Spectrum pool reassignment for wireless multi-hop relay systems

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Abstract— **We consider a wireless multi-hop relay system that operates on secondary sharing basis in licensed spectrum. The system employs OFDM-based spectrum pooling to avoid harmful interference to licensed systems. An OFDM-based spectrum pool comprises of a set of OFDM subchannels that correspond to white space spectrum regions. The proposed relay system comprises of a source node that transmits data to a destination node in multi-hops using multiple relay nodes. Communication over node pairs occurs on an available spectrum pool. At each receiving intermediate node, received data symbols are amplified, permuted and then forwarded on. Spectrum pool reassignment comprises determination of the permutation mapping at each intermediate node. We consider the problem of maximizing the system capacity by determining the optimum spectrum pool reassignment. We show that each relay can determine the optimum permutation mapping based on the *effective* signal to noise ratios in the previous hops and the signal to noise ratio in the next hop, while preserving system optimality.**

I. Introduction

It is widely recognized that radio spectrum under traditional spectrum management is utilized poorly over space and time, [5], [6], [21]. This is largely due to exclusive allocation of portions of spectrum to specific wireless systems and services. Given that such spectrum is licensed over large geographical regions and time spans, it is inaccessible to a different wireless system even if the licensed system is under-utilizing the spectrum. Solutions to improving spectrum utilization through secondary spectrum usage via cognitive radios and dynamic spectrum access have recently received attention [6], [10], [16], [19], [23]. In this paper, we consider a cognitive wireless multi-hop relay system that employs spectrum pooling for secondary spectrum sharing.

Wireless relay systems have been well-studied in literature with the implicit assumption that the communication system is a licensed system. Various relay system configurations have been analyzed for wireless flat fading channels in [2], [14], [15]. Dual-hop and multi-hop relay system configurations have been considered for frequency-selective fading channels in [4], [8], [9] and in the IEEE 802.16j Task Group standards development [13]. In contrast to these works, we consider a cognitive multi-hop relay system in this paper that operates in licensed spectrum on secondary sharing basis. The cognitive relay system comprises of a source node that uses a relay node for hopped communication to a destination node, with communication across nodes occurring on frequency regions that are not in current use by primary systems. Such a configuration is an effective solution for establishing communication between a source node and a desired destination node, where communication may otherwise not be possible due to various licensed systems operating in its communication range.

Under secondary spectrum sharing, the wireless relay system operates in licensed spectrum under the constraint that it limits the harmful interference caused to licensed systems. We consider orthogonal frequency division multiplexing (OFDM) based spectrum pooling to achieve this constraint. An OFDM-based spectrum pool comprises of a set of OFDM subchannels corresponding to white space spectral regions over which two cognitive radio nodes of the relay system can communicate without causing harmful interference to a licensed system. Since the distribution of licensed systems and their occupancy of spectral bands may differ geographically, different spectrum pools may be available between different node pairs in the multi-hop relay system.

In the multi-hop relay system under consideration, the source transmits data symbols over a certain set of subchannels in multi-hops over multiple relays. Each relay, in turn, amplifies the received frequency-domain data symbols, permutes them, and forwards them to the next node over an available set of subchannels. Finally, at the destination node the received data symbols are decoded. The permutation operation by each relay constitutes a reassignment of data from one set of subchannels to another permuted set of subchannels, and has an impact on the performance of the system.

The problem treated in this paper is the following. Given that the relays have knowledge of the signal to noise ratios (SNRs) over the subchannels in the spectrum pools, we want to determine the optimum spectrum pool reassignment that maximizes the capacity of the multi-hop relay system. We note that the problem of maximizing capacity under perfect knowledge of instantaneous SNRs for a 2-hop relay system was treated earlier in [20], and the closely related work [12]. In this paper, we generalize our results of [20], and show that a natural non-trivial generalization to multi-hop relay systems follows. In doing so, we obtain the capacity expression for an N-hop relay system that employs OFDM-based spectrum pooling. Applying results from majorization theory, we then

derive optimum permutation mappings that the relays have to choose in order to maximize system capacity. Our approach is based on characterizing the system capacity as a function of multi-dimensional vectors on which weak submajorization holds.

