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Johnson, Robert A.

Monterey, California: U.S. Naval Postgraduate School

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CENTRIFUGAL STRESSES IN ROTORS
OF ARBITRARY CROSS SECTION

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Robert A. Johnson

CENTRIFUGAL STRESSES IN ROTORS

OF ARBITRARY CROSS SECTION

by

Robert A. Johnson

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
AERONAUTICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

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This work is accepted as fulfilling

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IN

AERONAUTICAL ENGINEERING

from the

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ABSTRACT

Many rotors of recent design must run at the highest possible peripheral speeds, for example, gas or vapor turbines for space flight. Therefore, to reduce weight, a more accurate and adaptable solution is needed. Although there are many methods of determining the stresses in symmetrical disks it is desirable to have available an accurate solution for rotors of arbitrary cross section.

This paper presents a method for calculation of the centrifugal elastic stresses in rotors of arbitrary and unsymmetrical cross section. The method is essentially a digital computer solution for the elastic stress components.

The solution is restricted to the usual assumption of linearity of stress with strain. Also, the present solution does not take into account variations in Poisson's ratio and rotor density. Provision is made for externally applied stresses such as those due to blades located on the rotor periphery.

Setting up the problem consists only of preparing the data cards. An illustrative example is presented to show the manner of data card preparation.

Due to time limitations program results were not investigated but await further investigations.

This method, when complete, should reduce the time and expense now necessary in utilizing spin test facilities.

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LIST OF ILLUSTRATIONS

SYMBOLS

The following symbols are used:

P	Imaginary axial load, (1 lb./rad.)
Q	Imaginary radial load, (1 lb./rad.)
r	Radial distance, (in.)
z	Axial distance, (in.)
ν	Poisson's ratio
ρ	Mass density of rotor material, (lb. sec./in.)
σ_r	Radial stress, (lb./sq. in.)
σ_θ	Tangential stress, (lb./sq. in.)
σ_z	Axial stress, (lb./sq. in.)
τ_{rz}	Shear stress, (lb./sq. in.)
ω	Angular velocity of rotor, (rad./sec.)
r_0	Radial distance to assumed point of imaginary force application, (in.)
Φ	Airy's stress function
θ	Angle measured from vertical to r_o (also used as an axis) (rad.)
R	Distance from rotor boundary point to point of assumed load
ψ	Angle measured as shown in Fig. 1
ϕ	Angle measured as shown in Fig. 2
N	Normal
T	Tangential
Π	Stress dyadic [3]
$\vec{\iota}_r$	Unit vector as shown in Fig. 1

The following supplementary subscripts and superscripts are used for denoting values of the preceding symbols:

n -nth point station on the rotor periphery where the boundary conditions are known and used in the solution

m -mth point station on the imaginary boundary where ring forces are applied

' -denotes the axis system in connection with the assumed axial (P) force

Example of the use of a double subscript:

$\sigma_{r_{nm}}^{(P)}$ -the radial stress at point n caused by an axial (P) load application at point m

The following supplementary symbols denote combinations of the foregoing symbols arising in the analysis.

B -undetermined coefficient occurring in the stress function

$$\Phi = B(r^2 + z^2)^{1/2}$$

Relationship of coordinates in connection with the axial (P) load axis system defined in Fig. 1:

$$z' = z - z_o$$

$$(r')^2 = r^2 + r_o^2 - 2rr_o\cos\theta$$

$$R^2 = r^2 + r_o^2 - 2rr_o\cos\theta + (z - z_o)^2$$

$$\sin^2\psi = \frac{r_o^2\sin^2\theta}{r^2 + r_o^2 - 2rr_o\cos\theta}$$

$$\cos^2\psi = \frac{(r - r_o\cos\theta)^2}{r^2 + r_o^2 - 2rr_o\cos\theta}$$

Non-dimensionalized functions:

$$\xi_1 = \frac{r}{r_0}$$

$$\xi_2 = \frac{z'}{r_0} \qquad \xi_2 = \frac{z - z_0}{r_0}$$

$$\xi_3 = \frac{r'}{r_0} \qquad \xi_3^2 = \left(\xi_1^2 + 1 - 2\xi_1 \cos\theta\right)$$

$$\xi_4 = \frac{R}{r_0} \qquad \xi_4^2 = \left(\xi_1^2 + 1 - 2\xi_1 \cos\theta + \xi_2^2\right)$$

$$\sin^2\psi = \frac{\sin^2\theta}{\xi_1^2 + 1 - 2\xi_1\cos\theta} = \frac{\sin^2\theta}{\xi_3^2}$$

$$\cos^2\psi = \left(\frac{\xi_1 - \cos\theta}{\xi_3}\right)^2$$

$$\sigma_p = \frac{1}{8\pi(1-\nu)\,r_0^2}$$

Relationship of coordinates in connection with the radial (Q) load axis system defined in Fig. 2:

$$z'' = r\cos\theta - r_0$$

$$(r'')^2 = r^2\sin^2\theta + (z - z_0)^2$$

$$R^2 = r^2 + r_0^2 - 2r\,r_0\cos\theta + (z - z_0)^2$$

$$\sin^2\phi = \frac{(z - z_0)^2}{r^2\sin^2\theta + (z - z_0)^2}$$

$$\cos^2\phi = \frac{(r\sin\theta)^2}{r^2\sin^2\theta + (z - z_0)^2}$$

Non-dimensionalized functions:

$$f_5 = \frac{z''}{r_0} \qquad f_5 = f_1 \cos\theta - 1$$

$$f_6 = \frac{r''}{r_0} \qquad f_6^2 = f_1^2 \sin^2\theta + f_2^2$$

$$\sin^2\phi = f_2^2 / f_6^2$$

$$\cos^2\phi = f_1^2 \sin^2\theta / f_6^2$$

$$\sigma_Q = \frac{1}{8\pi(1-\nu)\, r_0^2}$$

The following combinations of the foregoing symbols are added for simplicity in the analysis and problem programming:

$$f_0 = \sin^2\theta \qquad D = \cos\theta$$

$$f_7 = (1-2\nu)/f_4^3$$

$$f_8 = f_4^5$$

$$f_9 = f_5 f_7$$

$$f_{10} = f_9 - 3 f_5 f_6^3 / f_8$$

$$f_{11} = f_9 + 3 f_5^3 / f_8$$

$$f_{12} = f_6 f_7 + 3 f_6 f_5^2 / f_8$$

$$f_{13} = f_2^2 / f_6^2$$

$$f_{14} = f_1 / f_6$$

$$f_{15} = f_1 f_2 f_0 / f_6^2$$

INTRODUCTION

Many rotors of recent design must run at the highest possible peripheral speeds. For example, gas or vapor turbines for space flight such as those proposed by Corliss [2]. The high speeds will allow fewer stages, hence less weight. Therefore a solution is needed that is more accurate, complete, and adaptable than the approximate methods used in the past.

Many of the problems in stress analysis, which are of practical importance, are concerned with solids of revolution deformed symmetrically with respect to the axis of revolution. Although there are many methods of determining the stresses in <u>symmetrical</u> disks, it is desirable to have available an accurate solution for rotors of <u>arbitrary</u> cross section.

One of the problems in the design of rotors is the determination of the stresses under operating conditions. Calculation of the elastic stress distribution is of major importance in determining the true stress distribution. Although the true stress distribution in an actual rotor may go beyond the proportional elastic limit of the material, the solution in this paper is restricted to the usual assumption of linearity of stress with strain.

Another difficulty is that the physical properties of the materials may vary with temperature and hence may have somewhat different values at different points in the rotor. The present solution does not take into account such variations, treating Poisson's ratio and rotor density

as constants.

The equations of equilibrium for the elastic stress distribution with polar symmetry are well known [1]. However, the solution may entail extremely lengthy and tedious calculations. Therefore, it necessarily becomes a computer problem.

This paper presents a method for calculation of the centrifugal elastic stresses in rotors of arbitrary and unsymmetrical cross section. It is but the first in a series of investigations to utilize the spin test facility located at the United States Naval Postgraduate School, Monterey, California.

The method is essentially a digital computer solution for the elastic stress components. A particular solution is known which satisfies the complete equilibrium and compatibility equations, and which includes the effects of centrifugal forces [1]. However, a complementary solution must be found such that, when superimposed upon the particular solution, the resulting combination will satisfy the required boundary conditions for the specific case under consideration. This complementary solution consists of a linear combination of functions which individually satisfy the stress field equations. The centrifugal force terms must be absent from the field equations when applied to the complementary functions.

The complementary stress components may be thought of as being produced by the application of ring loads in axial and radial directions. These loads are assumed to act at a number of points arranged along an imaginary boundary as depicted in Fig. 3. The stress distributions

corresponding to a ring load of unit intensity are unknown but may be found by suitable integration of the known results for a simple concentrated load. The resulting stresses, each multiplied by an as yet undetermined load amplitude, are summed in equations which define the prescribed boundary conditions. The known coefficients in these equations constitute a stress matrix. The unknowns of the matrix are, of course, the amplitudes of the various assumed ring loads. The unknowns may be found by computer iteration. The complete stress distribution is then fixed by substitution of the constants so obtained back into the equations of stress.

This computer solution has been developed so as to minimize engineering preparation and supervision. All details of the calculations, such as matrix generation and solution, are done by the computer. Setting up the problem, which consists only of preparing the data cards, requires very little time. Computer running time may be altered by varying the accuracy desired.

Use of the present program requires an input which defines the geometry and stresses, if any, at the rotor boundary, and which also specifies the assumed application points of the fictitious ring loads. The boundary stresses are unrestricted except for the fact that they must, of course, constitute a system of forces in equilibrium. Additional inputs consist of the material properties and rotor speed. All other operations are performed automatically by the computer.

In Appendix C of this paper an illustrative example is presented to show the manner of data card preparation. The example is self-explanatory

and may be used as a guide without reference to the analytical section of the report.

This method, when fully completed, should prove to be quite useful in the design of new rotors and investigation of present rotors. The computer solution would reduce much of the time and expense now necessary in utilizing spin test facilities.

