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About Determination of Stress Concentration in Bodies of a Complex Shape

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Abstract. The review of researches of stress concentration in elastic bodies with loaded ledges, which are carried out with separate analysis of each of the force factors, is provided. The known results of using of such an approach are shown – for example, the effects of geometrical concentrator's curvature are revealed, and on their base principally new nonpole gear systems are developed. The recommendations for significant refinement of the known method are given.

Introduction

Theoretical researches of the mechanism of stress concentration manifestation using the separate analysis for each of the force factors in elastic bodies with loaded ledges of a complex shape are especially valuable for the theory and practice of mechanical engineering. They can be widely used for strength calculations of details (for example, of a gear type) and for determination of geometrical shape of ledges with decreased stress concentration.

The approximate analytical method [1, 2] is constructed on the solution of the coupled problem of correlation analysis of two plane problems' combination (for example, [3] and [4] – based on the solutions [5] and [6], respectively) and well-known exact (for example, [7]) or numerical (for example, [8]) results. The method [1, 2] is characterized by consideration of each of the force factors (that occur in the loaded by localized load elastic bodies with ledges) separately, and this fact allows you to determine the degree of influence of each of them in every particular case. This is a prerequisite for the formation of the fullest possible understanding of the mechanism of stress concentration action.

The method, described in this paper, is developed as a superposition of solutions for separated force factors and based on local approximations of the elastic surface of a ledge, that allows to reduce the three-dimensional problem to a set of two-dimensional ones.

The solution of the first plane problem: determination of the force factors

It is based on the results, obtained for the force factors along the sealing line of a cantilever plate (Fig. 1) of infinite length under the acting of a concentrated force [6], and extended in [4], for example, to some cases of a distributed load, and also – to the problem of definition of force factors out of the sealing line (Fig. 2).

The first two-dimensional problem is to find an expression for the force factors of the elastic flat plate, which approximates the loaded area. To do this, we use the results [4] whereby the formula for the displacements in the case of loading an infinite elastic cantilever plate with a concentrated force is:

$$W_j^* = W_j^*(\xi,\eta,\zeta) = \frac{P^* A^2}{8\pi N} \int_0^\infty \frac{\varphi_j(\xi,\zeta,\nu)}{\nu^3 \Delta(\nu)} \cos(\nu\eta) d\nu, \tag{1}$$

and, respectively,

$$K_j^* = K_j^*(\xi,\eta,\zeta) = -\frac{\pi N}{P^* A^2} W_j^*(\xi,\eta,\zeta). \tag{2}$$

In the case of a distributed load, if it is understood as a set of concentrated forces (acting along a line parallel to the sealing and changing from point to point in a certain law), the displacement function is presented like $W_j = W_j(P_\Sigma, \xi, \eta, \zeta)$, but, as the variable P_Σ is itself a function of η, then in fact, W_j is a function of three real variables:

$$W_j = W_j(\xi,\eta,\zeta) = \int_0^{2l} W_j^*(\xi,\eta,\zeta)d(\eta) = \int_0^{\rho} W_j^*(\xi,\eta,\zeta)d(\eta), \quad (3)$$

and K_j respectively is:

$$K_j = \int_0^{\rho} K_j^*(\xi,\eta,\zeta)d(\eta), \quad (4)$$

where $\rho = 2l$ – a dimensionless value equal to the ratio of the length of the segment of a distributed load action to the width of a plate.

Considering further the case of parabolic load distributing

$$q = \frac{3P_\Sigma(\rho^2 - \varepsilon^2)}{4\rho^3}, \quad (5)$$

where ε – distance between the center of a segment and a point of load action, we use the formula (4) and the expression for K_j^* obtained by approximating the known results of [4] for the points along the lines of our interest. For example:

1. For the line ξ = 0.7, ζ = 0.7

$$K_j(0.7,\eta,0.7) = \int_0^{\rho}(0.1841 + 0.0178119\eta - 1.05561\eta^2 + 7.4532\eta^3 - 37.8405\eta^4 + \\ +123.111\eta^5 - 257.042\eta^6 + 343.651\eta^7 - 284.226\eta^8 + 132.275\eta^9 - 26.455\eta^{10})d(\eta) = \\ = 0.1841\rho + 0.00890595\rho^2 - 0.35187\rho^3 + 1.8633\rho^4 - 7.5681\rho^5 + 22.0185\rho^6 - \\ -36.7203\rho^7 + 42.9564\rho^8 - 31.5807\rho^9 + 13.2275\rho^{10} - 2.405\rho^{11}. \quad (6)$$