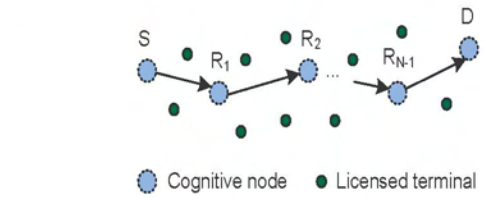
In Section II, we describe the multi-hop wireless relay system. The problem of spectrum pool reassignment for capacity maximization is formulated as an optimization problem in Section III. In Section IV, the optimum reassignment solution of the spectrum pools is presented. We show that the capacity as a function of the SNRs corresponding to the OFDM subchannels in the spectrum pools is a monotonic, super-modular function. For an objective function that belongs to this class, we then show that it is maximized when the subchannel SNRs in the spectrum pools are arranged with the same ordering. In Section V, we present simulation results to establish the effectiveness of the proposed spectrum pool reassignment method. Conclusions are drawn in Section VI.

Notations: For any $M \times 1$ vector $\mathbf{y} = [y_1, y_2, \ldots, y_M]^T$, we denote a permutation $\pi : \{1, 2, \ldots, M\} \rightarrow \{1, 2, \ldots, M\}$ on $\mathbf{y}$ by $\mathbf{y}^\pi$, given by $\mathbf{y}^\pi = [y_{\pi(1)}, y_{\pi(2)}, \ldots, y_{\pi(M)}]^T$. We denote by $\mathbf{y}^\downarrow = [y_{[1]}, y_{[2]}, \ldots, y_{[M]}]^T$ to be the resulting vector when the permutation $\pi = \downarrow$ orders the elements of vector $\mathbf{y}$ non-increasingly, i.e. $y_{[1]} \geq y_{[2]} \geq \ldots \geq y_{[M]}$.

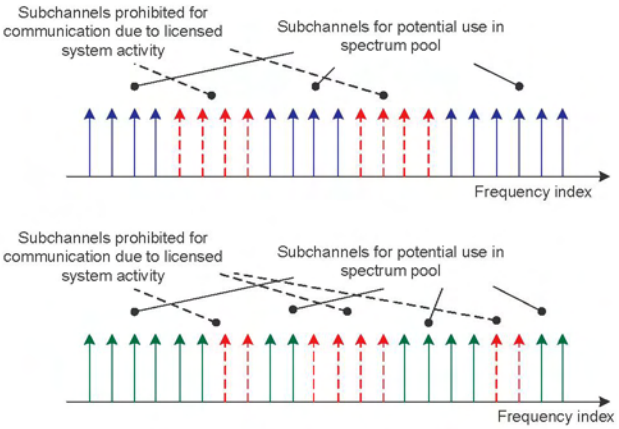
II. Multi-hop relay systems with spectrum pooling

The cognitive multi-hop relay system configuration under consideration is depicted in Figure 1(a). It comprises of a source node S that communicates data to a destination node D, via multi-hops using $N-1$ relay nodes $R_1, \ldots, R_{N-1}$. The system makes secondary spectrum use with communication between any pair of nodes occurring on white space spectrum portions. White space spectral portions correspond to frequencies where no licensed system is found to be operating. Given that the distribution of licensed user signal activity varies spatially, the white space spectrum portions available for operation between the various node pairs may be quite distinct from each other.

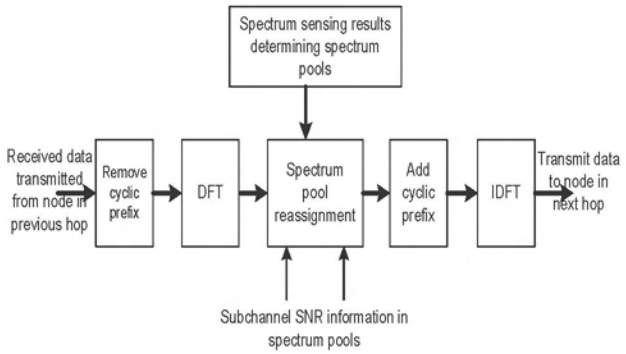
All nodes employ OFDM-based spectrum pooling as the underlying physical layer transmission technique. For a detailed treatment of OFDM-based spectrum pooling, refer to [22]. The wireless channel between any pair of nodes is modeled as a frequency-selective Rayleigh fading channel. The transmitter at a node applies an inverse discrete Fourier transform (IDFT) on the data to be transmitted and employs a cyclic prefix of length at least the impulse response length of the channel between the communicating nodes. The receiving node applies a DFT over the received data. It is well known [1] that an OFDM system over a frequency-selective fading channel using an IDFT-DFT pair and a cyclic prefix of length no less than the channel length is equivalent to a parallel set of OFDM subchannels that experience frequency-flat fading. We shall henceforth assume that the transmission between all node pairs occurs on parallel OFDM subchannels.



