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THE OPPORTUNITY COST OF DEFENSE INPUTS

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ABSTRACT. This article analyzes the difference between the budgetary expense and the opportunity cost of defense inputs. If inputs are obtained by the government from a market economy with undistorted prices, the price paid for the last unit of each input acquired equals the opportunity cost. However, taxes create a distortion between opportunity cost and unit price. An additional complication, discussed using the case of military personnel, is that pre-marginal units may have an opportunity cost lower than the unit price determined at the margin. Principles used to determine the social discount rate are also discussed in the analysis.

INTRODUCTION

Although "the cost of defense inputs" acquired from the civilian sector has been used interchangeably with "the budgetary expense for defense inputs," the two quantities are only equal under special circumstances. Cost has a precise meaning in public sector analysis, and the proper evaluation of defense alternatives requires a thorough understanding of its principles.

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As will be discussed, the cost of the defense inputs obtained from the civilian sector equals the value of the alternatives foregone when the inputs are acquired. If an input to defense is purchased from a competitive economy at an unchanging equilibrium price in which the demand price for an additional unit of the input equals the marginal opportunity cost, then the price paid by the government equals the amount that individuals in the economy are willing pay for the alternative uses of the resources which produce that input. As willingness to pay defines value, the defense sector would, in such a situation, pay an amount equal to the value foregone, that is, the opportunity cost of the input.

There are two factors, however, that would create a gap between the budgetary expense and the opportunity cost of an input. First of all, the market price of the input may not equal the supply price of the pre-marginal units. Second, if an input acquired by the government is taxed, a gap would be created between the price actually paid for a marginal unit of the input and the opportunity cost of that unit.

An important example of the pre-marginal supply price issue occurs with military personnel. The pre-marginal volunteers, whose supply price may be lower than the wage paid to the last volunteer, earn an economic rent. This economic rent--wage minus supply price for each pre-marginal individual--is not a cost required to transfer this input into the defense sector. It is an income transfer from the taxpayers to the pre-marginal volunteers over and above their opportunity cost. This income transfer does not, by itself, bring forth any additional personnel supply. As a result, it is sometimes said that rent is a transfer and not a cost. It is an income redistribution from taxpayers to military personnel.

The effect of taxes on the computation of opportunity cost is probably best illustrated for the situation when the government goes to the capital market to borrow funds for defense purchases. The annual budgetary payment for these funds is the market interest rate. However, if the income from investment and saving is taxed, the private investment projects foregone would receive a higher gross return before taxes than the market rate. Also, the extra saving induced would receive a lower net return after taxes than the market rate. To properly account for the opportunity cost of the funds, an adjustment must be made to the market interest rate.

Interestingly, the opportunity cost of inputs purchased by the government has been analyzed by Arnold Harberger who has derived operational formulae for implementing the opportunity cost methodology (Harberger, 1974). However, recent analysis suggests that there has not yet been full dissemination of these principles. For example, recent RAND analysis proposes a measure of the cost of defense activities that is incomplete (Neu, 1990).[(1)] An important purpose of this study, therefore, is to describe how the opportunity cost methodology applies to inputs acquired by the defense sector.

DEFINITION OF OPPORTUNITY COST

We first discuss the concept of opportunity cost as it relates to the "cost" of the inputs used to produce defense output. These inputs, which include the fuel used to operate military equipment, the cement used to produce missile silos, and the military personnel who volunteer, have alternative uses in the non-defense sector of the economy. In order to properly measure the cost of these defense inputs, a precise understanding of opportunity cost is needed.

The first clear statement of the "opportunity cost doctrine may have been made by Frederick von Viser. His Law of Cost has been stated: "What a thing really costs us is the sacrifice of the utility of those other things which we could have had from the resources that went into the one that we did produce" (Rukeyser, 1988).[(2)]

There are several issues to note from this definition. First of all, cost is not measured in terms of the either the quantity or market value of the resources employed, but rather in terms of the value of the "other things" sacrificed by producing the relevant amount of the good in question.

Second, the "other things we could have had" must refer to the best available alternative if the resources had not been employed in their current use. Cost should be measured using the thing that would be acquired if we did not select the good in question. Any alternative other than the next best would not reflect actual decision making and would underestimate the cost. Also, the issue is not what the resources would actually be producing if they are not in present employment, but rather what would be produced in the economy if these resources were not actually employed in their current use.