2. For the line ξ = 0.2, ζ = 0.9

$$K_j(0.2,\eta,0.9) = \int_0^{\rho}(0.0263 - 0.0166909\eta + 0.448487\eta^2 - 5.47033\eta^3 + 33.0295\eta^4 - \\ -117.459\eta^5 + 259.15\eta^6 - 358.548\eta^7 + 302.662\eta^8 - 142.471\eta^9 + 28.6596\eta^{10})d(\eta) = \\ = 0.0263\rho - 0.00834545\rho^2 + 0.149496\rho^3 - 1.36785\rho^4 + 6.6059\rho^5 - 19.5765\rho^6 + \\ +37.0214\rho^7 - 44.8185\rho^8 + 33.6291\rho^9 - 14.2471\rho^{10} + 2.60542\rho^{11}. \quad (7)$$

The equations of the type (6) and (7) allow to calculate [5] concentration of power factors (M_k, Q, P) and their corresponding stresses along the ledge under the influence of localized stress. The cases of ξ ≈ ζ are used [1] in the calculation of the additional (median height of the loaded ledge) geometrical concentrator. Cases of ξ ≈ (0.1 … 0.4) ζ are important for determining the stress concentration at the base of the ledge. The values of ξ and ζ are defined taking into account the conditions of sealing, found (for a gear example) with the help of finite elements method in [9].

These solutions (based on [2]) allow calculation of the force factors (and the stresses) to be produced (in the case of a localized load) beyond the sealing line of the ledge.

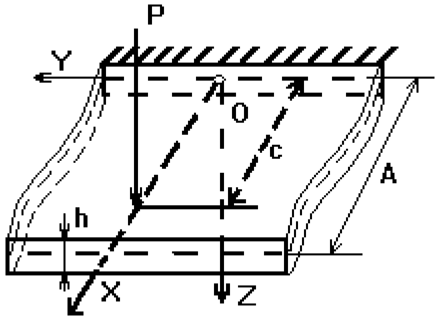

Fig. 1. Loading scheme of the cantilevered plate

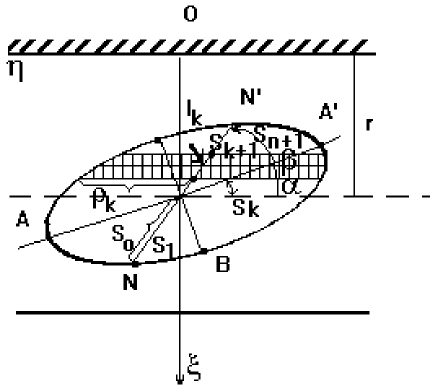

Fig. 2. Building of the equivalent loading

For example, accurate results, obtained by the conformal mapping of tooth-like ledges (open circles shown in Fig. 3) in [7], have been used.To realize the loading scheme of the simulating bar (Fig. 4) the method of input of the system of three force factors, replacing the external power, into the loading scheme of the ledge is used:

bending moment $M_k = K_{B1}'K_{S1}'K_{h1}'F_n H_k \cos(\alpha_d) - K_{B1}''K_{S1}''K_{h1}''F_n \sin(\alpha_d y_k)$,

where: F_n – an external force; α_d - an acute angle between the line of action of the force and the normal to the axis of symmetry of the tooth, the value of which depends on the position of the ledge target point and takes into consideration the additional moment of eccentric compression by the force P;

cutting force $Q = K_{B2}K_{S2}K_{h2}F_n \cos(\alpha_d)$;

compression force $P = K_{B3}K_{S3}K_{h3}F_n \sin(\alpha_d)$;

the influence coefficients here:

K_{Bi} – of the nature of the load distribution on the ledge surface;

K_{Si} - of the influence of neighboring concentrators;

K_{hi} – of the bottom of the model's hyperbolic recess [1, 2, 3].

The stresses determination for a geometrical concentrator

The results are obtained on the basis of method of local approximations [1, 2, 3] using solution [5] for an elastic bar with two hyperbolic recesses.

The solution of the second plane problem is the values of stresses, arising in the area of a geometric concentrator of an elastic ledge. To find them the ledge, being calculated, (solid line in Fig. 3 shows the profile of the tooth of the involute wheel $z = 25$, $\alpha = 20°$, $x = 0$) at any point $C(\beta, \gamma)$ of profile $y=y(x)$ in the area of its geometric concentrator is modeled with a bar (Fig. 4) with two-sided external (e.g. hyperbolic) recesses $y_h = y_h(x)$, whose parameters are determined by solving the system of equations

$$\begin{cases} y' = y_h'; \\ y'' = y_h''; \\ \dfrac{\beta^2}{a^2} - \dfrac{(\gamma + H_0)^2}{b^2} = 1. \end{cases}$$

