is satisfied and private signals of investors (most of them are institutional investors if we talk about sovereign debt) are at least as precise as common prior, i.e. $\beta \geq 1$. Thus, while a unique equilibrium is easily achieved in the absence of a rating agency, it is much less likely to be achieved when a rating agency is introduced.

If uniqueness is not achieved, instead of the single equilibrium, there will be three: two stable equilibria and one unstable equilibrium. With multiple equilibria, the volatility of capital flows may be increased further as shifts in credit ratings may trigger jumps from one equilibrium to another.

The switch from a situation with a unique equilibrium to multiple equilibria due to the introduction of a rating agency is illustrated in Figure 6. Equilibrium E is the unique equilibrium in the absence of the rating agency. When the rating agency is introduced, the equilibrium will shift to either of the stable equilibrium (A of C) unless equilibrium E is exactly equal to the unstable equilibrium B. In the case illustrated in Figure 6, θ is high enough that the equilibrium converges to C, with a higher probability of default. This need not be the case in general. For some initial θ, the addition of the rating agency will shift the equilibrium to A, in which case the addition of the rating agency lowers the probability of default in equilibrium. In this latter scenario, moving from E to A, a credit rating indeed gives "courage" to capital. However, this improvement comes at a cost — changes in ratings may shift equilibrium to the one with higher probability of default.

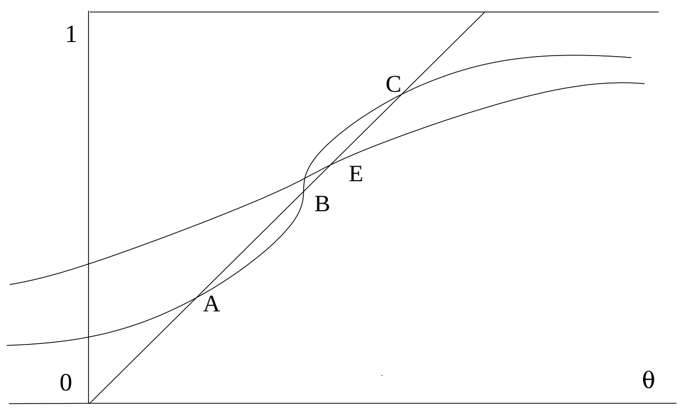


Figure 6: Equilibria (equations (4) and (7)) with and without a rating agency

Furthermore, Angeletos, Hellwig, and Pavan (2003) show that allowing the rating agency to act strategically (while limiting its action space) might lead to multiple equilibria even if the condition on β is satisfied. This gives us an additional reason to believe that introducing a rating agency into coordination game might bring back multiple equilibria.

4.2 Stylized facts revisited

Markets react to changes in sovereign ratings

As we have shown in Proposition 4, there is an independent effect of a rating agency announcement, "above and beyond" the effect of fundamentals. This is due to the fact that the credit rating is public information and therefore affects each investor's belief about other investors' beliefs. Even if the fundamentals $\bar{\theta}$ remained unchanged, the signal that the rating agency receives and therefore its announcement $\tilde{\theta}^A$ has an effect on θ^{A*} and also on the probability of default and the share of investors who will choose to roll over their debt.

While most empirical tests address the influence of credit ratings on the sovereign bond spreads, we fix the risk premium R. It is not determined within the model as part of equilibrium. One can interpret this risk premium (or spread) as set at the beginning of the period in consideration. At the end of the period, it is reasonable to assume that the risk premium will adjust according to the actual probability of default, that is, $R_{+1} = 1/(1 - p^A)$. Since a better announcement by the agency (higher $\tilde{\theta}^A$) lowers probability of default, it will lower the risk premium or spread the next period. Thus, the dynamic version of our model will generate a negative correlation between credit ratings and spreads, even if the fundamentals remain unchanged (in fact, even if the realization of fundamentals θ remain unchanged).

Rating agencies fail to predict crises

As shown on Figure 4, the myopic rating agency will be likely to underestimate the probability of default if its signal is not very precise. Thus, we might be observing that rating agencies that act myopically will consistently fail to predict crises if their information is not very good. This is consistent with the observation that rating agencies fail to predict sovereign debt crises, but do a much better job downgrading domestic private companies before the financial troubles surface. This difference could be due to the fact that rating agencies have much better information about private companies than they do about the sovereigns. In fact, the rating agencies admit that:

> ..It is important, though, that investors realize the limitations of this exercise [assigning sovereign ratings], which is necessarily far less certain than our ability to analyze either bank or corporate risks of default. Fitch (1998).