Finally, rather than to speak of the sacrifice of utility, the modern approach is to speak in terms of the money equivalent of the sacrifice. This money equivalent is the amount one is willing to pay to avoid the loss of the best alternative, that is, the amount one is willing to pay for this alternative when the relevant thing in question has been produced. As a result, Viser's Law of Cost might be restated as follows:

> What a thing really costs us is the amount we are willing to pay for the best alternative things we could have if the resources had not been employed in the production of the one that we did produce.

For this definition to be useful in economic analysis, it is necessary to relate it a social choice criterion. Employing this definition of cost supports the social objective of placing resources in the hands of those who value them the most in terms of willingness to pay. Thus, if the cost of a proposed defense project is evaluated using our definition of opportunity cost, the project will only be accepted if the amount that the government is willing to pay for the project is at least as great as the amount that individuals are willing to pay for the alternative things foregone. This criterion, therefore, insures that resources are directed to those who give them highest value.

A complication can arise in the implementation of this criterion if the amount one is willing to pay for an alternative depends on the wealth of the relevant individual. This creates a gap between the maximum one is willing to pay for a change and the money equivalent of *that* change. These wealth effects are likely to be small if expenditures on the good are only a small percent of total income. In our opportunity cost exposition, therefore, we ignore wealth effects.

COST OF DEFENSE INPUTS

We develop a method of introducing cost in economic analysis by supposing that the government purchases some quantity of a good, say fuel, which is competitively produced in the private sector and for which there is also private consumption. We assume that for the categories of petroleum-based fuel purchased--gasoline for government autos and trucks, diesel fuel for ground combat vehicles, and JP4 for aircraft--the price per BTU is approximately the same. This permits us to evaluate the government purchases using the demand and supply for petroleum-based fuel. The purchases of this good by the government are sufficiently large that the price of fuel rises as a result of the purchases. In the most general case, some consumption of fuel is displaced, and additional production occurs. For simplicity, we initially assume that there are no distortions resulting from taxes on fuel production. There are also no changes in the price of other goods as a result of the government's purchases. In the Appendix, we provide a formal treatment of the basic result developed in this section.

When analyzing the opportunity cost of a non-marginal activity, it is necessary to select a reference point (or status quo ante) from which the values foregone are measured. We take the civilian demand for petroleum-based fuel as given and analyze the alternatives foregone given this demand.

When civilian fuel consumption is reduced, resources that were being used to produce this displaced demand are now used to satisfy government demand. When producer supply increases, additional resources flow into the fuel production. The incremental cost of the resources used to increase the supply of fuel must equal the amount that individuals are willing to pay for the other goods no longer being produced.

Therefore, the cost to the private sector of the government purchases equals the amount individuals are willing to pay for the fuel consumption displaced by the government plus the amount individuals are willing to pay for the quantity of other goods that would be produced if the resources had not flowed into fuel production.

We start with the extreme situations in which the competitive supply curve is alternatively perfectly inelastic and perfectly elastic. This enables us to focus separately on the displacement of fuel consumption and expansion of fuel production. Then we address the more likely case in which the supply curve has the traditional upward slope.

Figure 1 depicts the situation in which supply is perfectly inelastic. The demand curve D_C is the civilian demand curve for fuel; the curve D_T is the demand curve including government demand D_M. Note that the effect of government purchases of indicated quantity Q_M is to increase price from P_0 to P_1. The shaded area with width equal to the quantity of displaced civilian fuel consumption equals the value of other fuel uses foregone as a result of the government purchases. This is because each point on a demand curve equals the maximum civilian consumers are willing to pay for a small unit change in fuel; the total shaded area, therefore, represents the total willingness to pay for the civilian fuel consumption foregone as a result of the government purchases. From this analysis, therefore, we see that, on the demand side, the opportunity cost of fuel equals the value of the alternative uses foregone.