ANALYSIS

Consider a rotor of homogeneous density ρ and of arbitrary cross section, as in Fig. 3, rotating at a constant angular velocity ω.

The assumption of polar symmetry means that the stress components are independent of the angle θ. Hence, all derivatives with respect to θ disappear. The shearing stress components $\tau_{r\theta}$ and $\tau_{z\theta}$ also disappear due to symmetry. Thus the governing differential equations of equilibrium in cylindrical coordinates are [1]

$$\begin{aligned} \frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} + \rho \omega^2 r &= 0 \\ \frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} &= 0 \end{aligned} \tag{1}$$

The major portion of the analysis will be concerned with the complementary solution for which Eqs. (1), by elimination of the centrifugal body force term, reduce to

$$\begin{aligned} \frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} &= 0 \\ \frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} &= 0 \end{aligned} \tag{2}$$

It is convenient to satisfy Eqs. (2) through the introduction of a stress function Φ. It may be verified by direct substitution that the conditions of equilibrium, Eqs. (2), are satisfied identically by the

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following relations

$$\sigma_r = \frac{\partial}{\partial z}\left(\nu \nabla^2 \Phi - \frac{\partial^2 \Phi}{\partial r^2}\right)$$

$$\sigma_\theta = \frac{\partial}{\partial z}\left(\nu \nabla^2 \Phi - \frac{1}{r}\frac{\partial \Phi}{\partial r}\right) \tag{3}$$

$$\sigma_z = \frac{\partial}{\partial z}\left[(2-\nu)\nabla^2 \Phi - \frac{\partial^2 \Phi}{\partial z^2}\right]$$

$$\tau_{rz} = \frac{\partial}{\partial r}\left[(1-\nu)\nabla^2 \Phi - \frac{\partial^2 \Phi}{\partial z^2}\right]$$

In addition, the compatability relations among the strains require [1] that the stress function Φ satisfy the biharmonic equation

$$\nabla^4 \Phi = 0 \tag{4}$$

Consider a family of functions of $\phi_1(r,z), \phi_2(r,z), \phi_3(r,z)$ ....
...$\phi_n(r,z)$, each one of which satisfies Eq. (4). Then any linear combination of these functions also satisfies this equation and therefore represents a possible stress function, that is

$$\Phi = \sum_{n=1}^{k} C_n \phi_n(r,z)$$

where C_1, C_2, C_3,...C_n represent arbitrary constants. It will be shown that these constants may be determined so as to satisfy a given set of boundary conditions. Upon substitution of the general stress function Φ into Eqs. (3), the resulting stress components are reducible to the form

$$\sigma_r = \sum_{n=1}^{k} C_n \Psi_n(r,z)$$

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$$[illegible]$$

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$$\sigma_\theta = \sum_{n=1}^{k} C_n \Xi_n(r,z)$$

$$\sigma_z = \sum_{n=1}^{k} C_n X_n(r,z)$$

$$\tau_{rz} = \sum_{n=1}^{k} C_n \Omega_n(r,z)$$

where ψ_n, Ξ_n, X_n and Ω_n are known functions if ϕ_n is a known function.

It can be seen in Fig. 3 that axial and radial ring loads are assumed applied at an imaginary boundary a preselected distance away from the actual rotor surface. This was done to avoid the introduction of infinite stresses along the actual rotor boundary.

The ring stress components are unknown but are found by suitable integration of the known results for a simple concentrated load. Timoshenko [1] shows that a stress function corresponding to a single concentrated load acting at a point in an indefinitely extended solid is

$$\Phi = B(r^2 + z^2)^{1/2} \tag{5}$$

This can be shown to satisfy the biharmonic equation. Upon substitution of this stress function Φ into the stress relations Eqs. (3), the corresponding stress components are found.

$$\sigma_r = B\left[(1-2\nu)z(r^2+z^2)^{-3/2} - 3r^2z(r^2+z^2)^{-5/2}\right]$$

$$\sigma_\theta = B\left[(1-2\nu)z(r^2+z^2)^{-3/2}\right] \qquad (6)$$

$$\sigma_z = -B\left[(1-2\nu)z(r^2+z^2)^{-3/2} + 3z^2(r^2+z^2)^{-5/2}\right]$$

$$\tau_{rz} = -B\left[(1-2\nu)r(r^2+z^2)^{-3/2} + 3rz^2(r^2+z^2)^{-5/2}\right]$$

It is seen that all of these stresses approach infinity when the point of load application nears the origin of coordinates. This explains the choice of an imaginary boundary some distance away from the actual rotor boundary for load application. In this connection, consider Fig. 4, where the element shown is a portion of a larger ring shaped element. The point n in Fig. 4 is considered to be the center of a small spherical cavity with the ring being part of the surrounding solid. It can be shown [1] that the resultant passes through the origin in the direction of the z-axis. Summing forces in the z-direction, the component of the surface force is

$$\bar{Z} = -(\tau_{rz}\sin\beta + \sigma_z\cos\beta) \qquad (7)$$

By substitution of Eqs. (6) into Eqs. (7) and by the use of the trigonometric identities

$$\sin\beta = r(r^2+z^2)^{-1/2}$$

$$\cos\beta = z(r^2+z^2)^{-1/2}$$

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the following relation is found.

$$\bar{Z} = B\left[(1-2\nu)(r^2+z^2)^{-1} + 3z^2(r^2+z^2)^{-2}\right]$$

The resultant axial force may now be found by integration, hence

$$2\int_0^{\pi/2} \bar{Z}\,(r^2+z^2)^{1/2} \cdot d\beta \cdot 2\pi r = 8B\pi(1-\nu)$$

From symmetry the resultant in the radial direction is zero.

Consider now a ring along which is applied a distributed axial load P, that is, a load of P pounds per radian. The magnitude of the force acting on an element $d\theta$ of the ring is then $P\,d\theta$. This may be treated essentially as an infinitesimal concentrated load acting at the point in question. Associated with this load is a corresponding stress function and stress distribution of the same form as derived above except, of course, that all quantities are now infinitesimals. In particular, the former constant B is now replaced by the infinitesimal dB. Also, from the above solution for the resultant axial force the following may be written

$$P\,d\theta = 8\,dB\,\pi(1-\nu)$$

Now, in order to obtain convenient stress influence functions and influence coefficients for subsequent use, specifically consider a unit ring load, that is, set P equal to one pound per radian. The previous relation now gives

$$dB = \frac{(1)\,d\theta}{8\pi(1-\nu)}$$

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This term is then substituted into the stress component equations in differential form. The first of Eqs. (6) becomes

$$d\sigma_r' = dB\left\{\frac{(1-2\nu)z'}{R^3} - \frac{3z'(r')^2}{R^5}\right\}$$

where R has already been defined. After substitution for dB

$$d\sigma_r' = \frac{(1)}{8\pi(1-\nu)}\left\{\frac{(1-2\nu)z'}{R^3} - \frac{3z'(r')^2}{R^5}\right\}d\theta$$

It is advantageous to change this into dimensionless form by dividing through by the square of the reference radius r_0 . Also let

$$\sigma_p = \frac{P}{8\pi(1-\nu)r_0^2} = \frac{(1)}{8\pi(1-\nu)r_0^2}$$

Hence the previously found stress distributions reduce to

$$d\sigma_r' = \sigma_p\left\{\frac{(1-2\nu)\xi_2}{\xi_4^3} - \frac{3\xi_2\xi_3^3}{\xi_4^5}\right\}d\theta$$

$$d\sigma_\theta' = \sigma_p\left\{\frac{(1-2\nu)\xi_2}{\xi_4^3}\right\}d\theta$$

(8)

$$d\sigma_z' = -\sigma_p\left\{\frac{(1-2\nu)\xi_2}{\xi_4^3} + \frac{3\xi_2^3}{\xi_4^5}\right\}d\theta$$

$$d\tau_{rz}' = -\sigma_p\left\{\frac{(1-2\nu)\xi_3}{\xi_4^3} + \frac{3\xi_3\xi_2^2}{\xi_4^5}\right\}d\theta$$

Next, vector equations are written to translate the coordinate system related to the axial load to that of the fixed system as in Fig. 1.

$$\vec{\tau}_r = \cos\psi\,\vec{\tau}_{r'} + \sin\psi\,\vec{\tau}_{\theta'} + 0$$
$$\vec{\tau}_\theta = -\sin\psi\,\vec{\tau}_{r'} + \cos\psi\,\vec{\tau}_{\theta'} + 0$$
$$\vec{\tau}_z = 0 + 0 + \vec{\tau}_{z'}$$

By the use of dyadics [3] the following relations may be written

$$d\sigma_r = \vec{\tau}_r \cdot d\Pi \cdot \vec{\tau}_r$$
$$d\sigma_\theta = \vec{\tau}_\theta \cdot d\Pi \cdot \vec{\tau}_\theta$$
$$d\sigma_z = \vec{\tau}_z \cdot d\Pi \cdot \vec{\tau}_z$$
$$d\tau_{rz} = \vec{\tau}_r \cdot d\Pi \cdot \vec{\tau}_z$$

where

$d\Pi$ =

	$\vec{\tau}_{r'}$	$\vec{\tau}_{\theta'}$	$\vec{\tau}_{z'}$
$\vec{\tau}_{r'}$	$d\sigma_r'$	0	$d\tau_{r'z'}$
$\vec{\tau}_{\theta'}$	0	$d\sigma_\theta'$	0
$\vec{\tau}_{z'}$	$d\tau_{r'z'}$	0	$d\sigma_z'$

The resulting stresses in the fixed coordinate system are

$$\begin{aligned} d\sigma_r &= \cos^2\psi\, d\sigma_r' + \sin^2\psi\, d\sigma_\theta' \\ d\sigma_\theta &= \sin^2\psi\, d\sigma_r' + \cos^2\psi\, d\sigma_\theta' \\ d\sigma_z &= d\sigma_z' \\ d\tau_{rz} &= \cos\psi\, d\tau_{r'z'} \end{aligned} \qquad (9)$$

