A Type-2 Fuzzy Rule-Based Expert System Model for Portfolio Selection

M.H. Fazel Zarandi, **E. Hajigol Yazdi**

Email: zarandi@aut.ac.ir, elahehajigol@gmail.com

Department of Industrial Engineering, Amirkabir University of Technology, P.O. Box 15875-3144, Tehran, Iran

Abstract:

This paper presents a type-2 fuzzy rule based expert system to handle uncertainty in complex problems such as portfolio selection. In a type-2 fuzzy expert system both antecedent and consequent have type-2 membership function. This research uses indirect approach fuzzy modeling, where the rules are extracted automatically by implementing a clustering approach. For this purpose, a new cluster analysis approach based on Fuzzy *C*-Means (FCM) is developed to generate primary membership of type-2 membership functions. A new cluster validity index based on Xie-Beni validity index is presented. The proposed type-2 fuzzy model is applied in stock market factors (such as risk, return, dividend,…) as the input variables. This model is tested on Tehran Stock Exchange (TSE). Through the intensive experimental tests, the model has successfully selected the most efficient portfolio based on individual investor. The results are very encouraging and can be implemented in a real-time trading system for stock.

Keywords: interval type-2 fuzzy set, fuzzy c-means clustering, validity index

1. Introduction

The stock market is a complex and dynamic system with noisy, non-stationary and chaotic data series (Peters, 1994). Due to complication and uncertainty of stock market, portfolio selection is one of the most challenging problems. Along with the development of artificial intelligence, especially machine learning and data mining, more and more researchers try to build automatic decision-making systems to predict stock market (Kovalerchuk & Vityaev, 2000). Among these approaches, soft computing techniques such as fuzzy logic, neural networks, and probabilistic reasoning draw most attention because of their abilities to handle uncertainty and noise in stock market (Vanstone & Tan, 2003, 2005). Though soft computing can somewhat reduce the impact of random factors, low-level data are so uncertain that they even behave purely randomly at some time (Peters, 1994). Human understanding of stock market integrates into the high-level representative model which reduces hidden ambiguity of low level data. The elements of financial market (such as risk, return, …) fluctuate because of the continues variety of strength between buying and selling side.

Portfolio management is a particular aspect of investment theory. There are a large number of ways of investing in the stock exchange. Thanks to the different techniques, it is possible to obtain the estimated profit and risk of a certain action. The goal of investment decision-maker is to select an optimal portfolio that satisfies the investor's objective. The difficulty of this problem is deciding about selecting which asset and its size in portfolio because of the uncertainty of their returns.

Quantitative approaches to portfolio optimization rely on how the return and risk of assets profitably are defined and measure. Each specified model provides the decision maker with a specific set of parameters. Since, Experts do not agree on which return model is most appropriate therefore they recommend different optimal portfolio choices.

The Modern portfolio analysis started from pioneering research work of Markowitz (1952). In modern portfolio selection theory, the well-known Markowitz's mean–variance approach requires minimizing the risk of the selected (stock) asset portfolio (measured by the variance of its return), while guaranteeing a pre-established return rate and the total use of the available starting capital.

After Markowitz work, scholar assumes returns of individual securities as stochastic variables and many researchers were focused on extending Markowitz's mean-variance models [13, 15, 36]. Moreover, the financial market is affected by several non-probabilistic factors such as vagueness and ambiguity in information which is characterized by linguistic descriptions such as high risk, low profit, high interest rate etc. so, Some authors have used possibility distributions to model the uncertainty on returns, while other authors have studied the portfolio selection problem using fuzzy formulations.

Tanaka and Guo replaced probability distributions of returns of securities with possibility distributions in their models. Carlsson introduced a possibilistic approach for selecting portfolios with the highest utility value under the assumption that the returns of assets are trapezoidal fuzzy numbers. In this paper, we develop an expert system for portfolio selection based on investor's propensity for risk.

The goal of this research is to develop a fuzzy modeling mechanism which is capable of implementing five objectives:

- generating a rule base automatically from numeric data,
- finding the optimal number of rules and fuzzy sets,

- optimizing the parameters of fuzzy membership functions,
- Reducing the impact of vagueness and ambiguity hidden in stock market situation
- Implementing the investor's personality for increasing the robustness of the system

To achieve these objectives, this paper proposes a fuzzy modeling paradigm by incorporating a modified FCM clustering associated with a proposed cluster validity measure, with considering interval type-2 fuzzy sets domain.

The rest of paper is organized as follows: Section 2 reviews the type-2 fuzzy sets and systems and their associated terminologies. In Section 3 the design approach of interval type-2 fuzzy logic system is presented. Section 4 presents the proposed interval type-2 fuzzy system for prediction of single stock price of an automotive manufactory in Asia. Finally, conclusions and comments on further research are appeared in Section 5.