(a) Cognitive multi-hop relay system



(b) Spectrum pools in two different hops



(c) Schematic of amplify-permute-forward scheme for spectrum pool reassignment at relay node

Fig. 1. An example schematic of the multi-hop relay system with OFDM-based spectrum pooling

In an OFDM-based spectrum pooling system, subchannels corresponding to the frequency regions in use by licensed systems are prohibited for secondary use. This is shown in Figure 1(b) where the dashed arrows indicate subchannel indices that are prohibited for use by the cognitive multi-hop relay system in order to ensure protection from harmful interference to the licensed systems. Licensed system activity may be determined using sophisticated detection algorithms [3], or by access to a spectrum usage database, or by licensed system assisted beaconing mechanisms [6]. Only the OFDM subchannels that correspond to white space spectral portions are pooled and utilized by the multi-hop relay system. This is shown in Figure

1(b) by the solid arrows indicating indices of the subchannels that are pooled together. Since there may be different licensed systems with different frequency occupancies operating in the detection range of the nodes, the spectrum pool corresponding to different node pairs may be different from each other.

A. Amplify-permute-forward operation at relay nodes

The source node S transmits data symbols to relay node R_1 on a certain pool of available OFDM subchannels of $S-R_1$ in the first time slot. In the next time slot, the relay R_1 amplifies the received data symbols, permutes them and then forwards them to the next relay node R_2 on a possibly different pool of OFDM subchannels of $R_1 - R_2$. Amplification is performed such that the power per subchannel is unchanged. Relaying is continued till relay R_{N-1} forwards the data to the destination node D, where it is finally decoded. The encoding at S, and the decoding at D, are performed independently on a per-subchannel basis.

B. Capacity of a multi-hop relay system with spectrum pooling

We assume that there are M subchannels in each of the spectrum pools corresponding to the node pairs. Denote $\mathbf{x} = [x_1, x_2, \ldots, x_M]^T$ to be the vector of data symbols transmitted from source S, with x_i being the symbol transmitted over the $\pi_0(i)$-th subchannel of spectrum pool $S - R_1$. We assume the symbols to be i.i.d. complex Gaussian with zero mean and unit variance. Without loss of generality, we assume that the source transmits the data symbols on OFDM subchannels that are indexed such that their subchannel SNRs are non-increasingly ordered. This choice of permutation mapping will be denoted $\pi_0 = \downarrow$, and the resulting subchannels indexed as $\{[1], [2], \ldots, [M]\}$. Let π_k denote the permutation applied by relay node R_k, $k = 1, \ldots, N-1$, subsequently. At each receiving node, the noise is modeled to be additive white Gaussian with variance σ_k^2. We assume that the noise at different receiver nodes are uncorrelated.

For a 2-hop relay system with data symbol x_i transmitted on subchannels $\pi_0(i)$ and $\pi_1(i)$, with respective subchannel SNRs $\rho_{\pi_0(i)}$ and $\rho_{\pi_1(i)}$, the effective subchannel SNR is given by [20]

$$SNR_{2,i} = \frac{\rho_{\pi_1(i)}\rho_{\pi_0(i)}}{1 + \rho_{\pi_1(i)} + \rho_{\pi_0(i)}}. \tag{1}$$

Here, subchannel SNR is defined as the SNR on a particular subchannel in a given single hop, while effective subchannel SNR is the resultant SNR that is obtained when data is transmitted over different subchannels in multiple hops.