Since all of the terms on the right side of Eqs. (9) have been expressed previously, substitution gives

$$d\sigma_r = \sigma_P \left\{ \frac{(1-2\nu) f_2}{f_4^3} - \frac{3 f_2 (f_1 - \cos\theta)^2}{f_4^5} \right\} d\theta$$

$$d\sigma_\theta = \sigma_P \left\{ \frac{(1-2\nu) f_2}{f_4^3} - \frac{3 f_2 \sin^2\theta}{f_4^5} \right\} d\theta$$

$$d\sigma_z = -\sigma_P \left\{ \frac{(1-2\nu) f_2}{f_4^3} + \frac{3 f_2^3}{f_4^5} \right\} d\theta$$

$$d\tau_{rz} = -\sigma_P \left\{ \left[\frac{(1-2\nu)}{f_4^3} + \frac{3 f_2^2}{f_4^5} \right] (f_1 - \cos\theta) \right\} d\theta$$

The above differential stress components may now be integrated around the complete ring to obtain the final stress components for the unit axial load. These stress components are those shown in Fig. 5. They are the individual stresses induced at point n on the rotor surface by application of fictitious unit axial loads at points m on the imaginary boundary. The final stress components due to the axial load (P) are

$$\sigma_{r_{nm}}^{(P)} = 2\sigma_P \int_0^\pi f_2 \left\{ \frac{(1-2\nu)}{f_4^3} - \frac{3(f_1 - \cos\theta)^2}{f_4^5} \right\} d\theta$$

$$\sigma_{\theta nm}^{(P)} = 2\sigma_P \int_0^\pi f_2 \left\{ \frac{(1-2\nu)}{f_4^3} - \frac{3 \sin^2\theta}{f_4^5} \right\} d\theta \qquad (10)$$

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$$\sigma_{z_{nm}}^{(P)} = -2\sigma_P \int_0^{\pi} f_2 \left\{ \frac{(1-2\nu)}{f_4^3} + \frac{3 f_2^2}{f_4^5} \right\} d\theta$$

$$\tau_{rz_{nm}}^{(P)} = -2\sigma_P \int_0^{\pi} \left\{ \left[\frac{(1-2\nu)}{f_4^3} + \frac{3 f_2^2}{f_4^5} \right] (f_1 - \cos\theta) \right\} d\theta$$

Derivation of the stress components due to an applied radial load (Q) is carried out in Appendix A. The coordinate relationships of Fig. 2 must be used. The solution, although more involved and tedious, is otherwise much the same. The final results are

$$\sigma_{r_{nm}}^{(Q)} = 2\sigma_Q \int_0^{\pi} \left\{ f_{14}^2 f_0^2 f_{10} - 2 f_{14} f_0 f_{12} D + f_0 f_{13} f_9 - D^2 f_{11} \right\} d\theta$$

$$\sigma_{\theta_{nm}}^{(Q)} = 2\sigma_Q \int_0^{\pi} \left\{ f_{14}^2 f_0 f_{10} D^2 + 2 f_0 f_{12} f_{14} D + f_{13} f_9 D^2 - f_0 f_{11} \right\} d\theta$$

$$\sigma_{z_{nm}}^{(Q)} = 2\sigma_Q \int_0^{\pi} \left\{ f_{10} f_{13} + f_{14}^2 f_0 f_9 \right\} d\theta$$

$$\tau_{rz_{nm}}^{(Q)} = 2\sigma_Q \int_0^{\pi} \left\{ f_{10} f_{15} - f_9 f_{15} - \frac{f_2}{f_6} f_{12} D \right\} d\theta$$

These stress components in the r and z coordinate system must now be transformed to indicate stresses in directions normal and tangential to the boundary surface at each point. The applicable equations are

$$\sigma_{N_{nm}} = \frac{\sigma_{r_{nm}} + \sigma_{z_{nm}}}{2} - \frac{\sigma_{r_{nm}} - \sigma_{z_{nm}}}{2}\cos(2\lambda) - \tau_{rz_{nm}}\sin(2\lambda)$$

$$\tau_{T_{nm}} = -\left(\frac{\sigma_{r_{nm}} - \sigma_{z_{nm}}}{2}\right)\sin(2\lambda) + \tau_{rz_{nm}}\cos(2\lambda) \tag{12}$$

where λ is the relative angle of the rotor surface with respect to the radial plane.

Next, the axial and radial unit loads applied at each point m are considered to be each multiplied by the as yet undetermined constants, P_m and Q_m. There are, of course, $2\cdot k$ terms for each point n, one for each of the k axial loads, and one for each of the k radial loads. The resulting terms are then summed. The boundary stress equations thereby obtained for the complementary solution are reducible to the form

$$\sigma_{N_n} = \sum_{m=1}^{k}\left\{P_m \sigma_{N_{nm}}^{(P)} + Q_m \sigma_{N_{nm}}^{(Q)}\right\}$$

$$\tau_{T_n} = \sum_{m=1}^{k}\left\{P_m \tau_{T_{nm}}^{(P)} + Q_m \tau_{T_{nm}}^{(Q)}\right\} \tag{13}$$

The boundary conditions must be known at each point n on the rotor. It is a necessary condition that the original boundary conditions given

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for the problem maintain the rotor in equilibrium. In most cases the boundary stresses will be zero except where known forces are applied such as those due to blade or shaft loading.

It is necessary at this point to introduce the centrifugal effects. A particular solution, which includes these effects, is derived in Appendix B. The centrifugal stresses are

$$\sigma_{r_n}^{(\omega)} = -\frac{\rho\omega^2 r_n^2}{3} - \frac{\rho\omega^2(1+2\nu)(1+\nu)z_n^2}{6\nu(1-\nu)}$$

$$\sigma_{\theta_n}^{(\omega)} = -\frac{\rho\omega^2(1+2\nu)(1+\nu)z_n^2}{6\nu(1-\nu)}$$

$$\sigma_{z_n}^{(\omega)} = \frac{\rho\omega^2(1+3\nu)r_n^2}{6\nu}$$

$$\tau_{rz_n}^{(\omega)} = 0$$

These centrifugal stress components are also substituted into the normal and tangential boundary stress equations, Eqs. (12).

The resulting boundary stresses of the particular solution, $\sigma_{N_n}^{(\omega)}$ and $\tau_{T_n}^{(\omega)}$ may now be superimposed upon the boundary stresses of the complementary solution and the sum set equal to the assigned boundary stresses. The resulting matrix consisting of 2·k equations may be summarized in the form below.

$$P_1\sigma_{N_{11}}^{(P)} + Q_1\sigma_{N_{11}}^{(Q)} + P_2\sigma_{N_{12}}^{(P)} + Q_2\sigma_{N_{12}}^{(Q)} + --- + P_K\sigma_{N_{1K}}^{(P)} + Q_K\sigma_{N_{1K}}^{(Q)} = \sigma_{N_1} - \sigma_{N_1}^{(\omega)}$$

$$P_1\tau_{T_{11}}^{(P)} + Q_1\tau_{T_{11}}^{(Q)} + P_2\tau_{T_{12}}^{(P)} + Q_2\tau_{T_{12}}^{(Q)} + --- + P_K\tau_{T_{1K}}^{(P)} + Q_K\tau_{T_{1K}}^{(Q)} = \tau_{T_1} - \tau_{T_1}^{(\omega)}$$

$$P_1\sigma_{N_{21}}^{(P)} + Q_1\sigma_{N_{21}}^{(Q)} + ----------- + P_K\sigma_{N_{2K}}^{(P)} + Q_K\sigma_{N_{2K}}^{(Q)} = \sigma_{N_2} - \sigma_{N_2}^{(\omega)}$$

$$P_1\sigma_{N_{K1}}^{(P)} + Q_1\sigma_{N_{K1}}^{(Q)} + --------- + P_K\sigma_{N_{KK}}^{(P)} + Q_K\sigma_{N_{KK}}^{(Q)} = \sigma_{N_K} - \sigma_{N_K}^{(\omega)}$$

$$P_1\tau_{T_{K1}}^{(P)} + Q_1\tau_{T_{K1}}^{(Q)} + -------- + P_K\tau_{T_{KK}}^{(P)} + Q_K\tau_{T_{KK}}^{(Q)} = \tau_{T_K} - \tau_{T_K}^{(\omega)}$$

This then fixes the final matrix which can now be solved for the necessary constant coefficients, P_m and Q_m. Once found, the coefficients may then be substituted back into the following stress equations thus obtaining the stresses desired.

$$\sigma_{r_n} = \sum_{m=1}^{k} \left\{ P_m \sigma_{r_{nm}}^{(P)} + Q_m \sigma_{r_{nm}}^{(Q)} \right\} + \sigma_{r_n}^{(\omega)}$$

$$\sigma_{\theta_n} = \sum_{m=1}^{k} \left\{ P_m \sigma_{\theta_{nm}}^{(P)} + Q_m \sigma_{\theta_{nm}} \right\} + \sigma_{\theta_n}^{(\omega)}$$

$$\sigma_{z_n} = \sum_{m=1}^{k} \left\{ P_m \sigma_{z_{nm}}^{(P)} + Q_m \sigma_{z_{nm}}^{(Q)} \right\} + \sigma_{z_n}^{(\omega)}$$

$$\tau_{rz_n} = \sum_{m=1}^{k} \left\{ P_m \tau_{rz_{nm}}^{(P)} + Q_m \tau_{rz_{nm}}^{(Q)} \right\}$$

The foregoing solution is in program form in Appendix D. Terms have been named to resemble as nearly as possible those used in the analysis. At specific points in the program where a change in the solution occurs, appropriate comment cards have been entered to assist in making minor changes if they are found desirable.

DISCUSSION AND RESULTS

The analysis just presented was formulated to solve for the centrifugal elastic stresses in rotors of arbitrary cross section. However, the thermal stress solution still remains to be found. This paper is but the first in a series of investigations to utilize spin test facilities.

The final stress component equations are believed to be correct, although, due to the limited time remaining, a thorough investigation of computer results has not yet been made.

