A Mathematics Improvement Program: Reaching the Struggling Intermediate Level Learner with a One-on-One Intervention

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CEURE **Center for Excellence in Urban and Rural Education**
School of Education
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Abstract. *Well-conceived early intervention programs can prevent failure in mathematics. This report describes a one-on-one mathematics tutoring program designed to improve understanding in mathematics, attitudes towards mathematics, and confidence. The Mathematics Improvement Program (MIP) curriculum is detailed and results from a case study with an urban third-grader are presented.*

Nothing saddens, disappoints or frustrates a teacher more than working with a child who after much success begins to struggle. Any third grade teacher can name a student under their tutelage who after success in the primary grades began to struggle in mathematics. Mathematics at the third grade level becomes more abstract, more complex, and for some students just more difficult. For example third grade mathematics requires students to perform operations with multiple digits. This would include multiplication and division with two and three digits. A strong sense of number and fluency with basic facts is necessary for success. The third grade mathematics curriculum also includes solving multi-step problems. Problem solving strategies such as making charts or graphs and analyzing them are necessary. Even choosing an efficient strategy from a menu is a difficult skill for third graders. Success in mathematics at this level is essential, for lack of understanding, lack of skills and loss of confidence will only bring more trouble in future grades. Many states also choose third and fourth grade as the opportunity to evaluate student achievement.

When students are not keeping up with classmates, making connections and building understanding, teachers must do something, yet finding the time to dedicate to

one student is difficult. One-on-one services may be available, but only for those students with an identified learning disability. For some struggling third grade math students there is no learning disability and therefore no services. Perhaps this is good, because a long term service may not be advantageous due to time and cost, whereas a short term intervention may be just the answer to sharpen skills, develop understanding, and rebuild confidence.

Well-conceived one-on-one early intervention programs can prevent future failure. Reading Recovery, an intervention program that involves one-on-one tutoring sessions, has been successfully implemented in many school districts, building students' abilities to read and comprehend (Shanahan and Barr, 1995). Reading Recovery is based on the assumption that early intensive intervention can prevent later reading failure. Mathematics Recovery is an early intervention program developed in Australia and based on the same premise as well as current research in mathematics education. A study conducted by Phillips, Leonard, Horton, Wright, and Stafford (2003) showed positive results for children in their second year of school and ranked in the bottom 25% of their class. Thirty minute one-on-one sessions were designed for each individual child based on their needs. The key was to develop the curriculum for each individual child. Tutors considered the child's weaknesses first and designed activities accordingly. Positive results included students who were working on grade level when they were not before, students showing self confidence, and students figuring out problems using strategies that made sense to them. The researchers discuss as limitations to their study the lack of comparable programs and the fact that any intervention program is better than none. Improvements in student performance may be due to any intervention, not a specific program.

The Mathematics Improvement Program

The program described here is different in many ways from Mathematics Recovery yet serves a similar purpose: to help struggling students before all confidence and ability are lost. The Mathematics Improvement Program (MIP) (McCarthy, 2005, October) strives to rebuild skills and understandings that may not have been developed

fully in the early years, resulting in problems as the curriculum becomes more difficult in third grade.

Candidates for the Program

The criteria for enrollment are clear and stem mainly from teacher identification. Test scores and grades are not as important as what the teacher observes during regular math instruction. Candidates for the program must have been successful in mathematics in previous years of schooling and preferably during the first half of third grade as well. The potential student would show recent signs of confusion, frustration, lack of confidence, and a drop in achievement as determined by the classroom teacher. The program would be implemented during the second half of third grade. Children with identified learning disabilities or behavior disabilities would be excluded from the program. After teacher identification, parental consent to participate should be obtained.

The Curriculum

Unlike Math Recovery, the Mathematics Improvement Program has prescribed curriculum with alterations made for individual children based on their strengths and weaknesses. It is expected that some areas will be easier for children than others and that is deemed okay, because the easier work will help to rebuild confidence.

Daily Number Definition. Number sense has a fundamental role in problem solving, computation, estimation, and measurement. If a child is struggling in mathematics it is first important to develop number sense.

Number sense is more than just counting and knowing place value. The National Council of Teachers of Mathematics in their publication Principles and Standards for School Mathematics (2000) believe that “instructional programs should enable students to understand numbers, ways of representing numbers, relationships among numbers, and number systems” (p. 32). They use the following expressions when elaborating on what is number sense: the ability to decompose numbers, a rich understanding of number, and how numbers are represented. VanDeWalle (2007) says that “number sense is a complex and multifaceted concept. A rich understanding of number, a relational understanding, involves many different ideas, relationships, and skills” (p.120). According to Schneider

and Thompson (2000) a student who has good number sense has a good understanding of number meaning and numerical relationships. To truly possess number sense one must possess flexible thinking and the ability to see relationships among numbers.

