Convergence of Sum-Product Algorithm for Finite Length Low-Density Parity-Check Codes

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Abstract — **The convergence behavior of an iterative decoder for low-density parity-check codes is investigated. The influence of finite length codes containing cycles in the factor graph is considered and different types of decoding errors are identified and classified.**

I. Introduction

Low-Density Parity-Check (LDPC) codes [1] decoded with the sum-product algorithm are capable of approaching channel capacity. This algorithm sends messages over the edges of a factor graph representation [2] of the code. For the asymptotic case of infinite block lengths, the convergence of the sum-product algorithm can be analyzed using *density evolution* [3] or *Extrinsic Information Transfer (EXIT) charts* [4]. The sum-product algorithm is optimal—in the sense of minimizing the bit error probability—for a cycle free factor graph. For block length tending to infinity, the assumption of cycle freeness is asymptotically fulfilled, since the probability of cycles decreases with increasing block length. For finite length LDPC codes, the influence of cycles can not be neglected. The number of iterations that can be calculated without violating the assumption is half the girth of the factor graph, i.e. half the length of the shortest cycle. For block lengths ranging from 10^3 to 10^4, in practice every digit of the codeword is involved in a cycle of length 10 or smaller implying a number of cycle free iterations of at most 5.

II. Convergence of the Decoding Process

For infinite length LDPC codes, the decoding process always converges. If the noise of the channel is below a certain threshold, the solution of the decoder will be a codeword. If the noise is above the threshold, the decoder will converge to a vector that is no valid codeword. In the presence of cycles, no such threshold exists. If the noise of the channel is very high or very low, the behavior of a decoder for finite lengths is similar to the behavior for infinite block lengths. In between these extremes, there is a noise region where the decoder tends to oscillate and therefore never converges. We distinguish between the following convergence behaviors:

- *Successful Decoding*: The decoder converges to a valid codeword that is equal to the transmitted one.
- *Error Type I*: The decoder converges but the solution is a vector that is no valid codeword. This error can be detected.
- *Error Type II*: The decoder oscillates and no decision can be made. This error can also be detected.
- *Error Type III*: The decoder converges to a vector that is a valid codeword but not the transmitted one. This type corresponds to an undetected error. From the point of view of the decoder, this error type can not be distinguished from the case of a successful decoding.

In the case of successful decoding (and error type III) the absolute values of the messages tend to infinity. This effect is also caused by cycles, because they induce a positive feedback. This behavior does not decrease the performance but has to be considered to avoid numerical problems.

III. Simulation Results

We simulated a regular LDPC code using an additive white Gaussian noise channel and divided the total block error rate into the three different types of error. The result is shown in figure 1. The dominating types of error at low and high signal to noise ratios are of type I and III respectively as in the asymptotic case. In the transition region the performance of the decoder is dominated by the oscillations.

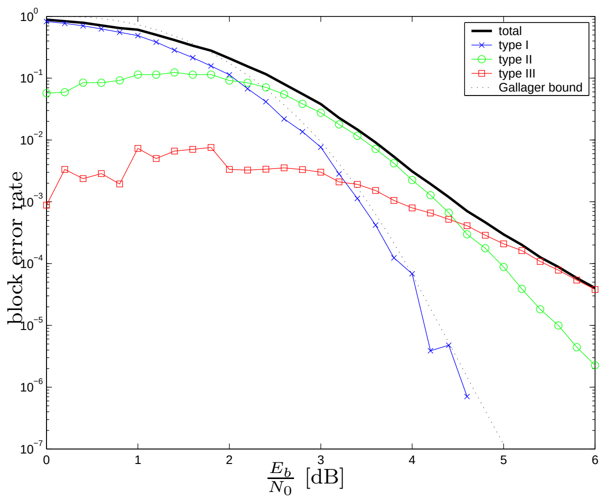

Fig. 1: Error Types of a Regular LDPC Code of Length 100 and Rate 0.5

Further results and visualizations of the decoding process can be found on the web at http://www.ftw.at/ldpc.

References

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