Only one cross section was investigated, again due to time limitations. A symmetrical disk was chosen so that symmetry could be checked in all pertinent values and the resulting stresses compared with those of thin disk theory.

The result of this paper is primarily the computer program of Appendix D. It has been designed to obtain the elastic stresses in a very short time and yet require very little engineering preparation and supervision.

There are certain changes in the program and in data preparation that may improve the accuracy of results. There are two primary changes that can be made. The TEST value of the integration program SIMCON may be decreased. The other change consists of varying the distance of the imaginary boundary from the rotor boundary. An optimum position is yet to be found that will provide a smooth load contour and yet produce the desired accuracy. Another similar means of improving the accuracy would be to increase the number of points under investigation. All such changes, of course, increase the computer running time. The present program can

investigate twenty five points in approximately fifteen minutes. Other changes in the program may reduce the running time even further.

It is felt that any minor errors now present in the computer program can be resolved. Once corrected, the program should give the stresses desired for any cross section.

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ACKNOWLEDGMENT

The generous assistance and encouragement of Professor T. H. Gawain of the United States Naval Postgraduate School is gratefully acknowledged.

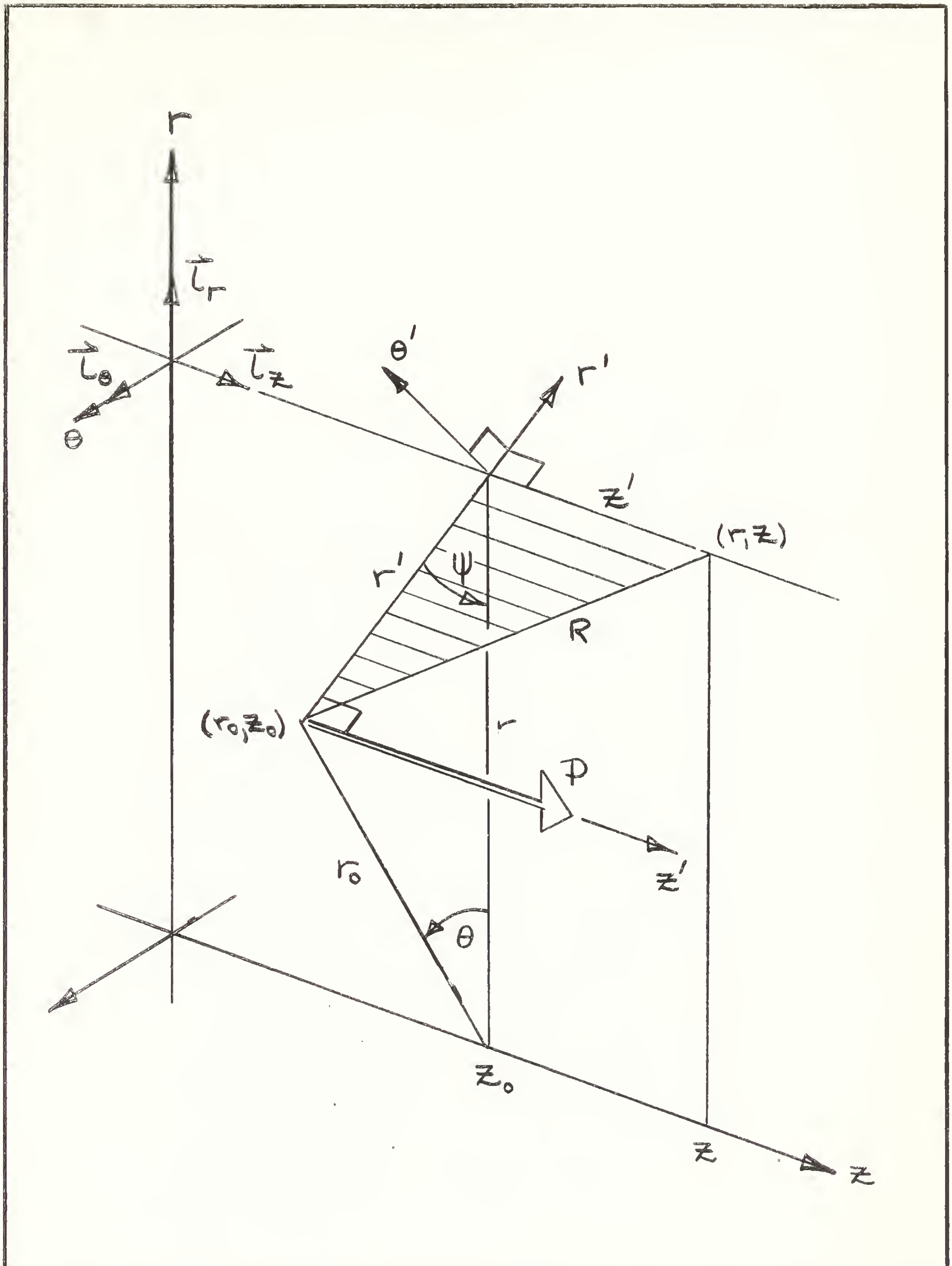

Fig. 1 Axial Load Coordinate System

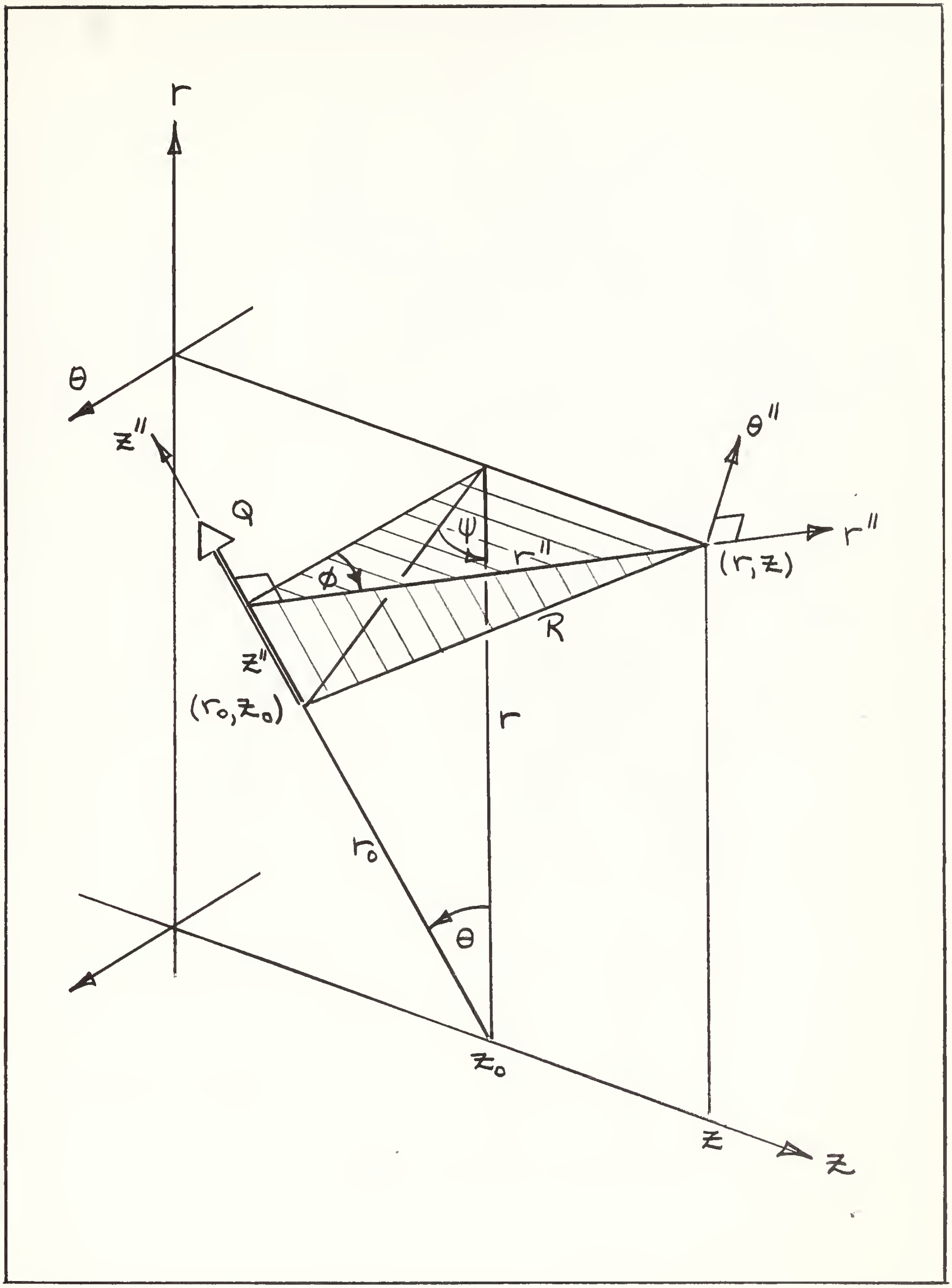

Fig. 2 Radial Load Coordinate System

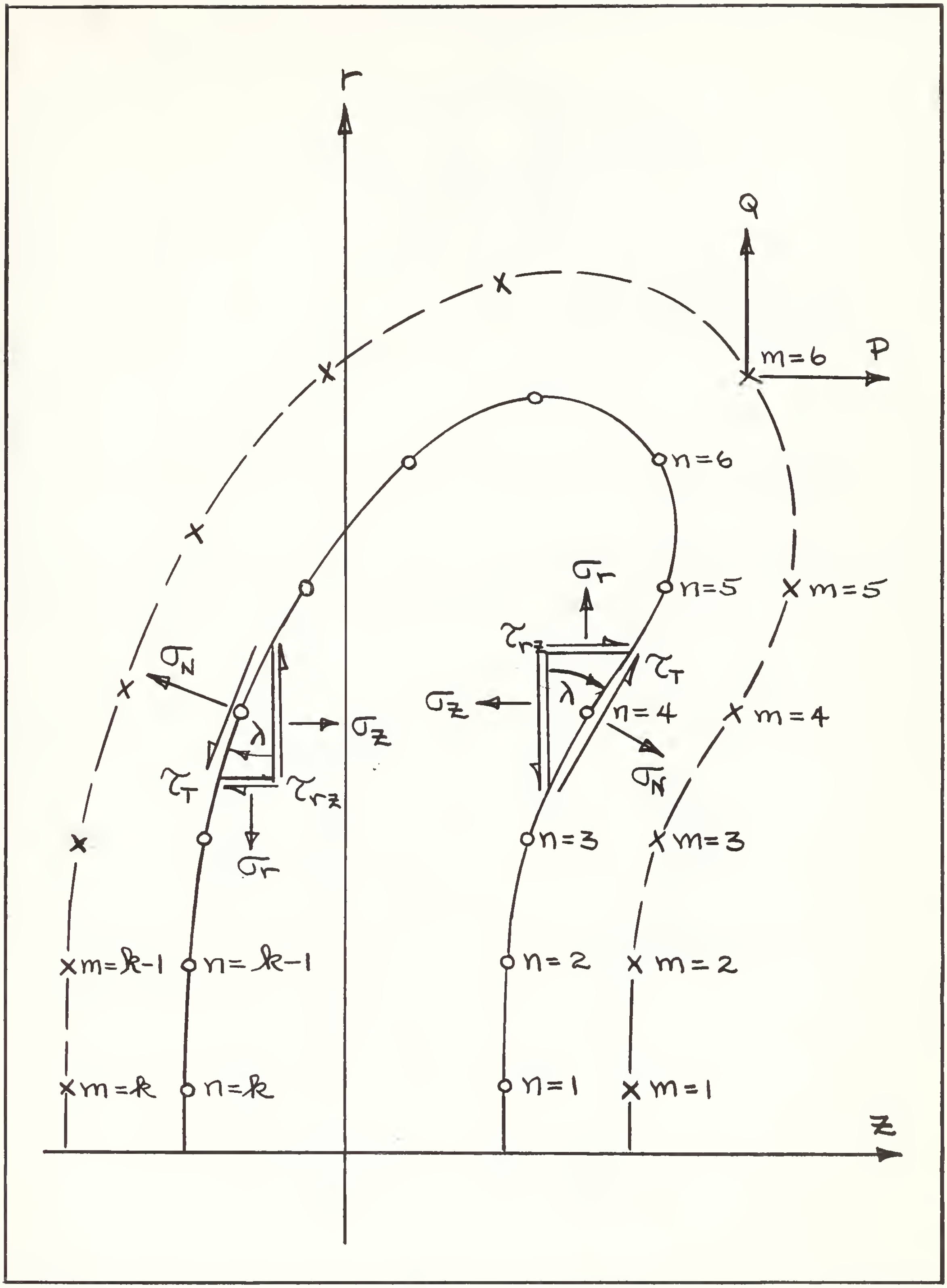

Fig. 3 Arbitrary Rotor Cross Section with Imaginary Boundary

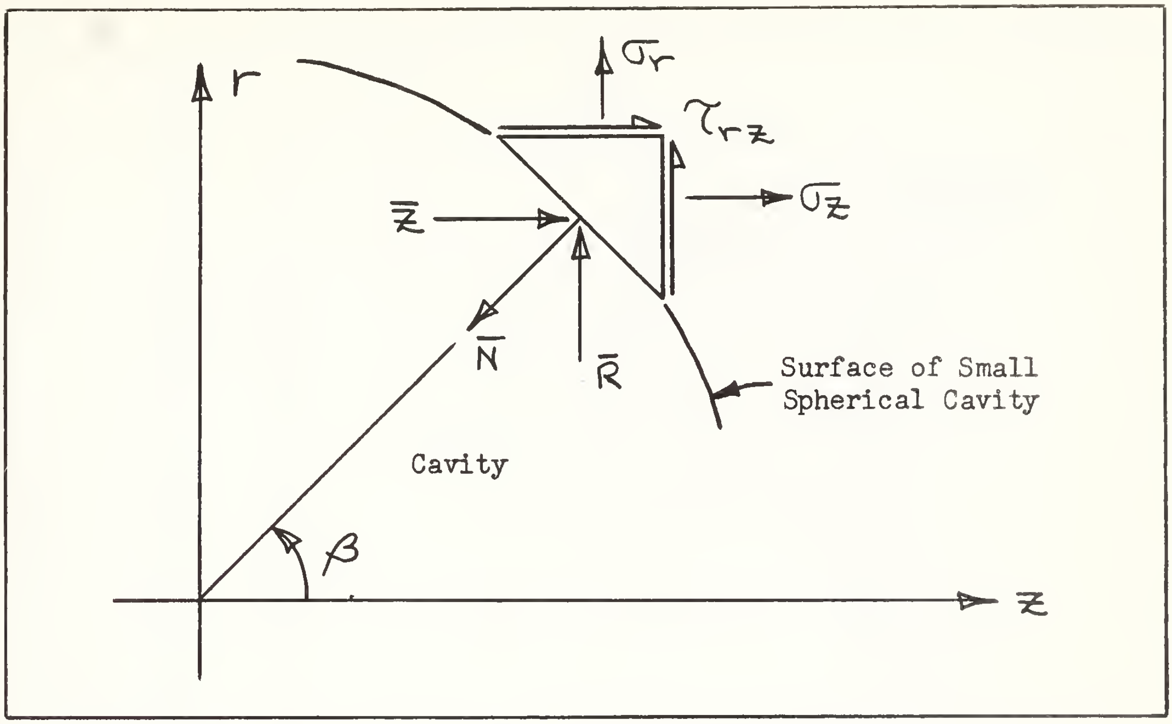

Fig. 4 Force at a Point Converted to a Ring Load

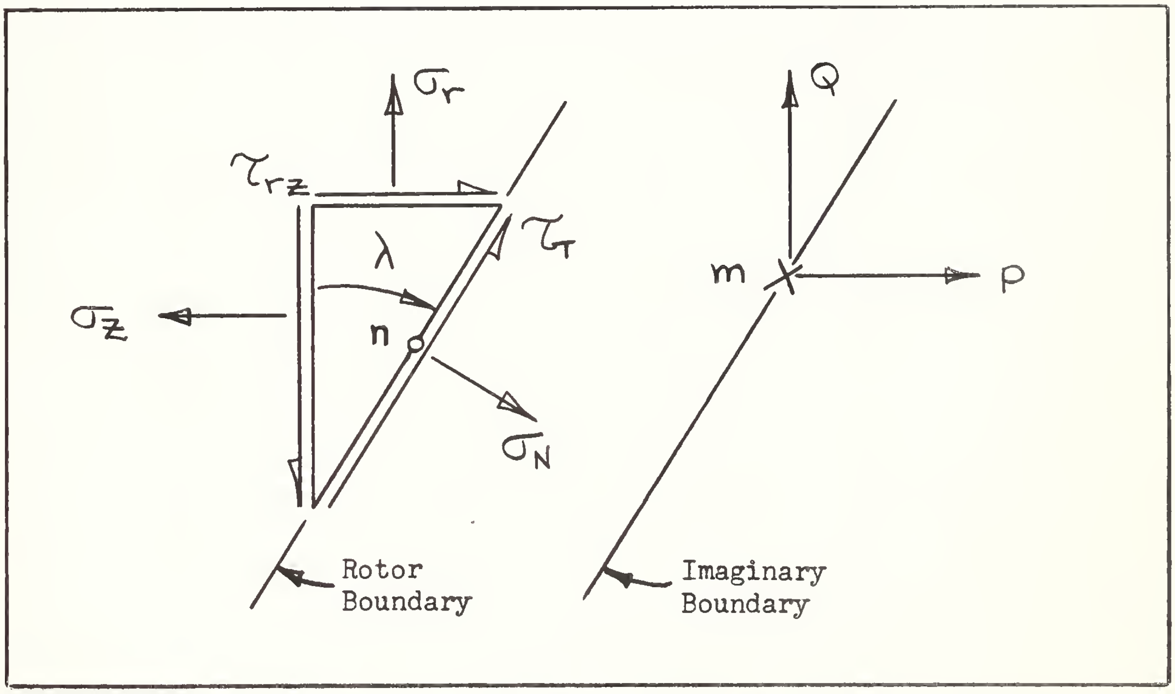

Fig. 5 Stress Components in an Element due to an Application of Fictitious Loads

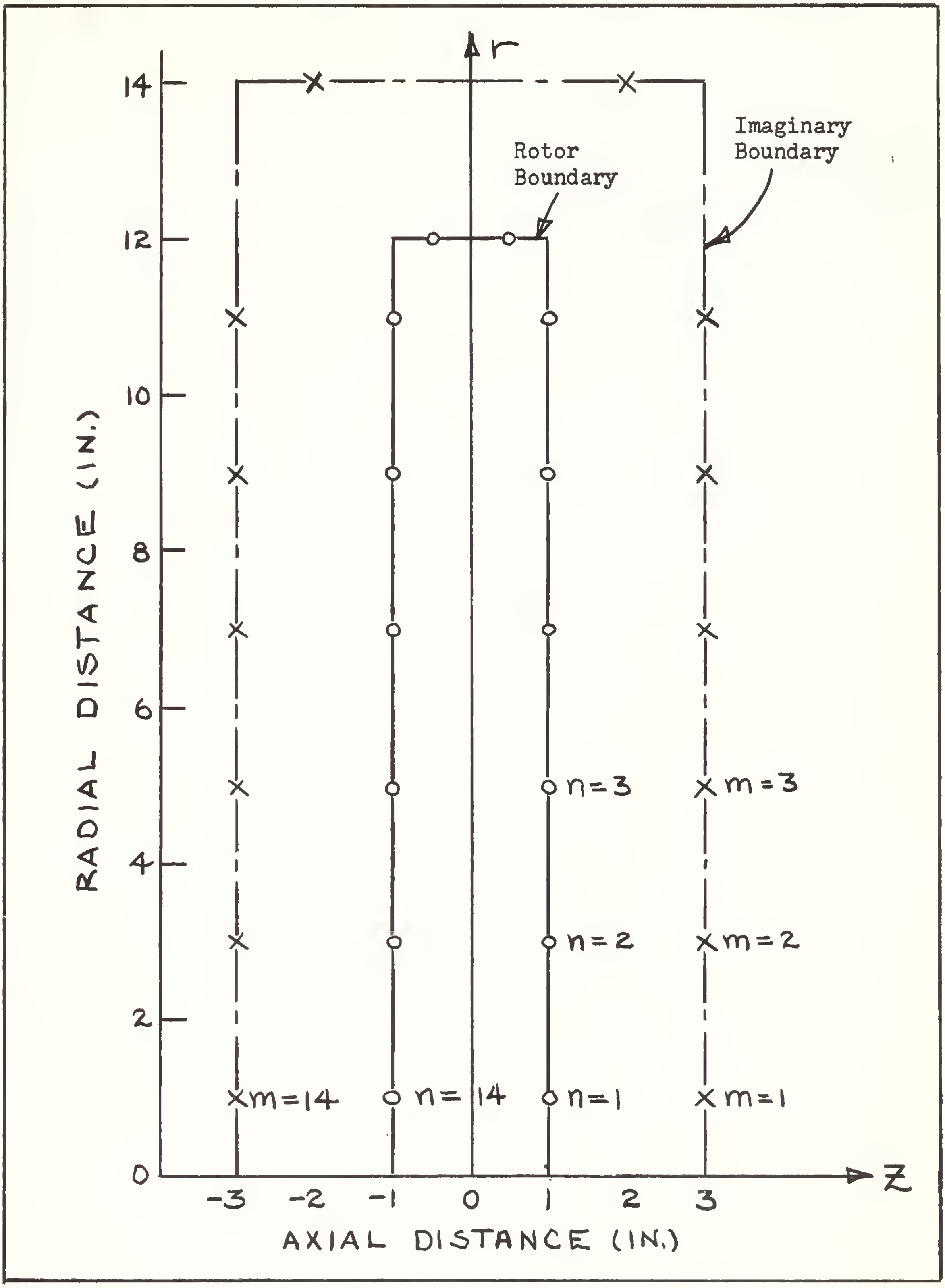

Fig. 6 Example Rotor Cross Section

APPENDIX A