Most third graders possess some number sense from their experiences with counting and whole number computation. Often for children at this grade level who find mathematics difficult, number sense is the weakness. For example, when given the question how many tens are in 600, they often answer 0 because the tens place has a zero. Few see that because there are 6 hundreds, there must be tens and that there are 60 of them. They have one way of looking at a number and it is not flexible. VanDeWalle (2007) expresses concern about students in grades four and five who seem to have little number sense and says they need lots of experiences with number to remedy this.

In order to counter this deficit in number sense, the inflexible thinking, and the problems it can cause in other areas of the mathematics curriculum, an activity was created called the Daily Number Definition. It takes only about 10 minutes, but is part of each MIP session. Each time the activity is completed a different number is given. The date on the calendar or the day of school can be used. Students then brainstorm different representations of the number. For example, for the tenth day of school the children may say 9+1, 2x5, 100/10. The teacher can record these on chart paper and then display them along the classroom wall making a number line. The student can then refer to previously completed numbers to help them come up with representations for the new number. Students can build from the expressions they know to new ones. This also encourages children to not only think about the daily number, but to consider the relationship it has to the numbers that have already been completed. The emphases are on representations and relationships. As VanDeWalle (2007) has stated, children have some understanding of number from their experiences and "…these ideas should be built upon as we work with children to help them develop new relationships" (p. 120).

Basic Fact Fluency. All third grade teachers have had experience with students who seem to have memorized facts without understanding the operation and struggle with applying the operations to word problems, or who seem to understand the operation but lack fluency with basic facts and lack strategies to solve basic facts other than finger counting. The National Council of Teachers of Mathematics states that fluency with

basic facts is essential (2000). According to VanDeWalle (2007), story problems are a good way to help students understand operations and make connections among the operations. He recommends using a variety of problem types. VanDeWalle also cautions against drilling basic facts too soon before strategies have been developed.

The basic fact fluency activity was developed to build an understanding of the operations, to develop strategies for solving basic fact problems that focus on using the facts the child already knows, and finally to build fluency with facts. Each session includes the reading of a word problem followed by discussion as to which operation would best solve the word problem. After solving the problem, an additional 10 facts of the same operation are given to the students. Strategies are discussed for solving facts the child is struggling with and additional practice using a strategy may be given at the discretion of the math improvement program teacher.

Problem Solving. Learning through problem solving is a goal of the National Council of Teachers of Mathematics (2000). This section of the Math Improvement Program is undergoing revisions, so some of the ideas expressed here have not been field tested. There are three critical features of the problem-solving section of the MIP. First is reading and understanding the problem. Questions similar to those used in reading instruction are discussed in order to connect the reading of math problems to what happens in reading other material. Questions include: Who is the story about? What is the problem? What facts do we know? and How can we solve the problem? It is expected that this pattern of questioning will eventually become internalized and the students will ask themselves these questions. Second, problems are sorted by similarity and the student is guided to look for patterns in word problems. Lastly, strategies such as drawing a picture, guess and check, and acting out the problem are taught using modeling and practice.

It is also important to note that problems will also be the primary vehicle for teaching math content. Just as word problems are the reason for understanding the operations and developing fluency with basic facts, the problems completed here will serve to introduce and review such ideas as perimeter, area, and operations with fractions and decimals.

Writing about Math. Writing provides a vehicle not only for expressing oneself but also for critical thinking and reflection. A writing component, completed at the conclusion of each session, is an integral part of the Mathematics Improvement Program This also provides data that allow the MIP teacher to assess strategies and math vocabulary the child has learned. Children's writing provides insight into their thinking. According to Kolstad, Briggs and Whalen (1996), mathematics in the real world is more of a speaking and reading enterprise than is typically done in math class. Integrating the language arts into the mathematics class is positive for children who find math difficult. O'Connell et al. (2005) conducted a study of writing is a tool for learning mathematics. Writing helped students become better problem solvers. Writing also helped students think about their own thinking. In their writing, students were describing their solutions to mathematics problems and the strategies they used.

There are two strategies that the MIP teacher can select to facilitate writing. The first is a journal where the child writes about the session and the teacher responds to the writing, including asking questions to facilitate more writing. The second is to make use of a graphic organizer such as the Four Square to help students reflect and write their ideas with less attention to mechanics such as essay and sentence structure. The organizer can be used later to facilitate the development of a paragraph or essay about the mathematics learned.