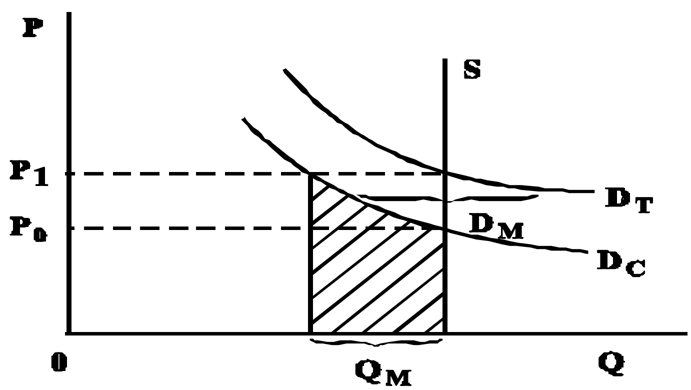

FIGURE 1
Opportunity Cost with Perfectly Inelastic Supply

At the other extreme, as depicted in Figure 2, is the case in which the supply curve for fuel is perfectly elastic. This could occur if there are no factors of production specialized to this industry, or when the industry remains too small a part of the economy to affect the prices of the inputs used when it expands.

Now when the government's demand is added to the other demand for fuel, additional fuel production of quantity Q_M takes place at constant price, P. There are, therefore, no alternative uses of fuel foregone as a result of the government's purchases.

There is, however, an opportunity cost associated with the additional production. Each point on the supply curve equals the supply price of additional production by the petroleum-based fuel industry. This supply price is the minimum price that must be received for members of the industry to produce the additional fuel output. The supply price, therefore, equals the marginal cost of new fuel production, and the identified shaded area equals the cost of the additional production. But this cost, which is just the amount that must be paid to transfer the resources into fuel production, is precisely equal to the amount that others are willing to pay for the resources. This

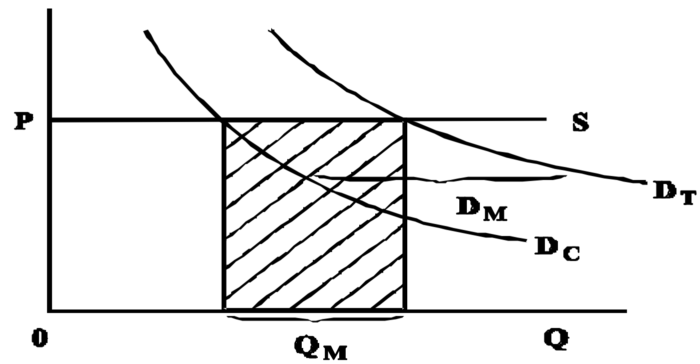

FIGURE 2
Opportunity Cost with Infinitely Elastic Supply

amount is the "value" of the alternative goods foregone. The indicated shaded area, therefore, equals the amount "we (the members of society) are willing to pay for the best alternative things we could have had not been employed in the production of the one that we did produce."

In Figure 3, we display the traditional situation in which there is upward sloping industry supply, S_T. This would occur because the

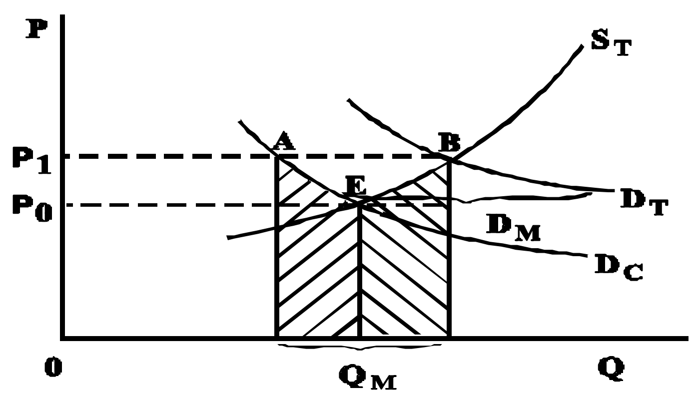

FIGURE 3
Opportunity Cost with Upward Sloping Supply

increase in fuel output would require the withdrawal of oil at increasing marginal cost, and also possibly because an expansion of refinery capability might entail an increase in the price of the inputs used to refine petroleum. Now, when the government adds its demand, D_M, to the original demand, and bids the price of fuel from P_0 to P_1, the opportunity cost equals the value of other fuel uses foregone from displaced civilian consumption plus the value of the other goods foregone as a result of the additional fuel production. The two shaded areas of the figure equal the opportunity cost of the government purchases.