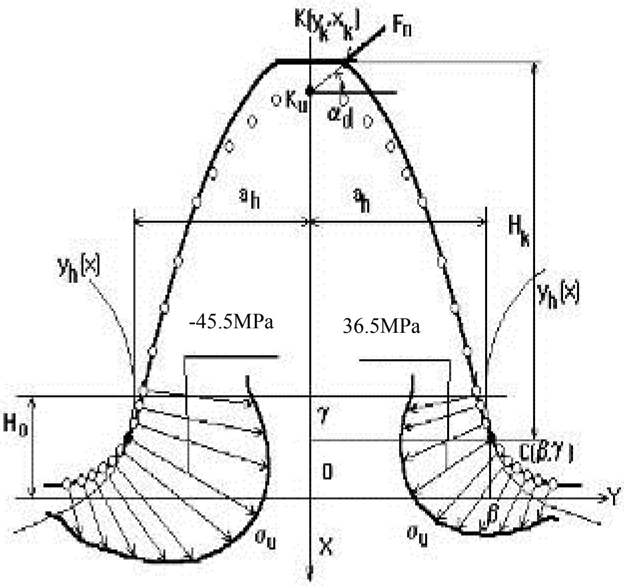

Fig. 3. Calculated form of a tooth being approximated with a bar with hyperbolic recesses

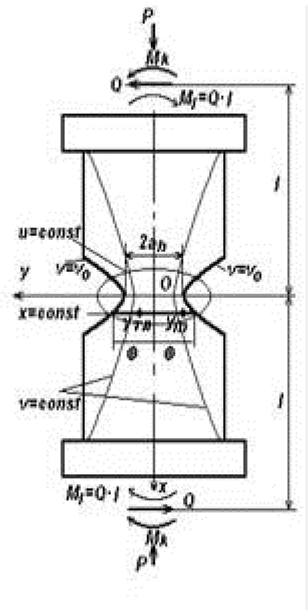

Fig. 4. Design scheme of the bar with hyperbolic recesses

In this case, the shoulder of the external force is assumed independent of the position of the minimum section of the bar.

Sizes of the semi-axes *a* and *b* of the approximating hyperbole and shift H_0 of its real axis along the symmetry axis of the ledge are:

$$H_0 = \frac{\beta \cdot y'}{\beta \cdot y'' + y'^2};$$

$$b = \sqrt{\frac{y'' \cdot (\gamma + H_0)^3}{y' - y'' \cdot (\gamma + H_0)}};$$

$$a = \frac{b \cdot y' \cdot \sqrt{b^2 + (\gamma + H_0)^2}}{\gamma + H_0}.$$

The attempts to use such models (for loaded ledges) are based on the definition of an external force shoulder, which is measured from the minimum section ($2ah$) of the modeling bar. In [1, 2, 3] the use of shoulder H_k, measured from the design section of the ledge, is justified. Compensation of fading impact of the bottom of a hyperbolic recess is realized by correction factors. These coefficients are found by the analysis of correlation of the method with the existing in literature exact values of stresses in dangerous sections of the ledge.

As the flat elastic analog we use an analytic solution obtained by G. Neuber [5] in the elliptical coordinate system (u, v) by using the three harmonic functions, which was developed about the same time by G. D. Grodsky, G. Neuber and P. F. Papkovich.

G. Neuber considered the hyperbolic recess $\pm v_o = const$ as the form of the geometrical concentrator and nominal stresses in the minimal section $x=0$ (of $2a$ width) σ_M (of the pure moment M bending), τ_Q (of the shearing force *Q* shift) and σ_P (of the force *P* compression) as the characteristic of loading of the bar of width *B* $\sigma_M = 1.5\frac{M_k}{a_h{}^2 B}; \tau_Q = \frac{Q}{2a_h B}; \sigma_P = -\frac{P}{2a_h B}$, and found the normal stresses σ_u and σ_v and the tangential stresses σ_{uv} separately for each of the loading forms, using the elliptical coordinates *u*, *v*:

- of bending of the pure moment M_k :

$$\sigma_{uM} = \frac{A_1}{h^2}\sin(2v)\cdot\left[-4+2\cdot(\cos^2(v)-\frac{\cos^2(v_0)}{h^2}\right];\sigma_{vM} = \frac{2A_1}{h^4}\sin(2v)\cdot(\cos^2(v_0)-\cos^2(v));$$

$$\tau_{uvM} = \frac{2A_1}{h^4}\sinh(2u_c)\cdot(\cos^2(v)-\cos^2(v_0));$$

- of the shearing force Q action:

$$\sigma_{uQ} = \frac{A_2}{h^2}\sinh(u_c)\cdot\sin(v)\cdot\left[-2+(\cos^2(v)-\frac{\cos^2(v_0)}{h^2}\right];$$

$$\sigma_{vQ} = \frac{A_2}{h^4}\sinh(u_c)\cdot\sin(v)\cdot\left[\cos^2(v_0)-\cos^2(v)\right];$$

$$\tau_{uvQ} = \frac{A_2}{h^4}\cosh(u_c)\cdot\cos(v)\cdot(\cos^2(v_0)-\cos^2(v));$$