Math Games. Characteristics of a child identified for the Mathematics Improvement Program include frustration, loss of confidence and a beginning dislike of mathematics. To remedy this, games can be used when time is available. Traditional or teacher-made games such a dominoes, Memory, Top It or War, and Bingo can be played. The discussions between the student and teacher during game playing can facilitate understanding. Games can also be sent home, so parents can play with the child.

Case Study: Mary

Mary was a student in a general education third grade class at an urban elementary school. Mary had been identified by her teacher as a child who was struggling in mathematics, although she had begun the school year successfully. She

lacked number sense according to her teacher. After obtaining parental permission Mary and I began to meet for 30 minute sessions about once a week.

Pre-assessment

For our first session, Mary completed an interview involving place value found in *Teaching Elementary Mathematics: A Resource for Field Experiences* (Smith, Lambdin, Lindquist & Reys, 2001). The goal of this interview was to investigate Mary's number sense. Some of the most interesting findings are discussed here.

The first task was for me to read a number and for Mary to write the number followed by Mary reading numbers I had written. Here are the numbers that Mary struggled with: for 348 she wrote 3048; for 12, 408 she wrote 1248; Mary read 4036 as "four hundred thousand thirty six." In this case Mary had difficulty with zeros, adding them when they were not needed and forgetting them when they were. Perhaps her understanding of the value of 0 and its role as a place holder was not well developed.

When Mary was asked to write a large number, she wrote 1,000. She was then asked to write a larger number; she wrote 2,000. This illustrates some understanding of value.

The next two tasks were most interesting when considering number sense. The first asked how many tens were in the following numbers: 40, 500, 6,000. Mary answered that there was one ten in 40, two tens in 500, and three tens in 6,000. This illustrates weak number sense especially with regard to place value. Although she knew there were increasingly more tens in the numbers as they got larger and that the number with more numerals was larger, she could not determine how many tens were in the number. The next task was to answer questions based on the number 6789. When asked how many hundreds, Mary replied 1. When asked how many tens, Mary again replied 1. Last was the question how many thousands, and Mary replied 2. It might be concluded that she views thousands as big and two as more than one.

It did not seem that Mary had a grasp of place value, but could read and write numbers. She lacked the ability to see numbers as parts, such as tens and hundreds and then as wholes. It was hoped that the Daily Number Definition activity would help Mary

develop flexibility with numbers, decomposing them and seeing the parts that make the whole.

The Tutoring Sessions

The goal of the program is to have 20 sessions of about 30 minutes each. Mary completed 13 sessions of 30 minutes each. She did not complete the full 20 because of her attendance and the ending of the school year. We simply ran out of time. Although some problem solving exercises were completed, the discussion is not included here because this section of the MIP curriculum is being redeveloped to produce a more systematic method of developing problem solving skills. Activities such as reading and sorting problems according to similarities were not part of this case study, but have been added to the program. This section of the curriculum is being redesigned in order to better develop students' problem solving abilities.

Daily Number Definition. In our first session of daily number definition Mary was presented with the number 10 and the direction to express 10 as many different ways as she could. Mary did not offer any at first, so I listed some examples (shown in Table 1 under Session 1.) After this, Mary did list seven correct expressions. It is interesting to note that Mary first listed 4+5 and I had listed 5+5. I then asked how much 4+5 was and she determined it was not 10, so we crossed it out. Mary's next example was 12-2. I had given 11-1 for an example. This illustrates that Mary was using my expression to develop another expression of the number ten, a goal of the activity. Mary also gave the examples 13-3, 14-4, 15-5, 16-6, and 17-7, building off of the expressions 11-1 that I had given and 12-2 that she developed. This illustrates the ability to use one expression to develop other expressions, another goal of the daily number definition.

In the second session Mary brainstormed expressions for the number 8 and again found a pattern with subtraction where she could raise both numbers by one and get the same result (see Table 1, Session 2). In the third session, Mary brainstormed expressions for the number 25 and again found a pattern to use, this time using addition. She was using the pattern 24+1 and then dropping the 24 and adding one to the other side to get 23 +2. She continued this way until she got to 15+10. She expressed that correctly, but then wrote 16+9 again, 14+8, 13 +7 and so on. We discussed this and again I asked her

to solve the equation to see if it equaled 25. When it did not I wrote 14+ and she filled in 11. Mary then correctly wrote the expressions of 13+12, 12+13, 11+14, and 10+15. Our discussion of patterns and relationships among the numbers was positive. For additional data on Mary's work in this area see Table 1. I wish I had pursued more two- and three-digit numbers, but as the end of the school year approached Mary's attendance was poor and we simply ran out of time.