The stresses produced at a point (z, r) by a concentrated radial load $Q d\theta$ at (z_o, r_o) may be found in the same manner as those due to an axial load. Therefore, let Q equal a one pound radial load per radian, then

$$dB = \frac{(1)\, d\theta}{8\pi(1-\nu)}$$

$$\sigma_Q = \frac{(1)}{8\pi(1-\nu)\, r_o^2}$$

then the stress distributions reduce to

$$d\sigma_r'' = \sigma_Q \left\{ \frac{(1-2\nu) f_5}{f_4^3} - \frac{3 f_5 f_6^3}{f_4^5} \right\} d\theta$$

$$d\sigma_\theta'' = \sigma_Q \left\{ \frac{(1-2\nu) f_5}{f_4^3} \right\} d\theta$$

$$d\sigma_z'' = -\sigma_Q \left\{ \frac{(1-2\nu) f_5}{f_4^3} + \frac{3 f_5^3}{f_4^5} \right\} d\theta$$

$$d\tau_{rz}'' = -\sigma_Q \left\{ \frac{(1-2\nu) f_6}{f_4^3} + \frac{3 f_6 f_5^2}{f_4^5} \right\} d\theta$$

Next, vector equations are written to translate the coordinate system related to the axial load to that of the fixed system as in Fig. 2.

$$\vec{\tau}_r = \cos\phi \sin\theta\, \vec{\tau}_r'' + \sin\phi \sin\theta\, \vec{\tau}_\theta'' + \cos\theta\, \vec{\tau}_z''$$

$$\vec{\iota}_\theta = -\cos\phi\cos\theta\,\vec{\iota}_r{}'' - \sin\phi\cos\theta\,\vec{\iota}_\theta{}'' + \sin\theta\,\vec{\iota}_z{}''$$

$$\vec{\iota}_z = \sin\phi\,\vec{\iota}_r{}'' - \cos\phi\,\vec{\iota}_\theta{}'' + (0)\,\vec{\iota}_z{}''$$

By the use of dyadics the resulting stresses in the fixed coordinate system are

$$d\sigma_r = \cos^2\phi\sin^2\theta\,d\sigma_r{}'' + \sin^2\phi\sin^2\theta\,d\sigma_\theta{}'' + \cos^2\theta\,d\sigma_z{}'' + 2\cos\phi\sin\theta\cos\theta\,d\tau_{rz}{}''$$

$$d\sigma_\theta = \cos^2\phi\cos^2\theta\,d\sigma_r{}'' + \sin^2\phi\cos^2\theta\,d\sigma_\theta{}'' + \sin^2\theta\,d\sigma_z{}'' - 2\cos\phi\cos\theta\sin\theta\,d\tau_{rz}{}''$$

$$d\sigma_z = \sin^2\phi\,d\sigma_r{}'' + \cos^2\phi\,d\sigma_\theta{}''$$

$$d\tau_{rz} = \cos\phi\sin\phi\sin\theta\,d\sigma_r{}'' - \cos\phi\sin\phi\sin\theta\,d\sigma_\theta{}'' + \sin\phi\cos\theta\,d\tau_{rz}{}''$$

And after substitution

$$d\sigma_r = \sigma_Q\left(\left\{f_1^2\frac{\sin^4\theta}{f_6^2}\left[\frac{(1-2\nu)f_5}{f_4^3} - \frac{3f_5f_6^3}{f_4^5}\right]\right\} + \left\{\frac{f_2^2\sin^2\theta}{f_6^2}\left[\frac{(1-2\nu)f_5}{f_4^3}\right]\right\} - \left\{\cos^2\theta\left[\frac{(1-2\nu)f_5}{f_4^3} + \frac{3f_5^3}{f_4^5}\right]\right\}\right.$$

$$-\left\{\frac{2f_1 \sin^2\theta\cos\theta}{f_6}\left[\frac{(1-2\nu)f_6}{f_4^3}+\frac{3f_6f_5^2}{f_4^5}\right]\right\}\Bigg)d\theta$$