2. Background:

2.1. Interval Type-2 Fuzzy Sets

To-date, because of reducing the computational complexity of using a general Type-2 fuzzy set in a Type-2 fuzzy logic system, most researchers implement an IT2 FS. The result is an IT2 FLS. In This paper, very simple introduction into IT-2 fuzzy modeling given for someone who is new to the field of an IT2 FLS to get into it very quickly as follow (Mendel & john 2002):

Definition 1. A type-2 fuzzy set, $\tilde{A}$ denoted, is characterized by a type-2 membership function, $\mu_{\tilde{A}}(x,u)$ x∈X as $\mu_{\tilde{A}}(x=x',u)$ $\equiv \mu_{\tilde{A}}(x') = \int_{u \in j_{x'}} 1/u$ Where x∈X and $u \in J_x \subseteq [0,1]$, i.e.,

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \quad (1)$$

in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$, $\tilde{A}$ can also be expressed as:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u)/(x,u) \qquad J_x \subseteq [0,1] \quad (2)$$

Definition 2. When all secondary grades as $\mu_{\tilde{A}}(x,u) = 1$ then is an interval T2 FS (IT2 FS).

Definition 3. Uncertainty in the primary memberships of an IT2 FS, $\tilde{A}$, consists of a bounded region that we call the footprint of uncertainty (FOU). It is the union of all primary memberships, i.e.,

$$FOU(\tilde{A}) = \bigcup_{x \in X} j_x \quad (3)$$

The upper membership function (UMF) and lower membership function (LMF) of $\tilde{A}$ are two T1 MFs that bound the FOU define as:

$$\bar{\mu}_{\tilde{A}}(x) \equiv \overline{FOU(\tilde{A})}, \forall x \in X \quad (4)$$

$$\underline{\mu}_{\tilde{A}}(x) \equiv \underline{FOU(\tilde{A})}, \forall x \in X \quad (5)$$

Note that for an IT2 FS , $j_x = [\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)], \forall x \in X$

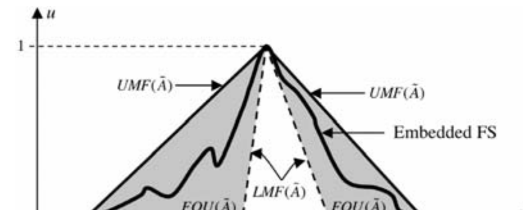

Fig1. Foot Print of Uncertainty for an interval type-2 fuzzy set (Mendel, 2002)

2.2. Interval Type-2 Fuzzy Logic Systems

A general T2-FLS as shown in fig.2 is very similar to the T1-FLS. The major structural difference being that the defuzzifier block of a T1-FLS is replaced by the output processing block in a T2-FLS. That block consists of type-reduction followed by defuzzification. Type-reduction maps a T2-FS into a T1-FS, and then defuzzification, as usual, maps that T1-FS into a crisp number.

The structure of the rules remains exactly the same in the T2 case, but now some or all of the FSs involved are T2. When all of the antecedent and consequent are IT2 FSs, then we call the resulting FLS, an interval T2 FLS (IT2 FLS).

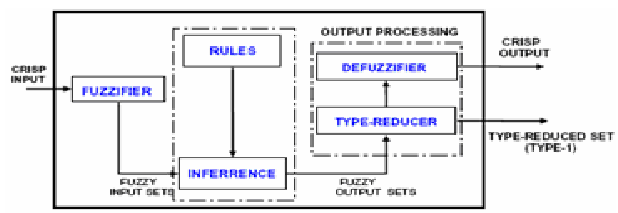

Fig. 2. The structure of a type-2 fuzzy logic system (Mendel, 2007)

3. Designing interval type-2 fuzzy logic system

There are different kinds of expert system based on functionality and structure of them but many early researches pay much more attention to rule-base reasoning. An Expert (Knowledge Based) System is a problem solving and decision making system based on knowledge of its task and logical rules or procedures for using knowledge. Both the knowledge and the logic can obtain in two manners so two kinds of expert system exist: direct and indirect expert systems. In direct approach, rules designed by considering experience of specialists in the purposed area. On the other hand, indirect approach designed system without implemented expert opinion straightly, but constructed with knowledge existence about problem and based on knowledge rules design.

Because of existing high degree of uncertainty in the information used for building rules such as different experts view points about purposed problem, we implement fuzzy expert system for solving our portfolio selection problem. So, we design an FLS too. Our basic workspace rest on Stock Market Exchange where investment parameters takes place in an imprecise and/or uncertain economic environment that early market condition changes cannot be predicted accurately. Then for most robustness we consider interval type-2 fuzzy expert system that covers much many uncertainties in the studying area. We have to develop an interval type-2 fuzzy system for portfolio selection. Fuzzy modeling can be identified as system description with fuzzy quantities. Since there is no general method for fuzzy logic set up and iterative procedure, to obtain the best performance of a FLS, the different parts of the FLS have to be identified. This stage of modeling is known as identification. According to Zadeh's definition, given a class of models, system identification involves finding a model which may be regarded as equivalent to the objective system with respect to input-output data. Identification of a fuzzy model is divided into two sections (Sugeno & Yasukawa, 1993): Structure Identification, and Parameter Identification. Structure Identification can be divided into two types, called type I and type II. Each type is divided into two groups. The structure identification of type 1 divided into subsystems:

