International Journal of Applied Electromagnetics and Mechanics 28 (2008) 55–59
IOS Press

Region of support calculations for a focused linear array of circular pistons

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Abstract. A geometric approach is developed to calculate the region of support for a focused linear array of circular pistons in a lossless medium. Equations are derived describing the boundaries of the region of support for a single piston. The model is expanded to examine the lossy case utilizing a Gaussian distribution to model attenuation in a viscous medium. These results are verified using the Fast Nearfield Method, and the numerical error is evaluated.

1. Introduction

In biomedical ultrasound, modeling focused linear arrays is useful in applications such as B-mode imaging [1]. Calculating the region of support for the waveform emitted by an array at a given instant in time allows the transient field to be computed more efficiently by narrowing the calculation space. Additionally, visualizing the region of support can be helpful when designing arrays of circular pistons. In this paper, a geometric method is derived and implemented for calculating the region of support in both lossless and viscous media. A reduction in computational overhead is achieved, especially in the lossless case.

2. Methods

2.1. Lossless model

In waveform modeling, Huygens' constructions provide the template for building a model of wave propagation. An array of circular pistons embedded in a lossless, homogeneous acoustic medium is considered. Each point on the surface of the piston is considered to be a source that radiates an outgoing spherical wave. At any particular point in time, a "snapshot" of the pressure wave can be obtained. In a cross-section taken in the axial-radial plane, this waveform takes the form of a semi-circle with radius r determined by the distance between pistons d. By superposition, the region of support due to a single piston can be represented as the sum of regions due to each point on the surface of the piston. Figure 1 shows a model of the expected boundaries of a single pulsed-piston with a pulse width of w μs. The dark region is the region of support as given by projecting Huygens' constructions from the center and edges of the piston. The light region represents the area behind the lagging edge of the pulsed wave, where the boundary is determined by the projection of the trailing edge of the waveform due to the extreme points on the piston.

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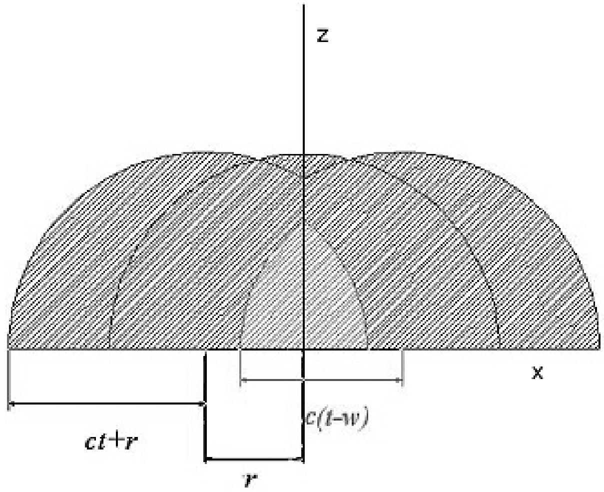

Fig. 1. xz-axis view of the region of support for a single piston.

2.2. Lossy model

In a lossy medium, the combined effects of diffraction and viscous diffusion are modeled by a Gaussian function multiplied by the unit step function $u(t)$ [5]

$$g(R,t) = \frac{u(t)}{2\pi R\sqrt{2\pi\gamma t}} \exp\left(-\frac{(t - R/c_0)^2}{2\gamma t}\right) \tag{1}$$

with the standard deviation proportional to the relaxation time γ of the medium, and a mean corresponding to the path length R divided by the speed of sound c_0 with a relaxation time $\gamma = 0.001\ \mu$s. The weakly dispersive, viscous media modeled by Eq. (1) yields an attenuation coefficient with a frequency-squared dependence. Since the region of support produced by this model is unbounded, a threshold must be chosen to define the boundaries of the region of support. Using Eq. (1) as the Green's function allows the threshold to be defined in terms of the number of standard deviations from the mean, where the standard deviation is given by $\sqrt{t\gamma}$.

3. Results

3.1. Lossless model

The leading axial boundary z_L of a single piston's region of support Ω as a function of lateral distance x is given by

$$z_L = \begin{cases} \sqrt{(c_0 t)^2 - (x + r)^2} & -(tc_0 + r) \leqslant x \leqslant -r \\ c_0 t & -r < x < r \\ \sqrt{(c_0 t)^2 - (x - r)^2} & r \leqslant x \leqslant (tc_0 + r) \end{cases} \tag{2}$$

where x is given as the lateral distance from the center of the piston in mm, and r is given as the radius of the piston in mm. After the piston has ceased emitting sound, the trailing axial distance z_T is calculated

Table 1
Reference field parameters

Elements	r	d	w	t	Focus(x, z)	f	c$_o$
16	0.3mm	0.9 mm	1.2 μs	33.33 μs	(0,50) mm	2.5 MHz	1.5 mm/μs

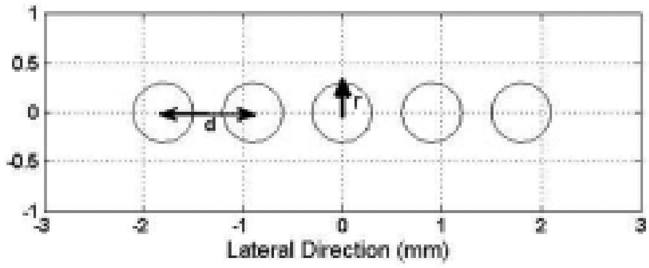

Fig. 2. Array geometry.

