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Registration of Point Clouds Based on Differential Geometry of Surface's Feature

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Abstract. A method for accurate registration on point clouds is presented in the paper. Manual alignment or the use of landmarks is avoided in the process of multi-view point clouds. Firstly, the differential geometric information is extracted from the point clouds. The extended Gaussian sphere and combination features are used to define the corresponding points of crude alignment. Secondly, the optimal algorithm，the point-to-point Iterated Closest Point, is applied to the accurate registration on point clouds. Thus, the complete point cloud can be obtained in the method.

Introduction

Registration algorithm in reverse engineering, complex surface inspection, computer vision and digital image processing has been widely studied. Variety of algorithms needs to have a reasonable classification, including: regional definition, feature extraction, feature representation, local matching and global matching five parts. Registration algorithm[1-6] can be divided as the inner feature algorithm and the outer feature algorithm according to the region defined, feature extraction and the feature expression of the point clouds.

The inner feature registration algorithm such as surface signature[2], spin-image[3], geometric histogram[4], harmonic shape image[5], splashes[6], is a dimension reduction approach. Through three-dimensional space into two-dimensional space, the registration of point clouds is changed as the image matching problem in two-dimensional space domain. The characteristics of the inner feature algorithms are storage capacity large, low accuracy and low robust.

The outer feature representation method is closely related to the coordinate system, the most common method is to use landmarks and optimal matching algorithm. Point cloud alignment accuracy depends on the accuracy of landmarks and the location of landmarks[7]. The golden standard for fine registration is the Iterative Closest Point algorithm and its variants[7,8]. ICP techniques either assume a rough alignment of the two point sets or run the algorithm by sampling the space of initial conditions.

In the following text, we will discuss the principle of registration, geometric feature information is extracted and the extend Gaussian sphere is used to define the corresponding points of crude alignment. In the end, some cases have been inspected.

Principle of Registration Algorithm

The registration algorithm in the paper includes four sections. Firstly, it is data processing, including removal of noise points, sampling and feature information extraction. Secondly, it is the calculation of differential geometry parameter, including nearest point search, Gaussian and mean curvature and center point calculation. Thirdly, crude alignment is done by means of differential geometric information. Fourthly, global registration of point clouds is done by ICP algorithm. Fig. 1 shows the steps of the method in paper.

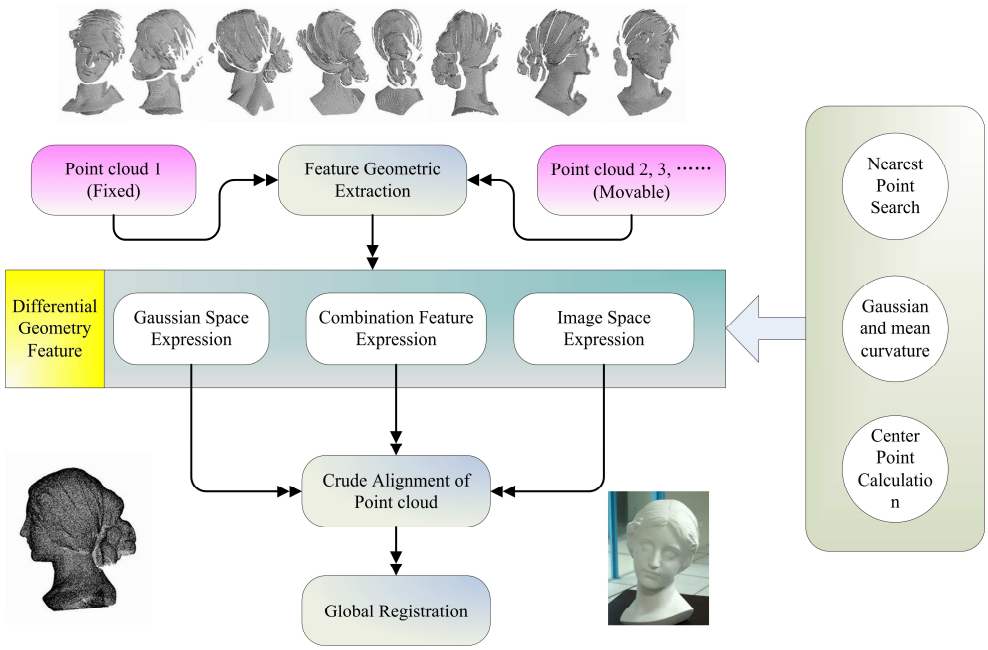

Fig. 1 Flow diagram of the registration based on differential geometry

A curve or surface may be expressed in Gaussian image. Gaussian image has complete geometry information. The sphere includes the normal vector, Gaussian curvature and mean curvature, so it is called the extended Gaussian sphere. Fig.2 is the example of Gaussian sphere expression about one brand car lamp surface. Fig.2(a) is the point cloud of the car lamp. Fig.2(b) is the normal vector of the point cloud. Fig.2(c) is the curvature of the point cloud. Fig.2(d) is Gaussian sphere expression of the point cloud.