$$d\sigma_\theta = \sigma_Q\Bigg(\left\{\frac{f_1^2\sin^2\theta\cos^2\theta}{f_6^2}\left[\frac{(1-2\nu)f_5}{f_4^3}-\frac{3f_5f_6^3}{f_4^5}\right]\right\}$$

$$+\left\{\frac{f_2^2\cos^2\theta}{f_6^2}\left[\frac{(1-2\nu)f_5}{f_4^3}\right]\right\}$$

$$-\left\{\sin^2\theta\left[\frac{(1-2\nu)f_5}{f_4^3}+\frac{3f_5^3}{f_4^5}\right]\right\}$$

$$+\left\{\frac{2f_1\sin^2\theta\cos\theta}{f_6}\left[\frac{(1-2\nu)f_6}{f_4^3}+\frac{3f_6f_5^2}{f_4^5}\right]\right\}\Bigg)d\theta$$

$$d\sigma_z = \sigma_Q\Bigg(\left\{\frac{f_2^2}{f_6^2}\left[\frac{(1-2\nu)f_5}{f_4^3}-\frac{3f_5f_6^3}{f_4^5}\right]\right\}$$

$$+\left\{\frac{f_1^2\sin^2\theta}{f_6^2}\left[\frac{(1-2\nu)f_5}{f_4^3}\right]\right\}\Bigg)d\theta$$

$$d\tau_{rz} = \sigma_Q\Bigg(\left\{\frac{f_1f_2\sin^2\theta}{f_6^2}\left[\frac{(1-2\nu)f_5}{f_4^3}-\frac{3f_5f_6^3}{f_4^5}\right]\right\}$$

$$-\left\{\frac{f_1f_2\sin^2\theta}{f_6^2}\left[\frac{(1-2\nu)f_5}{f_4^3}\right]\right\}$$

$$-\left\{\frac{f_2\cos\theta}{f_6}\left[\frac{(1-2\nu)f_6}{f_4^3}+\frac{3f_6f_5^2}{f_4^5}\right]\right\}\Bigg)d\theta$$

The above differential stress components may now be integrated around the complete ring to obtain the final stress components for the unit radial load. The final stress components due to the radial load (Q) are

$$\sigma r_{nm}^{(Q)} = 2\sigma_Q \int_0^{\pi} \Big\{ f_0^2 f_{10} f_{14}^2 - 2 f_0 f_{12} f_{14} D + f_0 f_9 f_{13} - f_{11} D^2 \Big\} d\theta$$

$$\sigma \theta_{nm}^{(P)} = 2\sigma_Q \int_0^{\pi} \Big\{ f_0 f_{10} f_{14}^2 D^2 + 2 f_0 f_{12} f_{14} D + f_9 f_{13} D^2 - f_0 f_{11} \Big\} d\theta$$

$$\sigma z_{nm}^{(Q)} = 2\sigma_Q \int_0^{\pi} \Big\{ f_{10} f_{13} + f_0 f_9 f_{14}^2 \Big\} d\theta$$

$$\tau rz_{nm}^{(Q)} = 2\sigma_Q \int_0^{\pi} \Big\{ f_{10} f_{15} - f_9 f_{15} - \frac{f_2}{f_6} f_{12} D \Big\} d\theta$$

APPENDIX B

The stress components due to centrifugal effects are obtained from a particular solution of the differential equations of equilibrium and compatability. The equilibrium equations are

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} + \rho \omega^2 r = 0$$
$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = 0 \qquad (13)$$

The following compatibility equations apply to the present case.