- Input candidate: as we know there is infinite number of possible inputs candidate for the inputs to a system, which should restricted to certain number. There isn't general solution for selecting finite number of inputs. So, we have to use heuristic methods to solve it. For our case i.e. portfolio selection variable can be grouped as follows:
  1. Linguistic variable that be apply for answering questions that identify investor's risk propensity.
  2. crisp Variables as stock market parameters;
- **Input variable:** In this section we find a set of input variables from the given input candidate which affects the inputs. There are some systematic models to solve this problem. These models are based on preassigned input-output variables. Sugeno and Yasukawa method used for selecting input variables among candidate variables.
- **Input- output relations:** In Input-output relation identification, number of fuzzy rules and how partitioning input space recognize. This problem is a combinational one. We need a heuristic method to find an optimal partition together with a criterion. These problems are solved by applying fuzzy clustering methods. For our purpose we use modified FCM that can be used by interval type-2 data.

3.1. Clustering the output space and determination of the number of rules

Clustering [1, 2, 5], a major branch of data mining, is used for discovering groups and identifying patterns and distributions of a given set of data. The goal of every clustering algorithm as an unsupervised classification is grouping data elements according to some (dis)similarity measures so that unobvious relations and structures in the data can be revealed. Since our data and centers of clusters are interval type-2 fuzzy sets, we use a distance measure base on indices, which is applicable for IT-2 sets. For achieving this aim, we introduce a determination index that can approximate each IT-2 fuzzy datum with type-1 one. We design an index that approximate each Interval type-2 fuzzy set by a type-1 one. The purposed distance measure is based on comparing each pair of fuzzy approximated variable by considering distances between their means (M_i, M_j) and standard deviations (Σ_i, Σ_j), separately. The proposed algorithm is as follow:

Step1: store unlabeled interval type-2 fuzzy data: $\tilde{x}_i \in \tilde{X}, i=1,2,...,n$, $\tilde{x}_i$ is an IT-2 fuzzy number for *i=1,...,n.*

Step2:choose

c : the number of cluster such that $1<c<n$

m : weightening exponential as fuzziness measure, $m>1$

ε : termination criterion , $0<\varepsilon<1$

initial fuzzy vector $\tilde{v}_{0j} \in \tilde{V}_0, j=1,2,...,c$

step 3 : approximate each $\tilde{X}, \tilde{V}$ with suggested indices as:

$$\tilde{X} \approx \alpha.\mu_{\tilde{X}} + (1-\alpha).\Sigma_{\tilde{X}} \quad (6)$$

and

$$\tilde{V} \approx \alpha.\mu_{\tilde{V}} + (1-\alpha).\Sigma_{\tilde{V}} \quad (7)$$

Step 4: iterate : for t=1 to To calculate U_t with V_t and μ_{ij} as:

$$\mu^t{}_{ij} = \frac{1}{\sum_{k=1}^{c}\left[\frac{\left[w_\mu^2 d_{ij\mu}^2 + (1-w_\mu)^2 d_{ij\Sigma}^2\right]}{\left[w_\mu^2 d_{ik\mu}^2 + (1-w_\mu)^2 d_{ik\Sigma}^2\right]}\right]^{\frac{1}{m-1}}} \quad (8)$$

calculate M_j, Σ_j based on U_t as:

$$M_j = \frac{\sum_{i=1}^{n}\mu_{ij}^m M_i}{\sum_{i=1}^{n}\mu_{ij}^m}. \quad (9) \qquad \Sigma_j = \frac{\sum_{i=1}^{n}\mu_{ij}^m \Sigma_i}{\sum_{i=1}^{n}\mu_{ij}^m}. \quad (10)$$

IF $E_t = \|(U_{t-1})-(U_t)\| \le \varepsilon$ stop, Else Next (t).

To determine an appropriate number of clusters N_c for a given data set, a cluster validity function needs to be selected here. Cluster validation refers to the problem whether a given fuzzy partition fits to the data all. So it is used for evaluating the clustering results by means of an objective function.