Basic Fact Fluency. The assessment of basic fact fluency yielded results common to many struggling third graders. Mary did better with addition facts than with multiplication facts. She struggled mostly with basic multiplication facts that contained the numerals 6, 7, or 8.

To develop basic fact fluency we completed a word problem at each session. Each word problem facilitated a discussion of the operation involved such as addition as putting together, multiplication as putting groups of things together, subtraction as taking away or comparing, and division as placing items in groups or determining how many items in each group. This was followed by 10 basic fact problems featuring the same operation. Facts were selected because they facilitated the discussion of strategies for solving the facts. Strategies such as making 10's for addition and subtraction, doubling the 2's facts to obtain the 4's facts in multiplication, and using multiplication facts to solve division problems were discussed and modeled.

Problem Solving. This section was weak and thus is being revised for the next case study. The problems were not sequenced well to build understanding. Revisions to this section have been discussed previously.

Writing About Math. Mary kept a journal throughout the intervention. She wrote about activities that we had done and what she liked about the sessions. This was also positive, not only because she reflected on what she had learned, but because she expressed positive experiences with math. Here is an example from her journal. "We subtracted numbers. We put numbers in order and look at what place the numbers are in. I liked playing Top It."

Math Games. This was one aspect of the intervention that worked extremely well, as suggested in the journal entry cited above. The five minutes we played games provided an opportunity to review concepts and to have fun. Mary enjoyed playing Race to 100, a

game where you try to get 100 base ten blocks or one flat by rolling a dice. This game helps build number sense. We also played War or as it is often called in math programs like Everyday Math, Top It. Comparing our numbers to determine who had more built number sense. We made a set of cards for Mary to take home and play with her family. The numbers used in this game included two and three-digit numbers. We played a matching game with fraction pictures and representations.

Case Study Results

This case study of the Math Improvement Program showed positive results. Post-intervention assessments showed growth in the areas of number sense and basic fact fluency. For example, on the basic fact fluency assessment for multiplication Mary scored a 64% on the pre-test and a 90% accuracy rate on the post-test. The classroom teacher reported at the midpoint of the intervention that she had observed improvement in Mary's number sense, basic fact fluency, class participation, and attitude.

Conclusion

As this intervention program continues to develop it is expected that additional case studies will also show positive results and ultimately fulfill the goals of the program: improved understanding in mathematics, improved attitude towards mathematics, and confidence. It is hoped that this short term intervention will assist students in meeting the goals of third grade mathematics and provide the foundation for mathematics achievement in the intermediate grades. It is expected that as students develop number sense, fluency with basic facts, problem solving abilities, and the ability to reflect on their own strategy use and thinking they will be more successful and more confident in mathematics now and in the future. New challenges in mathematics should not cause panic, but the student should now have strategies to apply to be successful.

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Table 1. Case Study Tutoring Sessions

session		
1	10	
mine	Mary's	
5+5	4+5	
11 --1	6+4	
2 x 5	13 --3	
5+2+3	14 --4	
20-10	15 --5	
ten	16 --6	
20/2	17 --7	
dime		

session		
2	8	
mine	Mary's	
6+2	4+4	
	5+3	
	2 x 4	
	10 --2	
	11--3	
	16 --8	25--17
	17--9	28--20
	18--10	29--21
	19--11	30--22
	20--12	31--23
	21--13	32--24
	22--14	
	23--15	
	24--16	

session		
3	25	Changes
		After
	Mary's	discussion
	24+1	
	23+2	
	22+3	
	21+4	
	20+5	
	19+6	
	18+7	
	17+8	
	16+9	
	15+10	
	14+8	14+11
	13+7	13+12
	12+6	12+13
	11+5	11+14
	10+	10+15

Session		
4	100	
	Mary's	
	99+1	25+25+25+25
	98+2	
	97+3	
	96+4	
	95+5	
	94+6	
	93+7	
	92+8	
	91+9	
	90+10	

Session		
5	50	
mine	Mary's	
	20+30	
	100-50	
24+1+24+1		
25+25		
25 x2		
10+15+20+5		
99+1-50		
	0+50	
	1+49	
	1 x 50	

session	
6	24
mine	Mary's
	24 x1
	2 x7-14+10
	2 tens 4ones
	23+1
	22+2
	21+3
	20+4
	19+5
	18+6

89+11
88+12
87+13
86+14
85+15
84+16
83+17

20+20+10
40+10
49+1