The result (1) can be generalized to an N-hop system by establishing the following recursion. Let $SNR_{N-1,i}$ be the effective SNR for an $(N-1)$-hop system corresponding to the transmitted data symbol x_i, which is forwarded in the N-th hop on subchannel $\pi_{N-1}(i)$, whose SNR is given by $\rho_{\pi_{N-1}(i)}$. Then the effective SNR corresponding to this set of subchannels of the N-hop relay system is given by

$$SNR_{N,i} = \frac{\rho_{\pi_{N-1}(i)} SNR_{N-1,i}}{1 + \rho_{\pi_{N-1}(i)} + SNR_{N-1,i}}. \tag{2}$$

Let $\rho_{\pi_{k-1}(i)}$ be the SNR on subchannel $\pi_{k-1}(i)$ in the k-th hop. Denote $\boldsymbol{\pi} = [\pi_0, \pi_1, \ldots, \pi_{N-1}]^T$ to be the permutation mapping vector. Let $\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}} = [\rho_{\pi_0(i)}, \rho_{\pi_1(i)}, \ldots, \rho_{\pi_{N-1}(i)}]^T$ be the vector of SNRs corresponding to subchannels over which data symbol x_i is transmitted over N hops. Using the tool of mathematical induction, the following result follows.

Lemma 2.1: For an N-hop relay system,

$$SNR_{N,i} = \frac{S_{N,i}(\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}})}{\sum_{k=0}^{N-1} S_{k,i}(\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}})}, \tag{3}$$

where

$$\begin{aligned}
S_{0,i}(\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}}) &= 1, \\
S_{1,i}(\rho_{N,i}^{\boldsymbol{\pi}}) &= \sum_{k=0}^{N-1} \rho_{\pi_k(i)}, \\
S_{2,i}(\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}}) &= \sum_{k<l}^{N-1} \rho_{\pi_k(i)}\rho_{\pi_l(i)}, \\
S_{3,i}(\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}}) &= \sum_{k<l<m}^{N-1} \rho_{\pi_k(i)}\rho_{\pi_l(i)}\rho_{\pi_m(i)}, \\
&\vdots \\
S_{N,i}(\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}}) &= \prod_{k=0}^{N-1} \rho_{\pi_k(i)}.
\end{aligned}$$

Note that the effective SNR depends on the subchannel SNRs in the individual hops, as well as the permutation mappings $\{\pi_k\}$ employed at the relays.

The effective capacity of the N hop relay system is simply the sum of the capacities over the OFDM subchannels and can be written as

$$C(\{\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}}\}) = \sum_{i=1}^{M} \log(1 + SNR_{N,i}), \tag{4}$$

where $SNR_{N,i}$ is given in (3).

III. Problem description

Each of the relay nodes performs spectrum pool reassignment by reassigning data on spectrum pools based on channel information in the form of subchannel SNRs. This has an impact on the system performance. We shall consider the instantaneous capacity as our system performance metric. Our goal is to determine the optimum permutations that maximize the capacity given in (4). That is, we want to determine the optimum permutation mapping vector $\boldsymbol{\pi}^\star$ that maximizes $C(\{\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}}\})$, given knowledge of the subchannel SNRs. Stated formally,

$$C(\{\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}^*}\}) = \max_{\boldsymbol{\pi}} \sum_{i=1}^{M} \log\left(1 + \frac{S_{N,i}(\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}})}{\sum_{k=0}^{N-1} S_{k,i}(\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}})}\right). \tag{5}$$

IV. Optimum spectrum pool reassignment

Before addressing the problem of choosing the optimum permutation mappings, we present some results that help us to characterize the objective function $C(\{\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}}\})$.

We begin by introducing a few definitions from majorization theory [7], [17] relevant to the development of our results.

Definition 4.1 (Weak submajorization): Given $\mathbf{x}, \mathbf{y} \in \mathcal{R}^M$, we say that $\mathbf{x}$ is said to be weakly submajorized by $\mathbf{y}$, written as

$$\mathbf{x} \prec_w \mathbf{y} \text{ if } \sum_{i=1}^{k} x_{[i]} \leq \sum_{i=1}^{k} y_{[i]}, k = 1, \ldots, M.$$

In particular, $\mathbf{x} \prec_w \mathbf{y}$ implies that

$$\sum_{i=1}^{M} x_i \leq \sum_{i=1}^{M} y_i. \tag{6}$$

Definition 4.2 (Super-modular function): A function ϕ on a real vector $\mathbf{z} \in \mathcal{R}^N$, with its second order partial derivatives existing, is super-modular if and only if its cross-derivatives are non-negative, i.e.

$$\frac{\partial^2 \phi}{\partial z_i \partial z_j} \geq 0, \text{ for } i \neq j.$$

That is, a function is super-modular if all the off-diagonal elements of its Hessian are non-negative.