$$\nabla^2 \sigma_r - \frac{2}{r^2}(\sigma_r - \sigma_\theta) + \frac{1}{1+\nu}\frac{\partial^2 \theta}{\partial r^2} = -\frac{2\rho\omega^2}{1-\nu}$$
$$\nabla^2 \sigma_\theta + \frac{2}{r^2}(\sigma_r - \sigma_\theta) + \frac{1}{1+\nu}\frac{1}{r}\frac{\partial \theta}{\partial r} = \frac{-2\rho\omega^2}{1-\nu}$$
$$\nabla^2 \sigma_z + \frac{1}{1+\nu}\frac{\partial^2 \theta}{\partial z^2} = -\frac{2\nu\rho\omega^2}{1-\nu} \qquad (14)$$
$$\nabla^2 \tau_{rz} - \frac{1}{r^2}\tau_{rz} + \frac{1}{1+\nu}\frac{\partial^2 \theta}{\partial r \partial z} = 0$$

The equilibrium equations may be satisfied by a stress function Φ in the form of polynomials [1]

$$\Phi = A_n(z, r)$$

[illegible]

Consider the following general stress equations.

$$\sigma_r = Br^2 + Dz^2$$

$$\sigma_\theta = Cr^2 + Dz^2$$

$$\sigma_z = Ar^2$$

$$\tau_{rz} = 0$$

These equations satisfy the second of Eqs. (13) as well as the fourth of Eqs. (14). The constants must now be adjusted so as to satisfy the remaining equations of equilibrium and compatibility. Substitution of Eqs. (15) upon reducing gives

$$A = \frac{\rho\omega^2(1+3\nu)}{6\nu}$$

$$B = -\frac{\rho\omega^2}{3}$$

$$C = 0$$

$$D = -\frac{\rho\omega^2(1+2\nu)(1+\nu)}{6\nu(1-\nu)}$$

The general centrifugal stresses then become

$$\sigma_r = -\frac{\rho\omega^2 r^2}{3} - \frac{\rho\omega^2(1+2\nu)(1+\nu)z^2}{6\nu(1-\nu)}$$

$$\sigma_z = \frac{\rho\omega^2(1+3\nu)r^2}{6\nu}$$

$$\sigma_\theta = -\frac{\rho\omega^2(1+2\nu)(1+\nu)z^2}{6\nu(1-\nu)}$$

$$\tau_{rz} = 0$$

APPENDIX C

Data Programming

The procedure of data programming will be given in steps so that an understanding of the analysis is unnecessary.

Consider the rotor cross section shown in Fig. 6.

Step 1. Select points n on the rotor boundary that are about equally spaced and best define the boundary. The boundary stresses must be known at each point n. The maximum number of points that may be chosen is twenty five.

Step 2. Construct an imaginary boundary around the rotor cross section. It could be at a distance from the rotor boundary equal to the distance between the chosen points n, for example, see page 18.

Step 3. Select points m on the imaginary boundary that are spaced in a manner similar to the points n. The number of points m must be equal to the number of points n.

Step 4. Determine the r and z coordinates of each point n and m.

Step 5. Determine the angles λ with the sign convention shown in Fig. 3. $(0 \leq \lambda \leq 3.1416)$

Step 6. Determine the normal and tangential boundary stresses at each point n.

All that remains is to prepare the data cards with the information obtained from the drawing and four other known constants. The four constants are:

n - The number of chosen points n

ω - The angular velocity in revolutions per minute

ν - Poisson's ratio

ρ - Rotor density

They are punched in that order on one card and in the following format;

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

(I 10, F 10.0, F 10.4, E 10.1).

All other data are taken from the drawing or are known. These values are:

r_n - The radial distance of each point n, (in.)

z_n - The axial distance of each point n, (in.)

λ_n - The angle λ which the rotor surface makes with the plane of rotation, (rad.)

σ_{N_n} - The normal boundary stress at each point n, (lb./sq. in.)

τ_{T_n} - The tangential boundary stress at each point n, (lb./sq. in.)

r_m - The radial distance of each point m, (in.)

z_m - The axial distance of each point m, (in.)

They are punched in that order and in the following format; (2F 10.2, F 10.4, 2F 10.1, 2F 10.2).

The prepared data for the rotor of Fig. 6 are a part of the program print out in Appendix D.