It is therefore possible that the reason rating agencies consistently fail to predict sovereign debt crises, as has been shown in the empirical literature, does not necessarily represent a bias or sluggishness in their response, but rather the quality of their information and the fact that they do not take full account of the coordination problem when making their forecasts.

Rating agencies may have over–reacted in the aftermath of the Asian crisis

Ferri, Liu, and Stiglitz (1999) argued that rating agencies gave the Asian countries excessively high credit ratings prior to 1998 and excessively low ratings following the crisis. Reisen and von Maltzan (1998) also argue that the rating downgrades following the Asian crisis were excessive given the changes in fundamentals. Without taking a stand on whether this did occur, our model provides some insight into how this situation might happen. Recall that the signal received by the agency is $\tilde{\theta}^A = \theta + \nu$, where we assumed that $\nu \sim N(0, 1/\alpha)$, with ν independently and identically distributed over time.

Agencies might prove "excessively optimistic" or "excessively pessimistic" if ν is not i.i.d., but instead is positively correlated with changes in θ. This might occur if preliminary estimates of fundamentals, such as productivity, are biased in a particular direction.

In this situation, countries which are growing and improving their fundamentals would experience even faster increases in their credit ratings. A sharp drop in fundamentals, as is likely to be the case following a crisis, would then lead to a negative ν and the rating agency would lower the credit rating by more than the change in "pure fundamentals" would indicate.

If this is the case, credit ratings would be procyclical and worsen lending cycles. This procyclicality of credit ratings is a concern under the Basel II accords as downgraded loans would require more capital and banks' ability to lend would be curtailed at times when monetary authorities are trying to ease financial conditions (Blum and Hellwig 1995, Carpenter, Whitesell, and Zakrajsek 2001).

4.3 Rating agencies can spread financial contagion

Our model also suggests that rating agencies could serve as a vector of contagion if the signal they receive about one country is affected by events in another country. Thus, if there is true contagion (through trade, financial markets, political clubs or other channels discussed in a broad contagion literature),[15] rating agency will provide an additional channel. To see this, suppose there are two countries: C and B, and country C is experiencing a crisis. If fundamentals (which in our case represent a country's ability and willingness to service its debt) are positively correlated across countries, the rating agency will get on average a "worse" signal about country B and therefore downgrade country B. This in turn will increase the probability of default in country B, since $\partial\theta^{A*}/\partial\tilde{\theta}^A < 0$, and will therefore provide an additional channel of contagion, beyond the effect of deteriorating fundamentals in country B.

In addition, a rating agency can spread financial contagion even if the fundamental links between the countries do not exist. This will be the case if the rating agency fails to predict a crisis in

[15] See, for example Glick and Rose (1999), Claessens, Dornbusch, and Park (2001), Pritsker (2001), Drazen (1999).

country C. When investors learn about this, they update their beliefs about the precision of rating agency's information. In particular, they will believe that α is lower than they previously thought. If previously the condition on the uniqueness of equilibrium was not satisfied, and country B was in a "good equilibrium", investors will start paying more attention to the private information they have about country B, and the system may switch to the unique equilibrium. In this unique equilibrium, the probability of default is likely to be higher than in a "good" equilibrium. Notice that this will occur in the absence of a rating action by the agency. Thus, the credibility of rating agency announcement could be another channel of financial contagion.

5 Conclusion

We use a global game model of sovereign debt roll–over to analyze the effects of introducing a rating agency to financial markets. The model helps explain a number of observations in recent empirical literature on the impact and behavior of sovereign credit ratings. We also find that if coordination game is a proper description of investors' behavior, there are a number of costs associated with introduction of a rating agency. These costs are mainly due to the fact that investors put less weight on their private signals when credit ratings are available.

It is important to emphasize that this paper is by no means a complete cost–benefit analysis of the role rating agencies play in financial markets. We merely point out some potential costs that can arise from introducing a rating agency in a coordination game by heterogeneous investors. One should not conclude on the basis of the paper that rating agencies are harmful, as this paper does not address the benefits that arise from provision of public information that might be costly to collect. We attempt such analysis in a related project, still in progress.

Nevertheless this paper does contain a caution regarding increased emphasis on credit ratings provided by rating agencies as has been suggested in the Basel II capital accord. Increasing the focus on a small number of publicly available signals may actually increase the volatility of global financial markets (Danielsson et al. (2001)). The model also highlights the importance of supplying high quality information directly to investors.

