Monotonicity Analysis for Constructing Qualitative Models

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Abstract. Qualitative models are more suitable than classical quantitative models in many tasks like Model-based Diagnosis (MBD), explaining system behavior, and designing novel devices from first principles. Monotonicity is an important feature to leverage when constructing qualitative models. Detecting monotone pieces robustly and efficiently from sensor or simulation data remains an open problem. This paper introduces an approach based on scale-dependent monotonicity: the notion that monotonicity can be defined relative to a scale. Real-valued functions defined on a finite set of reals e.g. the sensor data the simulation results, can be partitioned into quasi-monotone segments, i.e. segments monotone with respect to nonzero scale. We can identify the extrema of the quasi-monotone segments. This paper then uses this method to abstract qualitative models from simulation models for the purpose of diagnosis. It shows that using monotone analysis, the abstracted qualitative model is not only sound, but also parsimonious because it generates few landmarks.

1 Extended Abstract

Qualitative models are more suitable than classical quantitative models in solving many problems. Qualitative models are used in tasks such as diagnosis [7], explaining system behavior [4, 6, 8], and designing novel devices from first principles [9]. Building a qualitative model for a complex system requires significant knowledge and is a time consuming process. Thus automatic construction of qualitative model is a well motivated topic. This paper studies how to construct qualitative models from scattered data, such as sensor data or simulation data. The main contributions of this paper are:

- Our technique is shown to be robust and computational efficient at detecting monotone pieces from scattered data.
- We provide algorithms to generate the landmarks and abstract the model from the monotonicity analysis.
- The resulting qualitative model is shown to support diagnosis of dynamic faults.

1.1 How Monotonicity Analysis is Needed in Constructing Qualitative Models from Scattered Data

In many applications, we need to construct qualitative models from scattered data. For example, many systems have simulation models when they are designed or verified. The simulation data is a numerical description of system behavior. Model-based diagnosis transforms the numerical values into qualitative values and real functions into qualitative constraints.

Monotonicity is an important feature to leverage when constructing qualitative models. For model-based diagnosis, two kinds of qualitative models are used and both rely on the monotonicity of the function. On the one hand, we have the *Finite Relation Qualitative Model* (FRQ) [7, 10], where the qualitative relation is represented by tuples of real valued intervals. For the monotone segments of f, the tuples are determined by the bounding rectangle of f; that is, the smallest rectangle $\square = [x_a, x_b] \times [y_a, y_b]$ such that $y = f(x) \bigwedge x_a \leq x \leq x_b \Rightarrow (x, y) \in \square$. On the other hand, we have the *Qualitative Deviation Model* [2] where the qualitative relation is represented by the sign of deviation $[\Delta y]$ from a reference point x_{ref} defined as +,-, or 0, according to whether f is increasing, decreasing or flat. For example, if f is monotone increasing, we have $[\Delta y] = \mathrm{sign}(f(x) - f(x_{ref})) = \mathrm{sign}(x - x_{ref}) = [\Delta x]$.

Similarly, to explain human skill from log data, [8] uses the *Qualitative Constrained Function (QCF)* method. A qualitative constrained function takes the form $M^{s_1,\ldots,s_m} : \mathbb{R}^m \mapsto \mathbb{R}$, $s_i \in \{+, -\}$ and represents a function with m real-valued attributes strictly monotone increasing with respect to the i-th attribute if $s_i = +$, or strictly monotone decreasing if $s_i = -$. For example, $f = M^{+,-}(x, y)$ means f is increasing when x is increasing, and decreasing when y is increasing.

However, to be meaningful, the monotone segments need to be significant in relation to the problem. Especially when the data is in the presence of noise, the small fluctuations must be ignored. This requires that noise be removed by computatinally efficient methods such that the number of remaining segments is dependent only on the characterization of the noise. Existing methods do not meet these criteria.

1.2 The Limitation of Existing Analysis Methods

Given a finite series of measurements $x_1, x_2, \ldots, x_n$ over an interval I (a "time series"), we want to partition I into a small number of subintervals where for each subinterval the measures either go up or down, in a general sense.

Linear splines can be used to approximate the data [5], and then segment the domain based on the sign of the slope in a neighborhood of the data point. Alternatively, we could use linear regression to find segments that are closely approximated by a straight line; that is, we no longer require approximation by a continuous linear spline, but only piecewise approximation by straight lines forming a possibly discontinuous function. However, such linear fitting algorithms are relatively expensive or approximate [5].

Inductive learning is used in [8] to automatically construct qualitative models from quantitative examples. The induced qualitative model is a binary tree, called a qualitative tree, which contains internal nodes (called splits) and qualitatively constrained functions at the leaves. A split is a partition of a variable. The learning algorithm QUIN is an unsupervised learning algorithm. It determines the landmarks for the splits. The training data in QUIN is composed of all possible pairs of points from the quantitative data. This quadratic complexity limits the ability of the method to deal with complexity functions where thousands of data points are needed.

1.3 Scale-Dependent Monotonicity

We present the notion of scale-dependent monotonicity for detecting monotone pieces from scattered data.