- of the bar compression - stretching:

$$\sigma_{uP} = -\frac{A_3}{h^2}\cosh(u_c)\cdot\cos(v)\cdot\left[-2+(\cos^2(v)-\frac{\cos^2(v_0)}{h^2}\right];$$

$$\sigma_{vP} = \frac{A_3}{h^4}\cosh(u_c)\cdot\cos(v)\cdot(\cos^2(v)-\cos^2(v_0));$$

$$\tau_{uvP} = \frac{A_3}{h^4}\sinh(u_c)\cdot\sin(v)\cdot(\cos^2(v_0)-\cos^2(v)).$$

Applying for the stretching stresses $\sigma_{u\max}$ calculating the principle of the initial sizes' constancy and the superposition of forces, we obtain an expression for the free contour $v = v_0 = \arccos\left[b_h\cdot(a_h{}^2-b_h{}^2)^{-0.5}\right]$:

$$(\sigma_{uc})_v = v_0 = \frac{2}{h^2}(A_3\sinh(u)\cdot\cos(2v_0)-A_2\sinh(u)\cdot\sin(v_0)-2A_1\sin(2v_0)).$$

Then $\sigma_{u\max} = \max\{(\sigma_{uc})_v = v_0, \forall u_c > 0\}$, where the components of harmonic functions are[5]:

$$A_1 = -\frac{\sigma_M\sin^2(v_0)}{3(\sin(2v_0)-2v_0\cos(2v_0))};A_2 = -\frac{\tau_Q\sin(v_0)}{v_0-\sin(v_0)\cos(v_0)};A_3 = \frac{\sigma_P\sin(v_0)}{v_0+\sin(v_0)\cos(v_0)};$$

h – the coefficient of deformation of the orthogonal isometric ($h_u = h_v = h$) coordinates system; $h^2 = \sinh^2(u_c)+\cos^2(v)$.

Conclusion

Built as a superposition of solutions for different force factors and based on local approximations of the ledge's surface, this method is effective, economical and versatile.

The three-dimensional problem for each individual force factor is reduced to a combination of two two-dimensional problems. In three dimensions, the values of M_k, Q, P we obtain with the solution of the problem of the distribution of these force factors including the law (for example, semi-ellipsoidal) of the distribution of the external load and with specified position of the dealing line (outside the calculated section).

With the help of investigations of the mechanism of stress concentration phenomena for a stressed ledge by dividing the action of the three force factors a number of effects inherent to loaded ledges is found and tested: for example, unloading one [8] and the effect of improving the stressed state by decreasing the radius of curvature of the geometric concentrator of profile of the ledge [1, 2]. It is found that in the zones close, to the loading area, bevel of the geometrical concentrator affects the stress concentration the smaller, the smaller the profile angles in the region of the ledge are. If the geometrical concentrator is placed at the head of the ledge, the impact of eccentric compression becomes so large, that decrease of the ρ_p radius 25 times (up to ρ_p=0.2 mm) results reduction of $\sigma_{F\max}$ 4 times. This and the other identified effects are thoroughly checked by numerical experiments, for example, [8] (based on [9]). These effects are widely implemented in improved gears. The detailed qualitative analysis of the phenomena that occur in bodies with elastic ledges under the influence of a localized load have brought valuable results, turned afterwards into practical methods for calculating the parameters of the involute gears of different types for helicopters, locomotives, cars and other machines [10 – 12], for the creation of a fundamentally new type of gearing [13].

The performed analysis of the physical model of a detail with complex shape ledges formed the basis of the design of the recommendations for further development of the of the stress concentration mechanism's research method. For example, updates should be based on the correlation analysis of the results of the updated method and the known ones, which use not only maximum stresses (in the dangerous sections of the geometrical concentrators), but throughout the epure of stretching stresses in the surface.

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Nomenclature

A The width of the plate.
C The distance from the sealing of the plate to the center of the loaded area.
E Young's modulus.
$N = \frac{Eh^3}{12(1-\mu^2)}$ The flexural rigidity of the plate.
P^* The value of the concentrated load.
P_Σ The total value of the distributed load.
W^* The transverse displacement calculated midpoint (thickness) of the surface of the plate for the case of concentrated load.
W The transverse displacement calculated midpoint (thickness) of the surface of the plate for the case of distributed load.
c The arm of the concentrated force.
l The half-length of the segment along which the load is applied.
q The intensity of the load.
x, y Coordinates of the target point.
ξ, η, ζ The dimensionless parameters ($\xi = \frac{x}{A}, \eta = \frac{y}{A}, \zeta = \frac{c}{A}$);
μ Poisson's ratio.

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