Let $\alpha_{i,j}$, $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, M$, be NM non-negative real elements. Let ϕ be a real function of N real variables. Consider the $M \times 1$ vectors

$$\begin{aligned}\mathbf{u} \quad = \quad & [\phi(\alpha_{1,1}, \alpha_{2,1}, \ldots, \alpha_{N,1}), \phi(\alpha_{1,2}, \alpha_{2,2}, \ldots, \alpha_{N,2}), \\ & \ldots, \phi(\alpha_{1,M}, \alpha_{2,M}, \ldots, \alpha_{N,M})]^T,\end{aligned}$$

and

$$\begin{aligned}\mathbf{v} \quad = \quad & [\phi(\alpha_{1,[1]}, \alpha_{2,[1]}, \ldots, \alpha_{N,[1]}), \phi(\alpha_{1,[2]}, \alpha_{2,[2]}, \ldots, \alpha_{N,[2]}), \\ & \ldots, \phi(\alpha_{1,[M]}, \alpha_{2,[M]}, \ldots, \alpha_{N,[M]})]^T.\end{aligned}$$

Note that

$$\alpha_{i,[1]} \geq \alpha_{i,[2]} \geq \ldots \geq \alpha_{i,[M]}$$

for each $i = 1, 2, \ldots, N$.

The following result then holds [17, Chapter 6].

Lemma 4.1: $\mathbf{u} \prec_w \mathbf{v}$, for all $N \geq 2$ and non-negative real $\alpha_{i,j}$, if and only if the function ϕ is monotone and super-modular.

We now establish the connection between Lemma 4.1 and the problem at hand via the following result that characterizes our optimization function to a special class of functions.

Lemma 4.2: The function

$$\phi(\{\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}}\}) = \log\left(1 + \frac{S_{N,i}(\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}})}{\sum_{k=0}^{N-1} S_{k,i}(\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}})}\right), \tag{7}$$

with $\rho_{\pi_k(i)} \geq 0$ for all k, i, and $\boldsymbol{\pi}$, is monotonic and super-modular.

The proofs are omitted for brevity.

Let $\rho_{i,j}$ be the subchannel SNR on the $(i+1)$-th hop on subchannel j, and $\rho_{i,[j]}$ be the subchannel SNR such that

$$\rho_{i,[1]} \geq \rho_{i,[2]} \geq \ldots \geq \rho_{i,[M]}.$$

By virtue of Lemmas 4.2 and 4.1, it then follows that

$$\begin{aligned}& [\phi(\rho_{1,1}, \rho_{2,1}, \ldots, \rho_{N,1}), \phi(\rho_{1,2}, \rho_{2,2}, \ldots, \rho_{N,2}), \\ & \qquad \ldots, \phi(\rho_{1,M}, \rho_{2,M}, \ldots, \rho_{N,M})]^T \\ \prec_w \quad & [\phi(\rho_{1,[1]}, \rho_{2,[1]}, \ldots, \rho_{N,[1]}), \phi(\rho_{1,[2]}, \rho_{2,[2]}, \ldots, \rho_{N,[2]}), \\ & \qquad \ldots, \phi(\rho_{1,[M]}, \rho_{2,[M]}, \ldots, \rho_{N,[M]})]^T.\end{aligned}$$

Using (6), we then have

$$\begin{aligned}& \sum_{j=1}^{M} \log\left(1 + \phi(\rho_{1,j}, \rho_{2,j}, \ldots, \rho_{N,j})\right) \\ & \leq \sum_{j=1}^{M} \log\left(1 + \phi(\rho_{1,[j]}, \rho_{2,[j]}, \ldots, \rho_{N,[j]})\right).\end{aligned}$$

We can now specify the optimum choice of spectrum pool reassignments.

Theorem 4.1: The optimum selection of permutation mapping vector $\boldsymbol{\pi}^\star$ that maximizes the capacity $C(\{\boldsymbol{\rho}_{N,i}^{\boldsymbol{\pi}}\})$ in (5) is given by $\pi_k = \downarrow$, $k = 0, 1, \ldots, N-1$.