The model can also serve as a stepping stone for many potentially instructive extensions. Although the model is formulated in terms of investors' decisions on whether to roll over the debt, it can be reformulated in terms of the investment decision. This change would allow for richer dynamics that would endogenize the cost of debt or risk–premium, the size of the debt stock, and potentially even the cost of default. Another extension would be to introduce competing rating agencies. This would allow us to address the issues related to split ratings — a situation where different agencies assign different ratings to the same borrower.

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Appendix. Proofs and Solutions

Proof of Proposition 2

Taking implicitly the derivative of (4) with respect to β, obtain

$$\frac{\partial\theta^*}{\partial\beta} = -\frac{\phi\ \left((\theta^* - \bar{\theta})\sqrt{1+\beta} + \rho\right)}{2\,\beta\,\sqrt{1+\beta}(\sqrt{\beta} - \phi)},$$

where ϕ is standard normal density function. As $\phi \in (0; 1/\sqrt{2\pi})$ and $\sqrt{\beta} \geq \frac{1}{\sqrt{2\pi}}$, the denominator is positive and the derivative then is negative if and only if

$$(\theta^* - \bar{\theta})\sqrt{1+\beta} + \rho > 0 \ \text{ or } \ \theta^* > \bar{\theta} - \frac{\rho}{\sqrt{1+\beta}}.$$

However, θ^* is endogenous. We know that $\partial\theta^*/\partial\beta = 0$ if and only if $\theta^* = \bar{\theta} - \rho/\sqrt{1+\beta}$. We can substitute this expression back into (4) to obtain

$$\bar{\theta} = \frac{\rho}{\sqrt{1+\beta}} + \Phi\left(\frac{(2+\beta)\rho}{\sqrt{\beta(1+\beta)}}\right) \equiv T.$$

We know from Proposition 1 that $\partial\theta^*/\partial\bar{\theta} < 0$. Therefore we know that $\partial\theta^*/\partial\beta < 0$ if and only if $\bar{\theta} < T$, which gives us a necessary and sufficient conditions. Stricter sufficient conditions follow immediately. ■

Proof of Proposition 5

Taking implicitly the derivative of (6) with respect to β, obtain

$$\frac{\partial\theta^{A*}}{\partial\beta} = -\frac{\phi(1+\alpha)\ \left(\sqrt{1+\alpha+\beta}(\theta^{A*} - \theta^A)\ + \rho\right)}{2\,\beta\,\sqrt{1+\alpha+\beta}(\sqrt{\beta} - (1+\alpha)\phi)}.$$

As $\phi \in (0; 1/\sqrt{2\pi})$ and now $\sqrt{\beta} \geq \frac{1+\alpha}{\sqrt{2\pi}}$, the denominator is positive and the derivative then is negative if and only if

$$(\theta^{A*} - \theta^A)\sqrt{1+\alpha+\beta} + \rho > 0 \ \text{ or } \ \theta^{A*} > \theta^A - \frac{\rho}{\sqrt{1+\alpha+\beta}}.$$

We know that $\partial\theta^{A*}/\partial\beta = 0$ if and only if $\theta^{A*} = \theta^A - \rho/\sqrt{1+\alpha+\beta}$. We can substitute this expression back into (6) to obtain

$$\theta^A = \frac{\rho}{\sqrt{1+\alpha+\beta}} + \Phi\left(\frac{(2+2\alpha+\beta)\rho}{\sqrt{\beta(1+\beta)}}\right) \equiv TA.$$

We know from Proposition 4 that $\partial\theta^{A*}/\partial\bar{\theta} < 0$. Therefore we know that $\partial\theta^{A*}/\partial\beta < 0$ if and only if $\theta^A < TA$, which gives us a necessary and sufficient conditions. Stricter sufficient conditions follow immediately.

Taking implicitly the derivative of (6) with respect to α, obtain

$$\frac{\partial \theta^{A*}}{\partial \alpha} = \frac{\phi(\theta^{A*} - \tilde{\theta}^A + \frac{\rho}{2\sqrt{1+\alpha+\beta}})}{\sqrt{\beta} - (1+\alpha)\phi}$$