In Figure 4, we extract information from Figure 3 in order to focus on the situation facing the government. The government's demand curve, D_M, and the supply curve facing the government, S_M, are displayed. The supply curve facing the government equals the difference between the total industry supply curve, S_T, of Figure 3 and the civilian demand for fuel, D_C. In effect, the government faces an excess supply curve

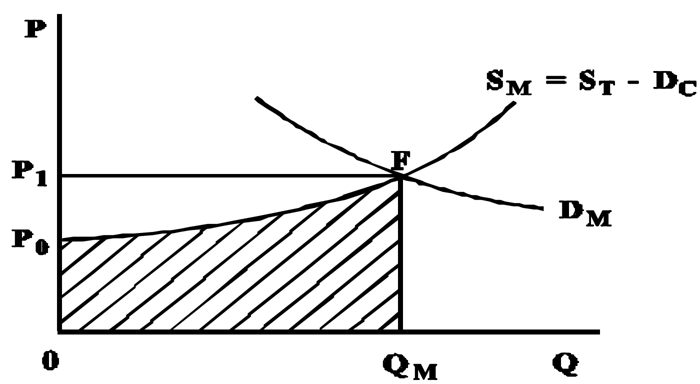

FIGURE 4
Opportunity Cost Computed using Supply facing Military

The reason why the government faces this excess supply curve is that at any price, the supply available to it is that part of total supply not demanded by the civilian sector. At the original price P_0 determined by the equilibrium balance between the civilian demand and total supply, the total supply is purchased by civilian consumers. At P_1, the government faces an excess supply of $S_M = S_T - D_C$. At this equilibrium price, S_M must equal D_M (because $D_C + D_M = S_T$). Figure 4, therefore, is simply another way of representing the determination of equilibrium.

The shaded area under the supply curve facing the government in Figure 4 must equal the two shaded areas of Figure 3. To see this consider the effect of a small (unit) increase in government demand that results in an imperceptible change in the equilibrium price that everyone pays. This increase in demand must be met with a combination of a decrease in civilian demand and increase in industry supply. The opportunity cost of the incremental demand equals the willingness of civilian consumers to pay for the fuel foregone plus the incremental cost of the additional production--a total just equal to the supply price facing the military. If we repeat this calculation for successive increments of government demand, with the price of each new increment rising slightly, the sum of the calculated opportunity costs must equal the computed area under the supply curve facing the government.

The government's total budgetary expense, P_1Q_M, clearly exceeds the opportunity cost of the fuel acquired. The difference between this expense and opportunity cost (areas P_1FP_0 of Figure 4 and ABE of Figure 3) is part of the producer's surplus generated by the government purchases. It is a transfer payment from taxpayers to the producers.

It is interesting that although the identified surplus is a transfer payment, it is actually classified in the GNP accounts as part of government purchases. One could convincingly argue that it should be included as a transfer payment in the government outlays accounts.

Another important implication of this analysis derives from the observation that if purchases by the government from a competitive industry do not affect the price of the input, then this price equals opportunity cost. It equals the value of civilian fuel uses foregone plus the value of other goods foregone when the government buys one more unit of the input.

We can now contrast this basic result with a situation in which a constant tax of t per unit is imposed on fuel production and the fuel producers make the tax payment to the government. Each supplier receives a net price, P_N, equal to the gross price, P_G, less the tax paid: $P_N = P_G - t$. At the margin, the opportunity cost of production equals P_N. The net price is the marginal cost of the resources employed and equals the value of other goods foregone. But with the tax, producers must receive a market price of P_G in order to obtain a net price of P_N, thereby covering opportunity costs on an additional unit of fuel production.

We graph this situation in Figure 5. The civilian and total demand curves are as before, but we now represent both the gross and net supply curves--the difference between the two being the tax per unit. The overnment's purchases of Q_M units of fuel now raise the gross (market) price

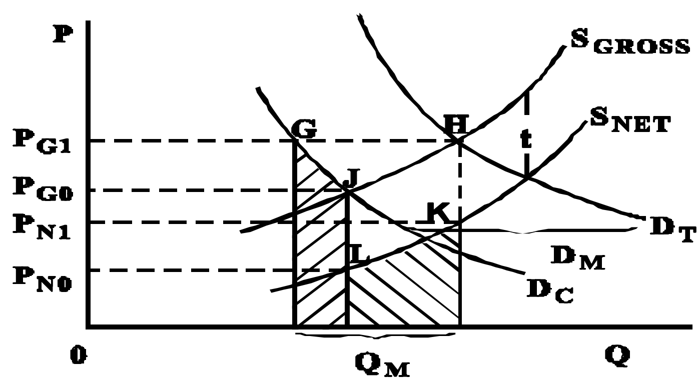

FIGURE 5
Opportunity Cost with Per Unit Tax on Input

from P_{G0} to P_{G1} and the net price from P_{N0} to P_{N1}. As the government pays the market price, the total budgetary expense equals $P_{G1}Q_M$. The shaded area under the demand curve equals the value of civilian fuel production foregone while the shaded area under the net supply curve equals the value of other goods foregone. The government's budgetary expense now exceeds the opportunity cost because of two distinct transfer payments.