That is, the system capacity of the multi-hop relay system is maximized when the subchannel SNR vectors in all hops are similarly ordered. If the subchannel SNR vector in the first hop is non-increasingly ordered, then all other subchannel vectors should also be non-increasingly ordered. Thus, given the spectrum pool information and the subchannel SNRs within the spectrum pools, each relay makes a choice of permutation mapping so that the subchannel SNRs in all pools are arranged in the same order. Further, due to the recursion (2), it follows that it is sufficient for any intermediate relay to only know the effective subchannel SNRs resulting from the previous hops in order to determine how the subchannel SNRs in the next hop should be ordered. This obviates the need for each relay to have SNR information pertaining to all the subchannels in every hop of the relay system.

V. Simulation results

We considered a 3-hop relay system, in a simple line topology with adjacent nodes separated by a fixed distance d. We modeled the wireless channel between all node pairs as a multipath channel, with a finite impulse response of length 16. The channel coefficients were assumed to be i.i.d. and randomly generated according to a complex Gaussian distribution with zero mean. The SNR of a single hop signal then follows an exponential distribution with average SNR $\gamma_{ave} = d^{-\beta}$, where $\beta = 2$ is the path loss exponent. We chose $d = 1$ in our simulations. We assumed a total of 1024 OFDM subchannels, of which only 256 (i.e., $M = 256$) are used for transmission in each of the spectrum pools.

In order to gauge the performance of the proposed reassignment scheme employed by the relay, we compared the achieved capacity under the optimum spectrum pool reassignment described in Section IV with a random reassignment scheme. In the random assignment scheme, 256 subchannels are chosen and randomly assigned from the spectrum pools by the relay. We also computed the system capacity if a direct transmission existed from source to destination, with 256

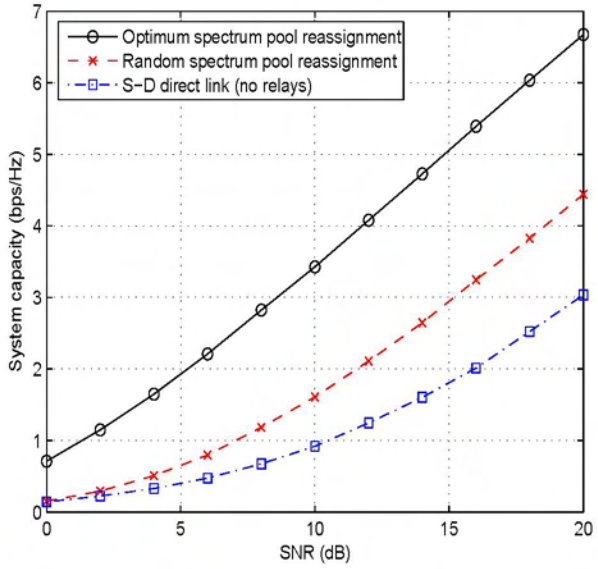


Fig. 2. Capacity gains with optimum spectrum pool reassignment

subchannels. The channel between the source and destination nodes is modeled similarly as described earlier, with a path loss exponent of 2 and distance between source and destination nodes to be $3d$.

The performance of these three schemes were compared in terms of system capacity, averaged over 1000 channel realizations, as the SNR was varied from 0 to 20 dB. The performance gains are evident from Figure 2 - for instance, to achieve a capacity of 3 bps/Hz, the multi-hop relay system requires an SNR of approximately 7 dB lesser with optimum reassignment of subchannels in the spectrum pools as compared with random subchannel reassignment. This gain is around 12 dB when the multi-hop relay system is compared with a direct source to destination transmission system.

VI. Conclusions

We considered the problem of optimum reassignment of OFDM subchannels that maximizes the capacity of a multi-hop relay system that employs OFDM-based spectrum pooling. We showed that the system capacity is maximized when each of the relays applies a permutation mapping such that the subchannel SNRs in all the spectrum pools are similarly ordered. The solution follows a natural generalization of the results in [20], with the interesting feature that each intermediate relay can base its choice of permutation mapping of the subchannels in the next hop on the knowledge of only the effective subchannel SNRs resulting in the previous hops.

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