(a) Point cloud (b) Normal vector (c) Curvature (d) Gaussian sphere

Fig. 2 Feature expression of a car lamp

Differential Geometric Information of Point Cloud

Nearest Point Search. Firstly, the k - neighborhood of point cloud data is determined, then the KD tree of the point cloud data is built, To traversal through the KD tree, the neighborhood points will be found, and then the normal vector at each point can be estimated. The normal vector N on a point P can be regarded as the normal vector of least square fitting surface based on the neighborhood point. The point O is the center of the neighborhood point P.

$$O = \frac{1}{k} \sum_{hi \in nbhd(P)} P_{hi} \tag{1}$$

Normal Vector Calculation. The normal vector of least square fitting surface is obtained by an optimization algorithm to make Eq. 2 minimum.

$$f = \sum_{hi \in nbhd(P)} \left\| (P_{hi} - O) \cdot N \right\| \tag{2}$$

The optimization problem can be transformed for to seek the smallest eigenvalue problem of a covariance matrix.

$$\begin{bmatrix} \sum_{hi}(x_{hi}-O_x)^2 & \sum_{hi}(x_{hi}-O_x)(y_{hi}-O_y) & \sum_{hi}(x_{hi}-O_x)(z_{hi}-O_z) \\ \sum_{hi}(y_{hi}-O_y)(x_{hi}-O_x) & \sum_{hi}(y_{hi}-O_y)^2 & \sum_{hi}(y_{hi}-O_y)(z_{hi}-O_z) \\ \sum_{hi}(z_{hi}-O_z)(x_{hi}-O_x) & \sum_{hi}(z_{hi}-O_z)(y_{hi}-O_y) & \sum_{hi}(z_{hi}-O_z)^2 \end{bmatrix} \tag{3}$$

Which, x_{hi}, y_{hi}, z_{hi} are the three coordinates, O_x, O_y, O_z are the weight of the three coordinates. The matrix is real a symmetric matrix, and Jacobian method is used to solve the eigenvalue. Let the minimum eigenvalue as $\lambda_{\min}$, the corresponding eigenvector is $V_{\min}$, then the normal vector of the least squares fitting surface at the point P is

$$N = V_{\min} \tag{4}$$

Curvature Calculation. Then the origin of local coordinate system is the point P, (u,v,h) is coordinate axis, the direction of the axis h is the normal vector of P, the other two axes lie on the tangent plane at the point P. The minimum fitting surface in the local coordinate system is expressed as Eq. 5.

$$h = au + bv + \frac{c}{2}u^2 + duv + \frac{e}{2}v^2 \tag{5}$$

Where $a = f_u, b = f_v, c = f_{uu}, d = f_{uv}, e = f_{vv}$

For a point cloud $P_i, i = 1,2,\ldots n$, an equation can be established as

$$AX = B \tag{6}$$

Where $A = \begin{bmatrix} u_1 & v_1 & \frac{u_1^2}{2} & u_1v_1 & \frac{v_1^2}{2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ u_n & v_n & \frac{u_n^2}{2} & u_nv_n & \frac{v_n^2}{2} \end{bmatrix}$ $X = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$ $B = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_n \end{bmatrix}$

Solving equation 6, we can get $X = \left[A^T A\right]^{-1} A^T B$, then the surface first and second base capacity express as following

$E = 1 + a^2$, $F = ab$, $G = 1 + b^2$

$$L = \frac{c}{\sqrt{1+a^2+b^2}}, \quad M = \frac{d}{\sqrt{1+a^2+b^2}}, \quad N = \frac{e}{\sqrt{1+a^2+b^2}} \tag{7}$$

Two principal curvatures satisfy the following equation

$$(1+a^2+b^2)k_n^2 - \left(\frac{(1+a^2)e + (1+b^2)c - 2abd}{\sqrt{1+a^2+b^2}}\right)k_n + \frac{ce-d^2}{1+a^2+b^2} = 0 \tag{8}$$

Solving the equation, we can get two principal curvature, and the results are substituted into the Eq.9 can get the two principal directions corresponding to the principal curvatures.

$$\begin{cases} (L - k_n E)du + (M - k_n F)dv = 0 \\ (M - k_n F)du + (N - k_n G)dv = 0 \end{cases} \tag{9}$$

So, Gaussian curvature is

$$K = k_{n1} \cdot k_{n2} = \frac{ce - d^2}{(1+a^2+b^2)^2} \tag{10}$$

Mean curvature is

$$H = \frac{k_{n1} + k_{n2}}{2} = \frac{(1+a^2)e + (1+b^2)c - 2abd}{2(1+a^2+b^2)^{3/2}} \tag{11}$$

Experiments and results

Fig.3 is the case of a brand car lamp's inspection. The left picture is original position of point cloud and CAD model. The middle picture is the registration of both data. The right picture is the result of deviation analyses. When crude alignment is finished, the biggest error value of Euclidean distance is 4.0mm, the mean error is 2.2mm and the stranded error is 0.7mm. When accurate registration is finished, the biggest error value of Euclidean distance is 2.0mm, the mean error is 0.467mm and the stranded error is 0.235mm. The results meet the requirements of surface inspection.

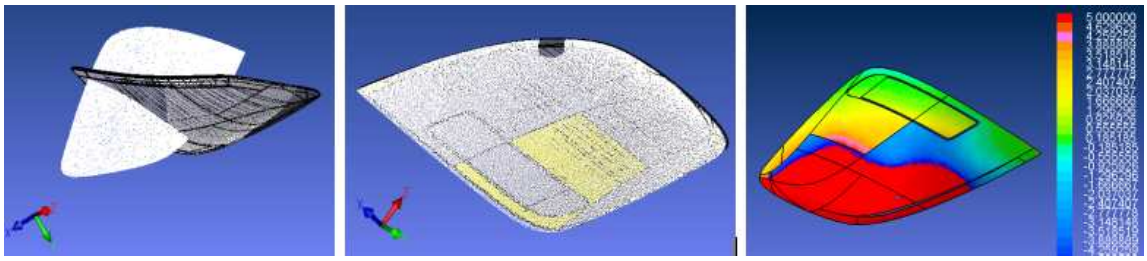

Fig. 3 Registration of point cloud and CAD model

Conclusions

The extended Gaussian sphere and combination features are used to define the corresponding points in crude alignment. The Iterated Closest Point method is applied to the accurate registration on point clouds. The experiment shows that the accuracy of algorithm is to meet actual requirements.

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