One transfer payment, area GHJ, is part of the increased surplus of the fuel producers. This payment is not needed to bring forth additional supply and does not represent any opportunities foregone. As before, it is a transfer from the taxpayers to the producers. The second transfer payment, area JHKL, equals the additional taxes paid by the fuel producers as a result of the government purchases. This part of the governments budgetary expense is returned as a tax payment, and therefore, is self canceling.

In many situations it is reasonable to assume that the government does not perceptibly affect either the gross or net price. If this is the case, there is a simple form of the opportunity cost price paid by the government. This shadow price, P_S, is simply a weighted average of the gross and net price, where the weights, W_G and W_N, are the proportions of the total acquisition quantities derived, respectively, from reduced civilian consumption and increased fuel production. We have

$$P_S = W_G P_G + W_N P_N. \tag{1}$$

The justification for this result is made clear by considering an example in which the government purchases one hundred million gallons of fuel, which are taxed at \$.50 per gallon. Suppose as a result of these purchases, there are forty million gallons of reduced civilian consumption and sixty million gallons of increased production. Also, we assume the gross price is \$1.50 per gallon and the net price is \$1.00 per gallon in the equilibrium that includes the government purchases.

The government pays the gross price, so the budgetary expense in this situation equals \$150 million. The shadow price of fuel, however, is \$1.20 = .4(\$1.50) + .6(\$1.00), and the opportunity cost of the government purchases equals \$120 million. The \$30 million dollar difference is the additional tax payment on the sixty million gallons of increased industry production.

The taxes paid on the forty million gallons of supplanted civilian production, \$20 million, should not be deducted when calculating opportunity cost. There is no change in this total when the government enters the fuel market. The tax collected on displaced consumption is lost, but replaced with the tax on government purchases. As we have indicated, the full price paid for the displaced civilian demand reflects opportunity cost.

OPPORTUNITY COST OF MILITARY MANPOWER

Another important application of this framework is in the area of military manpower. It is important to understand how much of the government's wage bill is an economic rent, and how much is not part of opportunity cost because of tax considerations. A complication in the analysis of personnel is that military personnel may receive a compensating differential arising from differences in job amenities between civilian and military employment.

We first analyze military manpower under the assumption that there is no difference in the supply price of an individual to military and civilian employment. Initially, therefore, there is no compensating differential required to equate the two types of employment. If there are no taxes on the demand or supply side, then the situation is logically equivalent to that depicted in Figures 3 and 4. We must now simply interpret the price axis as measuring the wage rate paid to military and civilian labor and the quantity axis as measuring the number of individuals in civilian and military employment.

Under this interpretation, Figure 3 depicts both the civilian and total demand for labor market from which military personnel are drawn. The difference between the two curves is the military's demand for personnel. We assume that leisure is the foregone alternative when labor is supplied to this labor market. In a manner similar to our discussion above, Figure 4 depicts the demand for military personnel and the supply of personnel facing the military.

Note that, as before, when the military adds its demand for labor to that of the civilian sector and the market wage is bid upward, there are civilian goods foregone. As before, the shaded vertical slab under the civilian demand curve equals the value of the foregone civilian goods. Also, as the military bids up the wage, there is an increase in the quantity of labor supplied from leisure. The shaded vertical slab under the total supply curve of Figure 3 equals the amount the additional workers are willing to pay for the leisure that is foregone--the opportunity cost of the additional labor provided.

As in the earlier discussion, the sum of these two areas is equal to the shaded area under the supply curve for miliary personnel in Figure 4. Also note that even though the DoD budgetary expense for labor equals the equilibrium wage times the labor hired by the military, the opportunity cost of military personnel only equals the area under the supply curve facing the military. The difference between the two areas, the triangle P_1FP_0 of Figure 4 equals an economic rent received by the military personnel. As such this area is not an transfer cost required bring labor into military employment; it is a transfer payment from the taxpayers to the military personnel.