Definition 4. Assume $\tilde{x}_i \in \tilde{X}, i=1,2,...,n$ is unlabeled interval type-2 fuzzy data and $\tilde{v}_{0j} \in \tilde{V}_0, j=1,2,...,c$ is initial IT-2 fuzzy cluster centers with their corresponding mean (M) and variance (Σ)and $1<c<n$, m>1, m Is a parameter which determines the fuzziness of the resulting clusters then state above we define distance measure as:

$$d_{ijM} = d(M_i, M_j) = \|M_i - M_j\| \quad (11)$$

$$d_{ij\Sigma} = d(\Sigma_i, \Sigma_j) = \|\Sigma_i - \Sigma_j\| \quad (12)$$

$$d^2 \equiv \|M_i - M_j\|^2 + \|\Sigma_i - \Sigma_j\|^2 \quad (13)$$

Definition 5. The variance of an Gaussian interval type-2 set with corresponding m_1, m_2, σ define as follow:

$$\Sigma_l = \frac{\sigma(\frac{m_1 - m_2}{2})e^{-\frac{1}{2}(\frac{m_1 - m_2}{2\sigma})^2}}{\sqrt{2\pi}Z(\frac{m_1 - m_2}{2\sigma})} + \sigma^2 + (\frac{m_1 - m_2}{2})^2 \tag{14}$$

$$\Sigma_r = \frac{2\sigma^2(m_2 - m_1) + \sqrt{\frac{\pi}{2}}\sigma(2\sigma^2 + m_1^2 + m_2^2) + \frac{(m_2^3 - m_1^3)}{3}}{m_2 - m_1 + \sqrt{2\pi}\sigma} - (\frac{m_1 + m_2}{2})^2 \tag{15}$$

$$\Sigma_{\mathrm{mod}\,ify} = \frac{\Sigma_l + \Sigma_r}{2} \tag{16}$$

Definition 6. Based on distance definition, modified Xie-Beni index for IT-2 fuzzy data can be defined as:

$$V_e(clust_j) = \frac{\sum_{i=1}^{n}[\tilde{x}_i - \tilde{v}_j]^2 \tilde{\mu}_{ij}^m}{\sum_{i=1}^{n}\tilde{\mu}_{ij}} = \sum_{i=1}^{n}(\mu_{ij}^m[\|M_i - M_j\|^2 + \|\Sigma_i - \Sigma_j\|^2]) \Big/ \sum_{i=1}^{n}\mu_{ij} \tag{17}$$

$$V_{clust} = \bigcup_{j=1}^{c} V_{e_e}(clust_j) = [v_l(clust), v_r(clust)] \tag{18}$$

$$v_l(clust) = Min(v_{clust}) \tag{19}$$

$$v_r(clust) = Max(v_{clust}) \tag{20}$$

$$V_{\mathrm{mod}\,ify} = \frac{v_l(clust) + v_r(clust)}{2} \tag{21}$$

$$\sigma_{clust} = \sqrt{V_{\mathrm{mod}\,ify}} \tag{22}$$

For separation we should maximize the minimum distance between cluster centers. So, we use:

$$\text{Separation} \approx \min_{fuzzyranking}\{\tilde{d}^2(\tilde{v}_i, \tilde{v}_j)\} \tag{23}$$

$$\chi_i = \frac{\sigma_{clust}}{n(\min_{fuzzyranking}\{\tilde{d}^2(\tilde{v}_i, \tilde{v}_j)\}} \tag{24}$$

4. Implementation of the proposed model in stock portfolio selection

In this section, we present an interval type-2 fuzzy model for portfolio selection in Tehran exchange stock market. We study on the 50 best companies of stock market. Input variables of the system are expected return, periodical rate of return, variance of rate of return, dividend of each share, liquidity coefficient of companies and variables that show decision risk preference (10 linguistic variables). The output of system is the percentage of each kind of shares to be bought by investor considering the above variables that can achieve the best performance portfolio. The data is modeled into a Multi-Input-Single-Output (MISO) system. The Output of the rulebase of system is risk investor propensity. Here, 'risk propensity' means the amount of risk an individual would feel comfortable with when they have invested their money. Two investors with the same profile characteristics may make very different financial choices. What's different about them is their level of risk-aversion, not their innate characteristics. By considering how people are unique, with different backgrounds, experiences, and beliefs, an investor who is Cautious by nature may feel comfortable with either a low level, a medium level, or a high level of risk. All these factors feed into the perception of risk. A simple way to assess investor's risk level will be applied. Some questions based on investor personal attitude used to handling risk and the total score is used for determined investor risk-aversion. User manifests his/her agreement with the criterion by using linguistic variables.

The steps of developing of the MISO model are as follows:

1. For variable selection, the Sugeno method (Sugeno variable selection algorithm) is used. For our interval type-2 system, we achieve 5 variables (expected return, periodical rate of return, variance of rate of return, dividend of each share, liquidity coefficient of companies) as inputs to our system further more investor linguistically answers to 10 questions are inputs to our system too.
2. Modified FCM algorithm (section 3-1) is implemented to cluster the questions which are base of measuring investor's risk propensity. Then, a new cluster validity index (a modification version of Xie-Beni index)is used to determinate the most suitable number of rules (*c*); since we cluster the average of answers given to each question then selecting interval type-2 fuzzy data look a good choice. To For reduce complexity, we estimate each IT-2 fuzzy set with an index. The purposed index is type-1 fuzzy set. The parameters of Modified FCM algorithm are assumed as follows: termination criterion ε = 10^{-2}, weighting exponent m= 1.3, c_{max} =4. The best number of clusters based on this cluster validity index is obtained by the minimum value of the cluster validity index that is happened in number 3, so our system contains 3 rules.
3. The output membership values are projected onto the input spaces to generate the membership values of inputs. We assume that inputs and output membership functions are Gaussian MF.
4. For generating interval type-2 fuzzy rule bases that the antecedent and consequent sets are interval type-2 sets, we use Gaussian Primary MF with Uncertain Mean and fixed standard deviation.

