Algorithms for Kernel in Permutation and P_4-free graphs

Moncef Abbas[a], Youcef Saoula[b]

[a]USTHB, Faculté de Mathématiques, LAID3, El-Alia, BP 32, Bab-Ezzouar,16111 Alger, Algérie

[b]Ecole Normale Supérieure, Département de Mathématiques, BP 92 Kouba, 16050 Alger, Algérie.

E-mail adress: Moncef_abbas@yahoo.com, saoula_y@yahoo.fr

Abstract: *In this paper, we give two polynomial algorithms which detect a kernel in permutation graphs and in $P_4 - free$ graphs with a normal orientation.*

Keywords: Orientation, kernel, permutation graph, $P_4 - free$ graph.

A graph is $P_4 - free$ if it has no subgraph with four vertices *a,b,c,d* and edges *ab,bc* and *cd*. For every undirected graph $G = (V, E)$, we call *orientation* of G any directed graph D with the same vertex-set as G and such that two vertices are adjacent in G if and only if there exists at least one arc between them in D. The orientation and the graph D will be not distinguished.

Let G be an undirected graph on n vertices and D an orientation for G. A clique C in G (resp. in D) is a subgraph of G (resp. of D) such that for every pair $\{u,v\}$ of vertices of C, the edge uv exists in G (resp. arc (u,v) or (v,u) is in D). A subset of vertices in G (resp. in D) is called *independent* if any two of its vertices are non-adjacent. A subset N of vertices of D is called *absorbing* if every vertex outside N has a successor in N. A *kernel* of D is a subset of vertices that is both absorbing and independent. An orientation D of G is said to be normal if every clique (not necessarily maximal) in D has a kernel. G is called a *permutation graph* if there exist: (i) A labeling $v_1, v_2, ..., v_n$ of its vertices and (ii) A permutation P of the integers 1,2,..., n under which v_i and v_j are adjacent in G if and only if $(P(i) - P(j)) / (i - j) < 0$.

Not every directed graph admits a kernel; in fact it was shown that deciding if a graph possesses a kernel is NP-complete problem; it would be interesting to elaborate polynomial algorithms which permit to find a kernel of graphs for which the existence problem is solved. It is known that the P_4-free graphs are permutation graphs. For a graph $G = (V, E)$ we give an algorithm which detect a kernel in the case of permutation graphs in $O\left(|V|^2 \cdot |E|\right)$ times; a fast algorithm to determine a kernel in the case of $P_4 - free$ graphs is given in $O(|V| \cdot |E|)$ times.

AMS Classification: 05C69, 05C85

References

[1] C. Berge, P. Duchet: Strongly perfect graphs, Annals of Discrete Mathematics 21 (1984) 57-61.

[2] M. Blidia, P. Duchet and F. Maffray: Note on kernels in perfect graphs, Combinatorica, 13 (2) (1993) 231-233.

[3] V. Chvatal: On the computational complexity of finding a kernel, Report CRM300, Centre de Recherche Mathématiques,Université de Montréal, 1973.