as a function of the lateral distance x

$$z_T = \begin{cases} \sqrt{(c_0(t-w)^2) - (x-r)^2} & x \leqslant 0 \\ \sqrt{(c_0(t-w)^2) - (x+r)^2} & x > 0 \end{cases} \tag{3}$$

where w is the pulse length. Verification of these results was carried out by comparing pressure fields calculated point-by-point using the time-space decomposition method [2]. Using MATLAB, snapshots are generated and then verified using the Fast Nearfield Method (FNM) [2] for a uniform set of array elements excited by a Hanning weighted toneburst. Figure 2 shows the array geometry considered. The array parameters are given in Table 1. A snapshot of the computed field is displayed on a logarithmic scale in Fig. 3, while the associated region of support is displayed in Fig. 4. The calculated error between the FNM and the geometrical calculation results is on the order of 10^{-13}. With these boundaries and conditions, the lossy region of support can be found using methods quite similar to those utilized in the lossless case. Figure 5 displays the region of support due to an array with parameters shown in Table 1.

For quarter-wavelength spatial sampling of 0.15 mm, the entire computation grid contains 255783 points, while the region of support depicted in Fig. 4 consists of 71300 grid points. Therefore, pressure computations shown in Fig. 3 are performed on less than 28% of the grid points, yielding an approximate factor of 4 speedup. Greater speedups are encountered for larger times due to the divergence of the wavefront.

3.2. Lossy model

An additional term of $n\sqrt{t\gamma}$ is added to Eqs (2) and (3) to adjust for the additional area encompassed by the area of diffraction, where n is the number of standard deviations from the lossless case, which defines the width of the band included in the region of support. The grey region represents the lossless case, whereas the black region characterizes the lossy region with width using $n = 2$. For greater accuracy, larger values of n may be chosen. Figures 5 and 6 demonstrate the difference due to a varying relaxation time as generated via the geometrical method.

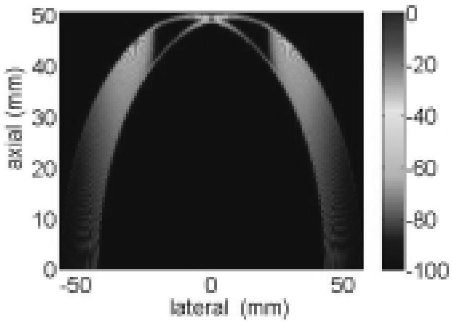

Fig. 3. Pressure field generated by FNM.

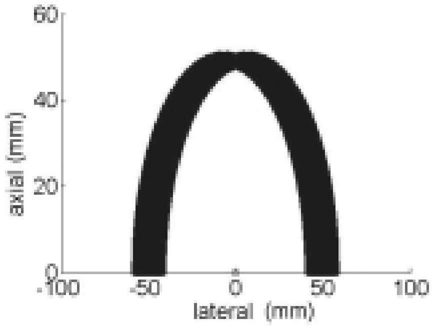

Fig. 4. Region of support in xz-plane generated by geometrical method.

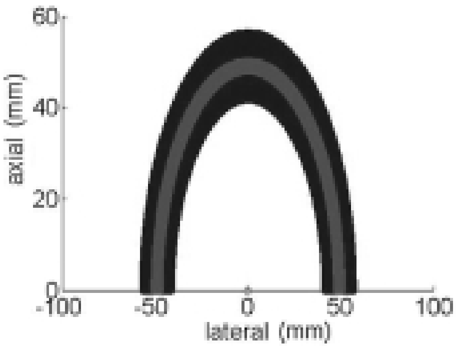

Fig. 5. Ideal (black) and lossy (grey) region of support with $\gamma = 0.1$ μs (extreme case).

4. Discussion

Large scale wave propagation studies require each element in the phased array to be modeled as a source, thus requiring the computation of an incident time-domain field. Since full-wave simulations may span hundreds of wavelengths in each dimension, the incident field must be computed on a very large volumetric grid for the entire time-history of the study. The proposed region of support method will vastly reduce the computational overhead associated with incident field calculation required as input into finite difference time-domain, k-space [3], or plane-wave time-domain codes [4]. The combination

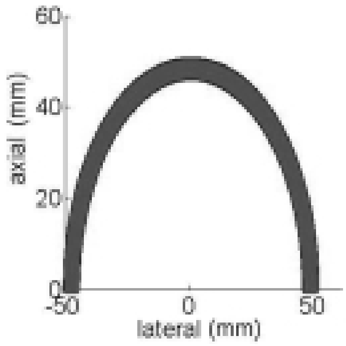

Fig. 6. Ideal and lossy region with $\gamma = 0.001\ \mu$s.

of fast incident and scattering field computation yields a high-fidelity simulation tool which captures the combined effects of focusing, scattering, and absorption. Future studies will consider strongly dispersive, tissue-like media with power-law loss [7]. The region of support calculation in dispersive media will enable array designs that compensate for pulse distortion due to frequency-dependent phase speed variations.

5. Conclusion

A method for using geometric methods to define the region of support due to a linear array of circular pistons has been proposed. Verification against a reference field has produced minimal error. The region of support calculation enables high-speed computation of complicated transient fields produced by focused and steered phased arrays.

Acknowledgments

This work was supported in part by NIH grant Nos. 1R01 CA093669 and 1R21 CA121235.

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