We have obtained 15 months stock information that we use 12 month information data for generating our rules and use 3 last period data for testing our model. After clustering, we use the optimum number of cluster 3. So, the number of rules is 3, too.

4.1. Proposed type-2 fuzzy model

We create an interval type-2 FLS based on type-1 FLS. Similar to the type-1 FLS the interval type-2 FLS uses singleton fuzzification; we have two options for selecting t-norm: min and product t-norm. Mamdani inference and centroid type-reduction are implemented. It also uses the same number of fuzzy sets and the same rules as the interval type-2 FLS. The only difference now is that the antecedent and consequent sets are interval type-2 which has a fixed standard deviation and uncertain means that takes on values in an interval, i.e., (Mendel, 2000).In fig 3 Gaussian Interval type-2 rule base for measuring investor risk propensity is shown.

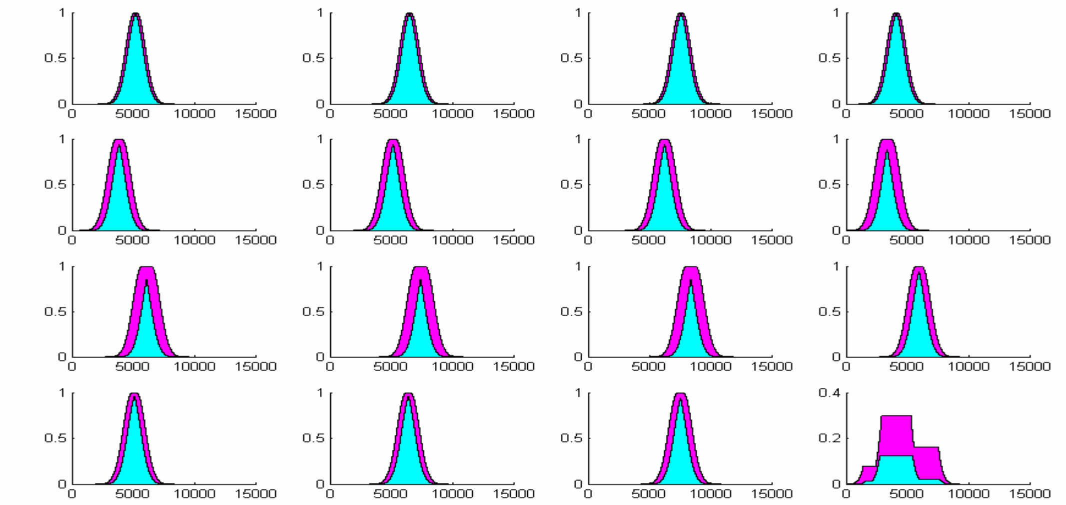

Fig. 3. Interval type-2 rule base of measuring investor risk propensity

As shown in fig.3, there are three inputs. Here, the inference system is mamdani type. The output of our FLS is risk propensity of investor. The advantage of system is determination risk aversion by considering the personality and action that be shown by investor in different stock market situations. So, by implementing this rulebase expert system each investor has a unique portfolio.

The output of our rulebase is an interval type-2 fuzzy set that must be type- reduced and then defuzzify. We use centroid type reduction and centroid defuzzifier. The result is a crisp number that is input to the portfolio selection model. We use compromise programming model in order to solve portfolio selection model.

4.2. Fuzzy portfolio selection model

In this section, portfolio selection problem formulated as an optimization problem with multiple objectives. We assume that an investor allocates his/her wealth among n assets offering random rates of return. We introduce some notations as follows:

r_i : The rate of expected return on the i -th asset,

x_i : The proportion of total funds invested in the i -th asset,

d_i : The annual dividend on the i -th asset,

r_i^{12} : The average 12-month performance of the i -th asset,

r_t : Risk aversion of investor as an instrument for measuring investor risk propensity level

In the proposed multi objective asset portfolio selection problem, we consider the following objectives and constraints.

4.2.1. Objectives

- **Short term return**

For n assets portfolio ($x_1, x_2, \ldots, x_n$) the expected short term return of the portfolio is expressed as:

$$R(x) = \sum_{i=1}^{n} r_i^{12} x_i \qquad (25)$$

r_{it} can be determined by historical data.