APPENDIX D

```
..JOB0254F,JOHNSON,R.A. PROBLEM TIME 15 MINUTES
      PROGRAM ROTOR
      DIMENSION RN(25),ZN(25),XLMDA(25),RM(25),ZM(25),
     1 PARTR(25),PARTX(25),PARTZ(25),SIGR(25),SIGX(25),
     2 SIGZ(25),TAURZ(25),COEF(50,51),X(50),SIGB(25),TAUB(25),
     3 SIGRP(1250),SIGXP(1250),SIGZP(1250),TAURZP(1250),
     4 SIGRQ(1250),SIGXQ(1250),SIGZQ(1250),TAURZQ(1250)
      COMMON RN,ZN,XLMDA,RM,ZM,IND,II,JJ,XNU,COEF
      READ 9,NN,RPM,XNU,RHO
C     NN  = NUMBER OF POINTS N ON ROTOR BOUNDARY
C     RPM = ANGULAR VELOCITY IN REVOLUTIONS PER SECOND
C     XNU = POISSONS RATIO
C     RHO = ROTOR DENSITY
 9    FORMAT(I10,F10.0,F10.4,E10.3)
      PRINT 120,NN,RPM,XNU,RHO
 120  FORMAT(1H1,2X5H NN =I4,3X6H RPM =F10.0,3X6H XNU =F10.4,
     1 3X6H RHO =E10.3)
      PRINT 100
 100  FORMAT(//4X1HN,13X2HRN,13X2HZN,10X5HLAMDA,11X4HSIGB,11X4HTAUB,
     1 13X2HRM,13X2HZM)
      NXN = NN**2
      N2N = 2*NN
      READ 10,(RN(I),ZN(I),XLMDA(I),SIGB(I),TAUB(I),RM(I),ZM(I),I=1,NN)
C     RN  = RADIAL DISTANCE TO POINT N
C     ZN  = AXIAL DISTANCE TO POINT N
C     XLAMDA = ANGLE OF ROTOR SURFACE WRTO PLANE OF ROTATION
C     SIGB = NORMAL BOUNDARY STRESS
C     TAUB = TANGENTIAL BOUNDARY STRESS
C     RM  = RADIAL DISTANCE TO POINT M
C     ZM  = AXIAL DISTANCE TO POINT M
 10   FORMAT(2F10.2,F10.4,2F10.1,2F10.2)
      PRINT 200,(I,RN(I),ZN(I),XLMDA(I),SIGB(I),TAUB(I),RM(I),ZM(I),
     1 I = 1,NN)
```

```
 200  FORMAT(//I5,2F15.2,F15.4,2F15.1,2F15.2)
C     CONVERT RPM TO RADIANS PER SECOND
      RDPS = RPM*.10472
      JV = 1
      DO 40 II = 1,NN
      A = 2.*XLMDA(II)
      B = COSF(A)
      C = SINF(A)
C     COMPUTE PARTICULAR SOLUTION STRESS COMPONENTS
 17   PARTR(II)=-(RHO*RDPS**2)*(((RN(II)**2)/3.)+((1.+2.*XNU)*(1.+XNU)
     1 *(ZN(II)**2)))/(6.*XNU*(1.-XNU))
      PARTX(II)=-((RHO*RDPS**2)*(1.+2.*XNU)*(1.+XNU)*(RN(II)**2))/
     1(6.*XNU*(1.-XNU))
      PARTZ(II) = ((RHO*RDPS**2)*(1.+3.*XNU)*(RN(II)**2))/(6.*XNU)
      DO 30 JJ = 1,NN
      SGP = 1./(8.*3.1416*(1.-XNU)*RM(JJ)**2)
      SGQ = SGP
      DO 20 IND = 1,8
C     COMPUTE COMPLEMENTARY SOLUTION STRESS COMPONENTS
      CALL SIMCON(0.,3.1416,.1 ,10,AREA,NOI,R)
      GO TO(1,2,3,4,5,6,7,8),IND
 1    SIGRP = 2.*SGP*AREA
      GO TO 20
 2    SIGXP = 2.*SGP*AREA
      GO TO 20
 3    SIGZP = 2.*SGP*AREA
      GO TO 20
 4    TAURZP =2.*SGP*AREA
      GO TO 20
 5    SIGRQ = 2.*SGQ*AREA
      GO TO 20
 6    SIGXQ = 2.*SGQ*AREA
      GO TO 20
 7    SIGZQ = 2.*SGQ*AREA
      GO TO 20
```

```
 8    TAURZQ =2.*SGQ*AREA
 20   CONTINUE
      LL = 2*II-1
      KK = 2*II
C     SUBSTITUTE COMPONENTS INTO BOUNDARY STRESS EQUATIONS
      SUMP = (SIGRP +SIGZP)/2.
      DIFP = (SIGRP - SIGZP)/2.
      SIGNP = SUMP - DIFP*B - TAURZP*C
      TAUTP = - DIFP*C + TAURZP*B
C     GENERATE TERMS OF MATRIX WRTO P
      COEF(LL,JJ) = SIGNP
      COEF(KK,JJ) = TAUTP
C     SUBSTITUTE COMPONENTS INTO BOUNDARY STRESS EQUATIONS
      SUMQ = (SIGRQ + SIGZQ)/2.
      DIFQ = (SIGRQ - SIGZQ)/2.
      SIGNQ = SUMQ - DIFQ*B - TAURZQ*C
      TAUTQ = - DIFQ*C + TAURZQ*B
C     GENERATE TERMS OF MATRIX WRTO Q
      COEF(LL,JJ+NN) = SIGNQ
      COEF(KK,JJ+NN) = TAUTQ
C     STORE STRESS COMPONENTS
      SIGRP(JV) = SIGRP
      SIGXP(JV) = SIGXP
      SIGZP(JV) = SIGZP
      TAURZP(JV) = TAURZP
      SIGRQ(JV) = SIGRQ
      SIGXQ(JV) = SIGXQ
      SIGZQ(JV) = SIGZQ
      TAURZQ(JV) = TAURZQ
      JV = JV + 1
 30   CONTINUE
C     SUBSTITUTE COMPONENTS INTO BOUNDARY STRESS EQUATIONS
      SPART = (PARTR(II) + PARTZ(II))/2.
      DPART = (PARTR(II) - PARTZ(II))/2..
      PARTS = SPART - DPART*B
```

```
      PARTT = -DPART*C
C     GENERATE TERMS OF MATRIX WRTO BOUNDARY AND CENTRIFUGAL STRESSES
      COEF(LL,N2N+1) = SIGB(II) - PARTS
      COEF(KK,N2N+1) = TAUB(II) - PARTT
 40   CONTINUE
      PRINT 300
 300  FORMAT(1H1,4X5HSIGRP,9X5HSIGXP,9X5HSIGZP,8X6HTAURZP,
     1 9X5HSIGRQ,9X5HSIGXQ,9X5HSIGZQ,8X6HTAURZQ,7X1HJ)
      PRINT 400,(SIGRP(J),SIGXP(J),SIGZP(J),TAURZP(J),
     1 SIGRQ(J),SIGXQ(J),SIGZQ(J),TAURZQ(J),J,J = 1,NXN)
 400  FORMAT(/8E14.6,I14)
      PRINT 600
 600  FORMAT(1H12X1HN,6X2HRN,8X2HZN,14X5HPARTR,15X5HPARTX,15X5HPARTZ)
      PRINT900,(J,RN(J),ZN(J),PARTR(J),PARTX(J),PARTZ(J),J = 1,NN)
 900  FORMAT(//I4,2F10.4,3F20.3)
C     SOLVE MATRIX
      CALL JORDAN(COEF,N2N,X)
      CALL TIME
      PRINT 350
 350  FORMAT(1H1,2X1HN,7X4HCOEF,1X4HX(N))
      PRINT 700,(N,X(N),N = 1,N2N)
 700  FORMAT(//I14,1E18.8)
C     COMPUTE FINAL STRESSES
      DO 50 MM = 1,NN
      SRP = 0.0
      SXP = 0.0
      SZP = 0.0
      TRZP = 0.0
      L = 1
      I = (MM - 1)*NN + 1
      J = I
 60   SRP = SRP + X(L)*SIGRP(I)
      SXP = SXP + X(L) *SIGXP(I)
      SZP = SZP + X(L)*SIGZP(I)
      TRZP = TRZP +X(L)*TAURZP(I)
```

```
      L = L + 1
      I = I + 1
      IF(L - NN)60,60,70
70    SRQ = 0.0
      SXQ = 0.0
      SZQ = 0.0
      TRZQ = 0.0
80    SRQ = SRQ + X(L)*SIGRQ(J)
      SXQ = SXQ + X(L)*SIGXQ(J)
      SZQ = SZQ + X(L)*SIGZQ(J)
      TRZQ = TRZQ + X(L)*TAURZQ(J)
      L = L + 1
      J = J + 1
      IF(L - N2N)80,80,90
90    SIGR(MM) = SRP + SRQ + PARTR(MM)
      SIGX(MM) = SXP + SXQ + PARTX(MM)
      SIGZ(MM) = SZP + SZQ + PARTZ(MM)
      TAURZ(MM) = TRZP + TRZQ
50    CONTINUE
      CALL TIME
      PRINT 500
500   FORMAT(1H13X1HN, 7X2HRN,10X2HZN,13X4HSIGR,11X4HSIGX,11X4HSIGZ,
     1 10X5HTAURZ)
      PRINT 800,(I,RN(I),ZN(I),SIGR(I),SIGX(I),SIGZ(I),TAURZ(I),
     1 I = 1,NN)
800   FORMAT(//I4,2F12.4,4F15.3)
C     PROGRAM ROUGH CHECK
      DO 69 K = 1,NN
      SGR = 437.*(144.-RN(K)**2)
      SGX = 262.*(240.-RN(K)**2)
      PRINT 1000,K,SGR,SGX
1000  FORMAT(1H1,/4H N =I2,4X6H SGR =F12.3,4X6H SGX =F12.3)
69    CONTINUE
      END
      FUNCTION F(X)
```

```
      DIMENSION RN(25),ZN(25),XLMDA(25),RM(25),ZM(25),COEF(50,51)
      COMMON RN,ZN,XLMDA,RM,ZM,IND,II,JJ,XNU,COEF
      D = COSF(X)
      E = SINF(X)
      F0 = E**2
      F1 = RN(II)/RM(JJ)
      F2 = ((ZN(II)-ZM(JJ))/RM(JJ))
      F3=SQRTF((F1**2)+1.-2.*F1*D)
      F4=SQRTF((F1**2)+1.-2.*F1*D+F2**2)
      F5=F1*D-1.
      F6 = SQRTF((F1**2)*(F0)+F2**2)
      F7=(1.-2.*XNU)/F4**3
      F8 = F4**5
      F9 = F7*F5
      F10 = F9 -(3.*F5*(F6**3))/F8
      F11 = F9+(3.*(F5**3))/F8
      F12 = F7*F6+(3.*F6*(F5**2))/F8
      F13 = (F2**2)/(F6**2)
      F14 = F1/F6
      GO TO(1,2,3,4,5,6,7,8),IND
1     F = F2*(F7-(3.*((F1-D)**2))/F8)
      RETURN
2     F = F2*(F7-(3.*E**2)/F8)
      RETURN
3     F = -(F2*(F7+(3.*F2**2)/F8))
      RETURN
4     F = -(F7 + (3.*F2**2)/F8)*(F1 - COSF(X))
      RETURN
5     F = ((F14**2)*(F0**2))*F10-2.*F14*F0*D*F12+F13*F0*F9-(D**2)*F11
      RETURN
6     F = (F14**2)*F0*(D**2)*F10+2.*F14*F0*D*F12+F13*(D**2)*F9-F0*F11
      RETURN
7     F = F13*F10+(F14**2)*F0*F9
      RETURN
8     F15 = F1*F2*F0/F6**2
```

```
      F = F15*F10-F15*F9-F2*D*F12/F6
      RETURN
      END
C     D1 UCSD SIMCON
C     REVISED OCTOBER 1962
      SUBROUTINE SIMCON(X1,XEND,TEST,LIM,AREA,NOI,R)
      NOI=0
      R1=10.0
      ODD=0.0
      INT=1
      V=1.0
      EVEN=0.0
      AREA1=0.0
19    ENDS=F(X1)+F(XEND)
2     H=(XEND-X1)/V
      ODD=EVEN+ODD
      X=X1+H/2.
      EVEN=0.0
      DO 3 KI = 1,INT
21    EVEN=EVEN+F(X)
      X=X+H
3     CONTINUE
31    AREA=(ENDS+4.0*EVEN+2.0*ODD)*H/6.0
      NOI=NOI+1
34    R=ABSF((AREA1-AREA)/AREA)
      IF(NOI-LIM)341,35,35
341   IF(R-TEST)35,35,4
35    RETURN
4     AREA1=AREA
46    INT=2*INT
      V=2.0*V
      GO TO 2
      END
      SUBROUTINE JORDAN (A,N,X)
      DIMENSION A(50,51),X(50)
      K =N+1
```

```
      F = F15*F10-F15*F9-F2*D*F12/F6
      RETURN
      END
C     D1 UCSD SIMCON
C     REVISED OCTOBER 1962
      SUBROUTINE SIMCON(X1,XEND,TEST,LIM,AREA,NOI,R)
      NOI=0
      R1=10.0
      ODD=0.0
      INT=1
      V=1.0
      EVEN=0.0
      AREA1=0.0
 19   ENDS=F(X1)+F(XEND)
 2    H=(XEND-X1)/V
      ODD=EVEN+ODD
      X=X1+H/2.
      EVEN=0.0
      DO 3 KI = 1,INT
 21   EVEN=EVEN+F(X)
      X=X+H
 3    CONTINUE
 31   AREA=(ENDS+4.0*EVEN+2.0*ODD)*H/6.0
      NOI=NOI+1
 34   R=ABSF((AREA1-AREA)/AREA)
      IF(NOI-LIM)341,35,35
 341  IF(R-TEST)35,35,4
 35   RETURN
 4    AREA1=AREA
 46   INT=2*INT
      V=2.0*V
      GO TO 2
      END
      SUBROUTINE JORDAN (A,N,X)
      DIMENSION A(50,51),X(50)
```

```
      K =N+1
11    IF (K-1)13,6,15
15    D=0.
      DO 2 I=2,K
      IF(ABSF(A(I-1,1))-D) 2,2,3
3     L =I-1
      D = ABSF(A(L,1))
2     CONTINUE
4     IF(L-1)5,6,5
5     DO 7 J=1,K
      D =A(L,J)
      A(L,J) = A(1,J)
7     A(1,J) = D
6     DO 8 I =1,N
8     X(I) = A(I,1)
      IF (K-1)12,13,12
12    DO 10 J=2,K
      D = A(1,J)/X(1)
      DO  9 I=2,N
9     A(I-1,J-1)  = A(I,J) -X(I)*D
10    A(N,J-1) =D
      K = K-1
      GO TO 11
13    CONTINUE
      RETURN
      END
      END
```

DATA

14	11500.	.3333	.720E-3			
1.00	1.00	.0000	.0	.0	1.00	3.00
3.00	1.00	.0000	.0	.0	3.00	3.00
5.00	1.00	.0000	.0	.0	5.00	3.00
7.00	1.00	.0000	.0	.0	7.00	3.00
9.00	1.00	.0000	.0	.0	9.00	3.00
11.00	1.00	.0000	.0	.0	11.00	3.00
12.00	.50	1.5708	.0	.0	14.00	2.00
12.00	-.50	1.5708	.0	.0	14.00	-2.00
11.00	-1.00	.0000	.0	.0	11.00	-3.00
9.00	-1.00	.0000	.0	.0	9.00	-3.00
7.00	-1.00	.0000	.0	.0	7.00	-3.00
5.00	-1.00	.0000	.0	.0	5.00	-3.00
3.00	-1.00	.0000	.0	.0	3.00	-3.00
1.00	-1.00	.0000	.0	.0	1.00	-3.00

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