If we now apply an income tax to labor, a gap is created between the gross and net supply curves. An individual's gross supply price to employment includes the income tax paid; the net supply price equals the supply price after taxes. This is the amount required to coax out labor from leisure, and therefore, is the opportunity cost. The only difference between this situation and the one depicted in Figure 5 is that an income tax is typically an *ad valorem* tax rather than a per unit excise tax, and the gross and net supply curves would no longer be vertically parallel.

For simplicity we assume that the demanders of labor do not pay a tax. The demand price of these users, therefore, equals, in market equilibrium, the gross supply price of labor. When we add the military to civilian demand in Figure 5, the value of civilian goods foregone is shown to be the shaded area under the demand curve; the value of the leisure foregone equals the shaded area under the net supply curve. As a result, if the additional labor comes from civilian employment the opportunity cost of each worker displaced equals the relevant demand price; if labor comes from leisure the opportunity cost of each additional worker equals the relevant supply price.

Of course, it is unreasonable to assume that an individual's supply price for military service is the same as for civilian employment. Unlike fuel, cement, or other material inputs to defense, different employments yield different amenities for individuals. As a result, a compensating differential is required to equalize the level of well-being obtained from military and civilian employment.

To take account of the compensative differential between civilian and military work, we focus on the supply curve facing the military and the demand for military personnel. Figure 6 depicts the supply curve, S_M, facing the military. The demand curve, D_M, for military personnel and the resulting accessions, L_M, and military wage, W_M, are also indicated.

Each point on the supply curve represents the supply price of a particular individual and includes the relevant compensating differential. The supply curve facing the military arrays individuals in the labor pool from low to high supply price. Also indicated on the diagram is a dashed line C_M. Points on this line represent the civilian wage forgone for each individual, so we are now assuming that all personnel enter the military from civilian employment. For simplicity, we have assumed that those with the lowest supply price receive a positive compensating

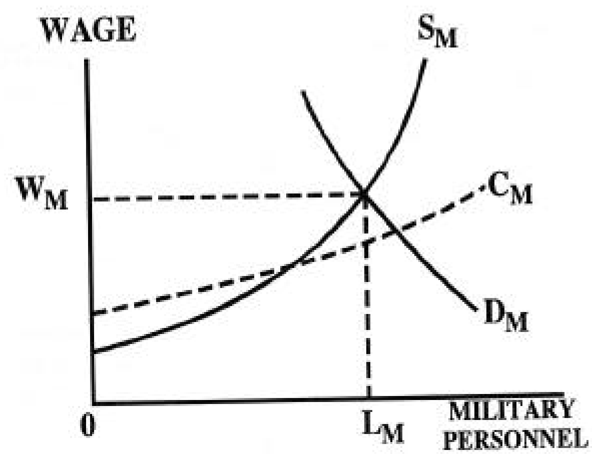

FIGURE 6
Military Personnel Demand and Supply and Civilian Wage Foregone

differential when entering the military, those with the highest supply price receive a negative compensating differential. Also, the size of the compensating differential increases with the military supply price.

We can think of the compensating differential as that component of the supply price of an individual that must be paid to compensate for the change in job amenities between civilian and military status. As a result, job amenities can be thought of as a distinct good that each individual values. Figure 7 plots the size of the compensating differential against military personnel. Because this total is taxed (or reduces taxes, if negative), we have depicted both the gross and net compensating differential curves. It is the after tax or net compensating differential curve that must be paid to induce the individual to accept the change in job amenities. The net compensating differential, therefore, reflects opportunity cost. In Figure 7, the sum of the positive and negative cross-hatched areas equal the total opportunity cost associated with the change in job amenities when the military selects the number of personnel, L_M, at which demand equals supply.