Given an ordered set of measurements $\{x_k\}$ and some tolerance value $\delta > 0$, we say that the data points are *not going down* or are *upward monotone*, if consecutive measures do not go down by more than δ, that is, are such that $x_i - x_{i+1} > \delta$. However, this definition is not very useful because measures can repeatedly go down and thus the end value can be substantially lower than the start value. A more useful definition of *upward monotonicity* would be to require that we cannot find two successive measures x_i and x_j $(j > i)$ such that x_j is lower than x_i by δ $(x_i - x_j > \delta)$. This definition is more useful because in the worse case, the last measure will be only δ smaller than the first measure. However, we are still not guaranteed that the data does in fact increase at any point. Hence, we add the constraint that we can find at least two successive measures x_k and x_l $(l > k)$ such that x_l is greater than x_k by at least δ $(x_l - x_k \geq \delta)$.

To summarize, given some value $\delta > 0$, we say that a sequence of measures is *upward* δ*-monotone* if no two successive measures decrease by as much as δ, and at least one pair of successive measures increases by at least δ. Similarly, we say that a set of measures is *downward* δ*-monotone* if no two successive measures increase by as much as δ, and at least two measures decrease by at least δ.

This generalized definition of monotonicity was introduced in [1] using $\delta-$pairs: a $\delta-$pair is a pair of successive data points x_i, x_j $(j > i)$ such that $|x_i - x_j| \geq \delta$, and for any k such that $i < k < j$ then $|x_k - x_i| < \delta$ and $|x_k - x_j| < \delta$.The direction of a $\delta-$pair is *upward* (positive) if $x_j > x_i$ and *downward* (negative) if $x_i > x_j$. Notice that $\delta-$pairs having opposite directions cannot overlap.

A sequence of successive measurements x_i, x_{i+1}, ..., x_j is $\delta-$monotone if it contains at least one $\delta-$pair and all $\delta-$pairs have the same sign. We say that the measurements are *upward* $\delta-$monotone (positive) if all $\delta-$pairs are positive and *downward* $\delta-$monotone (negative) if all $\delta-$pairs are negative. A $\delta-$structure is a disjoint partition of the measurements into $\delta-$monotone sets of alternating sign. For a given sequence of measurements, there may be several $\delta-$structures: think of a curve going up, then flat, and then down again – does the flat part of the curve belong to the first or last segment? However, despite some degree of freedom in setting the boundaries between $\delta-$monotone segments, all

$\delta-$structures have the same number of segments. A $\delta-$structure can be computed in linear time ($O(n)$) and constant space ($O(1)$) by scanning sequentially through the measurements, starting a new segment every time we encounter a $\delta-$pair having sign opposite to that of the current segment.

The full paper addresses the following issues:

- choosing the tolerance δ;
- an algorithm of linear time complexity $O(n)$ to compute δ monotone segments;
- joint δ structure for multiple data sets;
- multidimensional case.

1.4 Building Finite Relation Qualitative Model for Diagnosis from Simulation Model

A qualitative model is a description of the system that conveys all physically possible situations and that is as concise as possible with respect to the purpose of model-based problem solving. For Model-based diagnosis, a qualitative model should be fine enough to distinguish the faulty behavior, and also abstract enough to include only the information relevant to the diagnosis task.

Intuitively, a fault is discriminable when the faulty behavior is disjoint from the normal behavior, and the discrepancy is projected on some observables. [3] presents 3 classes of fault discrimination: non-discriminable (ND), deterministically discriminable (DD) and possibly discriminable (PD). A fault is deterministically discriminable if the projection of the faulty mode on some observables is disjoint with the projection of the normal mode on the same set of observables:

$$SIT_{DD} : Proj_{\{obs_1,\ldots,obs_i\}}(R_r) \cap Proj_{\{obs_1,\ldots,obs_i\}}(R_f) = \phi \qquad (1)$$

In (1), R_r and R_w are the normal and faulty models respectively. $\{obs_1, \ldots, obs_i\}$ is the set of observables sufficient to detect the fault, a subset of the total collection of observables. Under certain conditions, if there is at least one observable such that the projections of the two modes can be differentiated, the fault is detectable.

Our model abstraction is an iteratively refining process. It starts from no partition, i.e. the variables take values from the whole domains. The abstracted model is the coarsest in this case. If detectability is not satisfied, the domain of a character variable (as in [3]) is split into two sub-intervals by a newly generated landmark. And the other variables are partitioned accordingly. In this manner, a finer relation is derived. This procedure continues recursively until faults are deterministically discriminablewithin at least one subdivision, or the partition is finer than a preset threshold.

The full paper covers the following issues:

- qualitative relation on δ-monotone segments;
- generating initial landmarks;
- parameter changes;

- multidimensional case;
- an algorithm of qualitative model abstraction;
- supporting the diagnosis of dynamic faults;
- optimizing the abstraction by choosing proper character variables.

The full paper also gives an example on how to abstract the qualitative model from Matlab/Simulink model for diagnosis.

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