- **Annual dividend**

The annual dividend of a portfolio is expressed as

$$D(x) = \sum_{i=1}^{n} d_i x_i \qquad (26)$$

- **Risk**

The semi-absolute deviation of return of the portfolio x = ($x_1, x_2, \ldots, x_n$) below the expected return over the past period t , t = 1, 2, . . . , T can be expressed as:

$$V_t(X) = \left| \min\{0, \sum_{i=1}^{n} (r_{it} - r_i) x_i\} \right| = \frac{\left| \sum_{i=1}^{n} (r_{it} - r_i) x_i \right| + \sum_{i=1}^{n} (r_i - r_{it}) x_i}{2} \qquad (27)$$

So, the expected semi-absolute deviation of return of the portfolio x = ($x_1, x_2, \ldots, x_n$) below the expected return is given by:

$$V(X) = \frac{1}{T} \sum_{t=1}^{T} V_t(X) = \sum_{t=1}^{T} \frac{\left| \sum_{i=1}^{n} (r_{it} - r_i) x_i \right| + \sum_{i=1}^{n} (r_i - r_{it}) x_i}{2T} \qquad (28)$$

Use V(x) to measure the portfolio risk. After that we define a utility function based on risk and return of the portfolio and investor's risk propensity measured by rulebase expert system as follows:

$$U = R(X) - \frac{1}{r_t} V(X) \qquad (29)$$

We should maximize this utility function.

- **Liquidity**

Liquidity is the degree of probability of being able to convert an investment into cash without any significant loss in value. We design a reduced index by implemented heuristic information about companies liquidity degree.

$$L(x) = \sum_{i=1}^{n} L_i x_i \quad (30)$$

Constraints

- Capital budget constraint on the assets:

$$\sum_{i=1}^{n} x_i = 1 \quad (31)$$

Of all the assets in a given set, the investor would like to pick up the ones that in his/her subjective estimate are likely to yield the greatest "portfolio effect". It is not necessary that all the assets in the given set may configure in the portfolio as well. Investors' would differ as regards the number of assets they can effectively handle in a portfolio.

- No short selling of assets:

$$x_i \geq 0, \forall i = 1,...,n. \quad (32)$$

4.2.2. The decision problem

The constrained multi-objective portfolio selection problem is now formulated as:

Maximize $U = R(X) - \frac{1}{r_t} V(X)$

Maximize $D(x) = \sum_{i=1}^{n} d_i x_i$

Maximize $E(L(x)) = E\left(\sum_{i=1}^{n} L_i x_i\right)$

Subject to

$$\sum_{i=1}^{n} x_i = 1$$

$$x_i \geq 0, \forall i = 1,...,n. \quad (33)$$

4.2.3. Compromise programming model for purposed portfolio selection

Within the field of multi-criteria decision making, Compromise programming (CP) is as a technique based on the selection of those efficient solutions nearest to the ideal point, which is composed of the optimum values of each individual objective function. Zeleny states that alternatives that are closer to the ideal are preferred to those that are farther from it because being as close as possible to the perceived ideal is the rationale of human choice. In the context of this work, this leads us to the concept of closeness between fuzzy numbers.

4.2.4. Calculating fuzzy ideal and fuzzy anti-ideal solutions

In order to calculate the ideal solution it is necessary to obtain the individual optimum of each objective. It is necessary to solve each fuzzy single objective problems with constant constraints. We calculate ideal and anti-ideal solution for each 3 problem, respectively (U^*, D^*, L^*) and (U_*, D_*, L_*) that anti-ideal solutions will be obtained by minimizing the objective functions separately. After obtaining fuzzy ideal and anti-ideal solutions next step is minimization of y distance between the objective vector and the ideal solution.

For designing the compromise model of our model we should use the concept of individual regret. All of the objective have attribute of the type ''more is better'' so we will define the individual regret of the portfolio's utility to the ideal utility as:

$$d_1(x) = \frac{U^* - U_P(X)}{ran_U} \quad (34)$$

So the individual regret for second and third objective function defines as:

$$d_2(x) = \frac{D^* - D_P(X)}{ran_D} \quad (35)$$

$$d_3(x) = \frac{L^* - L_P(X)}{ran_L} \quad (36)$$

In order to obtain compromise programming objective function we will aggregate and weight the individual regrets using a weighted objective function. Then the following problem can be formulated:

Min $L_1 = \left[\sum_{j=1}^{3} w_j d_j(x)\right]$

s.t.

$$\sum_{i=1}^{n} x_i = 1 \quad (37)$$

$x_i \geq 0, \forall i = 1,..., n.$ Compromise solution corresponding with distance minimizes the weighted sum of the individual regrets from the ideal (U^*, D^*, L^*).

The Weighted system is based on the importance of criterion and specified its primary value by expert. The most importance causes the most weight. In the table 1 shown the amount of purposed weight that is used for optimization the compromise programming model.