As $\phi \in (0; 1/\sqrt{2\pi})$ and $\sqrt{\beta} \geq \frac{1+\alpha}{\sqrt{2\pi}}$, the denominator is positive and the derivative is negative if and only if

$$\theta^{A*} < \tilde{\theta}^A - \frac{\rho}{2\sqrt{1+\alpha+\beta}}.$$

The derivative is equal to zero if $\theta^{A*} < \tilde{\theta}^A - \frac{\rho}{2\sqrt{1+\alpha+\beta}}$. We can substitute this back into (6), however, in general we would not be able to find a closed form condition on $\tilde{\theta}^A$. The expression simplifies, however, if we assume $\tilde{\theta}^A = \bar{\theta}$ and therefore $\tilde{\theta}^A = \bar{\theta} = \theta^A$. Since both ε and ν are mean zero, this will be true on average, as $\bar{\theta}$ is unconditional mean of $\tilde{\theta}^A$. In this case we can obtain the condition

$$\tilde{\theta}^A = \bar{\theta} = \theta^A = \frac{\rho}{2\sqrt{1+\alpha+\beta}} + \Phi\left(\frac{1+\alpha+2\beta}{2\sqrt{\beta}\sqrt{1+\alpha+\beta}}\ \rho\right) \equiv TAa$$

Once again, since $\partial\theta^{A*}/\partial\theta^A < 0$, $\partial\theta^{A*}/\partial\alpha < 0$ if and only if $\tilde{\theta}^A = \bar{\theta} = \theta^A > 0$. ■

Proof of Proposition 6

Using equations (4) and (6) we can see that $\theta^* > \theta^{A*}$ if and only if

$$\theta^* - \bar{\theta} + \sqrt{1+\beta}\,\rho > (1+\alpha)\theta^{A*} - \bar{\theta} - \alpha\tilde{\theta}^A + \sqrt{1+\alpha+\beta}\,\rho.$$

The equality $\theta^* = \theta^{A*}$ holds if

$$\tilde{\theta}^A = \theta^* + \frac{\sqrt{1+\alpha+\beta} - \sqrt{1+\beta}}{\alpha}\ \rho \equiv TI.$$

By Propositions 1 and 4, θ^* does not depend on $\tilde{\theta}^A$ while θ^{A*} is decreasing in $\tilde{\theta}^A$, therefore $\theta^* > \theta^{A*}$ if and only if $\tilde{\theta}^A > TI$. ■

Proof of Proposition 7

$$\frac{\partial D^*}{\partial \bar{\theta}} = -\frac{\partial \theta^*}{\partial \bar{\theta}} = \frac{1}{\sqrt{\beta} - \phi\left(\frac{1}{\sqrt{\beta}}\ (\theta^* - \bar{\theta} + \sqrt{1+\beta}\ \rho)\right)},$$

$$\frac{\partial D^{A*}}{\partial \bar{\theta}} = -\frac{\partial \theta^{A*}}{\partial \bar{\theta}} = \frac{1+\alpha}{\sqrt{\beta} - (1+\alpha)\,\phi\left(\frac{1}{\sqrt{\beta}}\ \left((1+\alpha)\theta^{A*} - \bar{\theta} - \alpha\,\tilde{\theta}^A + \sqrt{1+\alpha+\beta}\ \rho\right)\right)}.$$

At a point where $\theta^* = \theta^{A*}$,

$$\Phi\left(\frac{1}{\sqrt{\beta}}\ (\theta^* - \bar{\theta} + \sqrt{1+\beta}\ \rho)\right) = \Phi\left(\frac{1}{\sqrt{\beta}}\ \left((1+\alpha)\theta^{A*} - \bar{\theta} - \alpha\,\tilde{\theta}^A + \sqrt{1+\alpha+\beta}\ \rho\right)\right),$$

and therefore

$$\phi\left(\frac{1}{\sqrt{\beta}}\ (\theta^{*}-\bar{\theta}+\sqrt{1+\beta}\ \rho)\right)=\phi\left(\frac{1}{\sqrt{\beta}}\ \Big((1+\alpha)\theta^{A*}-\bar{\theta}-\alpha\,\tilde{\theta}^{A}+\sqrt{1+\alpha+\beta}\ \rho\Big)\right)\equiv\phi(⁂).$$

Thus,

$$\partial D^{A*}/\partial\bar{\theta}-\partial D^{*}/\partial\bar{\theta}=\frac{1+\alpha}{\sqrt{\beta}-(1+\alpha)\,\phi(⁂)}-\frac{1}{\sqrt{\beta}-\phi(⁂)}=\ (1+\alpha)\,\phi(⁂)+\alpha\sqrt{\beta}-(1+\alpha)\phi(⁂)=\alpha\sqrt{\beta}>0,$$

since $\alpha,\beta>0$. ■