Objective function 1 utility function	Objective function 2 dividend	Objective function 3 liquidity
0.6	0.2	0.2

Table 1. Purposed weights apply for compromise programming model

This weight tune by running system several times and this can be viewed as a good choice. Data of stock market call from an excel sheet and by applying lingo 8 we solve portfolio selection problem. For an investor that given the following answer to the question as stated in table 2 the output of rulebase system (investor's risk propensity) is 8.6(the range of risk propensity of an individual investor is between 0 and 10). So this investor is a risky one. Optimum portfolio is constructed in the manner the investor achieve the best profit from investigation under his/her risk prefers. The result of purposed model brief in table 2 as follow:

Symbol	x_{26}	x_{37}	x_{42}	x_{45}
Ratio	1.96%	1.6%	85.97%	10.47%

Table 3. Results of designed compromised programming model

By comparing result with the information of these stocks we can consider it's a best one, because by means of this model we select stocks with high profit by consuming investor risk aversion level and high liquidity ratio and appropriate dividend.

1	2	3	4	5	6	7	8	9	10
Slightly agree	Makes no difference	Makes no difference	Highly agree	Highly agree	Makes no difference	Slightly agree	Highly agree	Slightly agree	Highly agree

Table 2. A case answers of an individual investor given to 10 questions

5. Conclusions and future works:

The focus of this paper was to develop an interval type-2 fuzzy expert system for stock portfolio selection of the 50 most appropriate companies from the Tehran Stock Exchange (TSE).

After investigating the system domain, the inputs and output of the system were determined. Then the stock portfolio selection model was designed in a multiple-input-single-output (MISO) system that has 3 inputs and one output (investor risk propensity).

We have used indirect approach to fuzzy system modeling by implementing the proposed cluster validity index for determining the number of rules in fuzzy clustering approach. Then Sugeno and Yasukawa method was used to selecting variables for rule-base fuzzy logic systems. After that, the output membership value was projected onto the input spaces to generate the membership values of input variables, and the membership functions of inputs and output were tuned. Then, the type-2 Mamdani inference system was implemented that the result is interval type-2 fuzzy number. For generating interval type-2 fuzzy rule base the Gaussian Primary MF with Uncertain Mean and fixed standard deviation was used; then interval type-2 inference engine, product or min t-norm optionally, sum s-norm and the centroid Type Reduction was used after that centroid defuzzification was done to get the result. Finally, we have implemented the proposed fuzzy model in stock portfolio selection of the 50 most appropriate companies from the Tehran Stock Exchange (TSE).

For validation of system we used last three month data and compared its result with the result of the proposed type-2 fuzzy model.

The proposed system shows its superiority with respect to robustness, flexibility. The system may be used by institutes to purpose optimum stock portfolio by considering personality of their clients. Also it can be used by individual investor for more profitable trading in stock market.

6. Reference:

[1] M.R. Anderberg, Cluster Analysis for Application, Academic Press, NewYork, 1973.

[2] J.C. Bezdek, Fuzzy mathematics in pattern classification, Ph.D. Dissertation, Cornell University, Ithaca, NY, 1973.

[3] J.C. Bezdek, Cluster validity with fuzzy sets, J. Cybernet. 3 (1974) 58–73.

[4] J.C. Bezdek, Pattern Recognition with Fuzzy Objective Function Algorithms, Plenum Press, NewYork, 1981.

[5] J.C. Bezdek, R.J. Hathaway, M.J. Sabin, et al., Convergence theory for fuzzy *c*-means: counter-examples and repairs, IEEE Trans. Systems, Man Cybernet. SMC17 (1987) 873–877.

[6] [13] J.C. Bezdek, N.R. Pal, Cluster validation with generalized Dunn's indices, in: N. Kasabov, G. Coghill (Eds.), Proc. 1995 Second NZ Internat.

[7] J.C. Bezdek, N.R. Pal, Some new indices of cluster validity, IEEE Trans. Systems, Man and Cybernet. 28 (1998) 301–31.

[8] Chung.F ,Rhee.H ,Hwang.C, 2002, "An interval type-2 fuzzy perceptron",0-7803-7280-IEEE

[9] J.C. Dunn, A fuzzy relative of the ISODATA process and its use in detecting compact, well-separated clusters, J. Cybernet. 3 (1974) 32–57.

[10] Emami, M.R. ,Tu¨rksen,I.B., Goldenberg, A.A, 1999 ," A unified parameterized formulation of reasoning in fuzzy modeling and control" , Fuzzy Sets and Systems 108 , 59-81

[11] Y. Fukuyama, M. Sugeno, A new method of choosing the number of clusters for the fuzzy *c*-means method, in: Proc. Fifth Fuzzy Systems Symp., 1989, pp. 247–250.

[12] I. Gath, A.B. Geva, Unsupervised optimal fuzzy clustering, IEEE Trans. Pattern Anal. Mach. Intell. 11 (1989) 773–781.

[13] Gupta, P. , Mehlawat, M.K. , Anand Saxena, A. , 2007, "Asset Portfolio Optimization Using Fuzzy Mathematical Programming", Mathematical Programming, Information Sciences

[14] Hisdal E., "The IF THEN ELSE statement and interval-values fuzzy sets of higher type," Int. J. Man-Machine Studies, vol. 15, pp. 385–455, 1981.

[15] Huang.X , 2007," Mean-semi variance models for fuzzy portfolio selection", Journal of Computational and Applied Mathematics

[16] Huang.X , 2007," Risk curve and fuzzy portfolio selection", Computers and Mathematics with Applications

[17] John, R. I., "Fuzzy Sets of Type-2" J. of Advanced Computational Intelligence, 3(6), 499-508, 1999a.

[18] John, R. I., "Type 2 Fuzzy Sets" Expert Update, Vol. 2, No 2, Summer 1999, ISSN 1465-4091, 1999.

[19] Karnik.N.N , Mendel.J.M, 1999," Type-2 Fuzzy Logic Systems", IEEE TRANSACTIONS ON FUZZY SYSTEMS, VOL. 7, NO. 6, DECEMBER 1999

[20] Karnik, N. N. and J. M. Mendel, "Applications of Type-2 Fuzzy Logic Systems: Handling the Uncertainty Associated With Surveys," in Proc. FUZZ-IEEE'99, Seoul, Korea, August 1999.

[21] Karnik, N. N. and J. M. Mendel, "Centroid of a type-2 fuzzy set," Information Sciences, vol. 132, pp. 195-220, 2001.

[22] Karnik, N. N. and J. M. Mendel, “Introduction to Type-2 Fuzzy Logic Systems,” in Proc. 1998 IEEE FUZZ Conf., pp. 915-920, Anchorage, AK, May 1998a.
[23] Karnik, N. N. and J. M. Mendel, “Type-2 Fuzzy Logic Systems: Type-Reduction,” in Proc. IEEE Conference on Systems, Man and Cybernetics, pp. 2046-2051, San Diego CA, Oct., 1998.
[24] Karnik, N. N., J. M. Mendel and Q. Liang “Type-2 Fuzzy Logic Systems," IEEE Trans .on Fuzzy Systems, vol. 7, pp. 643-658, Dec. 1999.
[25] D.W. Kim, K.H. Lee, D. Lee, On cluster validity index for estimation of the optimal number of fuzzy clusters, Pattern Recognition 37 (2004), 2009–2025.
[26] Liang, Q. , Mendel, J.M.,2000,”Interval Type-2 Fuzzy Logic Systems: Theory and Design”, IEEE TRANSACTIONS ON FUZZY SYSTEMS, VOL. 8, NO. 5,
[27] Liu, N.K. and Lee, K.K, "An Intelligent Business Advisor System for Stock Investment", Expert Systems, vol.14, p.129-139, 1997.
[28] Mendel.J.M,2007, “Advances in type-2 fuzzy sets and systems”, Information Sciences 177,84–110
[29] Mendel, J. M. and H. Wu, “Properties of the Centroid of an Interval Type-2 Fuzzy Set, Including the Centroid of a Fuzzy Granule, Proc. IEEE FUZZ Conference, pp. 341-346, Reno, NV, May 2005.
[30] Mendel, J. M. and R. I. John, “A Fundamental Decomposition of Type-2 Fuzzy Sets,” Proceedings of Joint 9th IFSA World Congress and 20th NAFIPS Int’l. Conf., Vancouver, British Columbia, Canada, July 25-28, 2001.
[31] Mendel, J. M. and R. I. John, “Type-2 Fuzzy Sets Made Simple,” IEEE Trans. on Fuzzy Systems, vol. 10, pp. 117-127, April 2002.
[32] Mendel, J. M. R. I. John and F. Liu, “On Using Type-2 Fuzzy Set Mathematics to Derive Interval Type-2 Fuzzy Logic Systems,” Proc. North American Fuzzy Info. Processing Society (NAFIPS), pp. 528-533, Ann Arbor, MI, June, 2005.
[33] Mendel, J. M., “Computing With Words, When Words Can Mean Different Things to Different People,” in Proc. of Third International ICSC Symposium on Fuzzy Logic and Applications, Rochester Univ., Rochester, NY, June 1999.
[34] Mendel, J. M and John, R. I and Liu, F, Interval Type-2 Fuzzy Logic Systems Made Simple, IEEE Trans on Fuzzy Systems, pp. 1-14, 2006.
[35] N.R. Pal, J.C. Bezdek, On cluster validity for fuzzy *c*-means model, IEEE Trans. Fuzzy Systems 3 (3) (1995) 370–379.
[36] Vercher.E, Bermúdez .J.D, Segura.J.V, 2007, “Fuzzy portfolio optimization under downside risk measures”, Fuzzy Sets and Systems 158, 769 – 782
[37] X.L. Xie, G. Beni, A validity measure for fuzzy clustering, IEEE Trans. Pattern Anal. Mach. Intell. 13 (1991) 841–847.