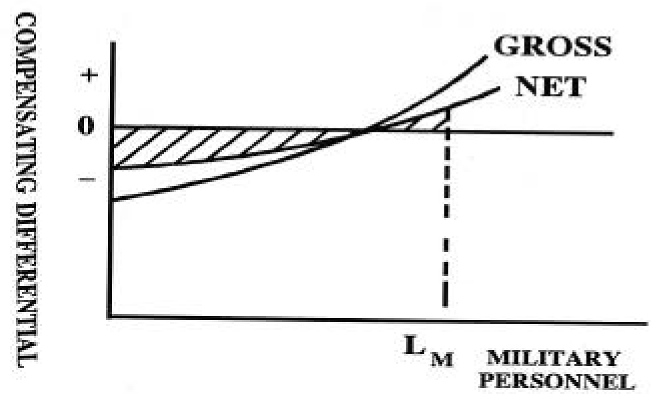

FIGURE 7
Gross and Net Compensating Differential

We can now summarize the components of the opportunity cost of military personnel when there is an income tax on labor and a compensating differential that equates miliary and civilian employment. This cost equals the sum of the following elements: the gross value of civilian goods foregone because of the labor displaced from civilian employment; the net value of leisure foregone when the military labor comes from leisure; and the sum of the net compensating differentials paid to the military personnel who supply their services.

The first two components of opportunity cost are based on the same underlying logic as before. Each additional unit of reduced civilian production is valued at the gross demand price, and there is no change in the income taxes as the military displaces the civilian demand. The second component is an opportunity cost because net supply price equals the value of leisure foregone to the relevant individual. The net compensating differential is an opportunity cost because it equals the value of the change in amenities as measured by the individual.

Sometimes it is helpful to distinguish between the opportunity cost experienced by the individual who supplies labor to the military and the opportunity cost to society, that is, to all the individuals in society. The calculated opportunity cost total and economic rent are exactly the same whether one is viewing cost from the standpoint of the individual or society.

For simplicity, let us ignore taxes and consider the case of an individual who earns a civilian wage of $10,000 and enlists in the military at a wage of $15,000. If this person's compensating differential is $2,000 his/her supply price to the military is $12,000 and an economic rent of $3,000 is earned. In the individual's personal cost-benefit calculus, there is a loss of a $10,000 civilian job and also the loss of civilian job amenities valued at $2,000. The total opportunity cost, therefore, is $12,000. Because the individual receives a benefit or military wage of $15,000, the benefit exceeds the cost by $3,000, the rent obtained by the individual.

From the standpoint of society, $10,000 is both a loss to the individual who enlists and part of the $15,000 wage benefits received in the military. For this individual, therefore, the $10,000 cancels. When analyzing costs and benefits from the standpoint of society, one should eliminate these self-canceling costs and benefits. The employment of this individual in the military rather than in the civilian sector should be assessed using the value of final goods gained and foregone in the society.

The work in the civilian sector is valued at $10,000 by those willing to pay for the goods produced by this individual. When the individual enlists, civilian goods valued by individuals in society at $10,000 are foregone. Another final good foregone is the civilian job amenities. The individual who enlists values these at $2,000--the amount that must be paid to compensate for lost amenities. The total opportunity cost to society, therefore, equals $12,000. If the military wage is $15,000, the value of an additional person's labor in the military is $15,000. This is the social benefit obtained from this individual in the military, and implies that the benefits exceed the costs by $3,000. This is the same economic rent obtained from this person's military service as when the perspective of the individual is used.

SOCIAL DISCOUNT RATE

It is appropriate to complete our discussion of opportunity cost by considering the perennial social discount rate question. A recent *OMB Circular* (1992) states that a discount rate of 7 percent should be employed on government investment projects because "This rate approximates the marginal pretax rate of return on an average investment in the private sector in recent years."

Typically a defense project generates a stream of benefits and costs that occur over time. Because of the time value of money, these dollars are not comparable unless they are first discounted at an appropriate discount rate. Because we need to compare this marginal defense project against the civilian alternatives foregone, the appropriate discount rate should be equal to the "rate of return" foregone as a result of this project. Then, if the present value of the defense project calculated using this procedure is higher, the rate of return on the defense project must be greater than the rate of return foregone.

Of course, for most defense projects, costs and benefits are not commensurable. The cost of a new weapons system can be measured in dollars, while the benefits can only be measured using some military effectiveness measure. In this situation, cost-effectiveness is used to help evaluate the program. But, to sensibly determine whether the subjective benefits from the military effectiveness achieved are greater than the dollar cost of the project, one must first discount the costs using an appropriate interest rate.

One issue that needs to be resolved is the source of funds for the marginal defense project. Although defense purchases are financed through both taxes and borrowing, it is probably reasonable to assume that the government obtains the resources for the marginal defense project by borrowing funds from the capital market.

When the government enters the capital market and bids up the interest rate, private investment is displaced and private saving may be increased. However, the returns to both investment and saving are taxed, and may also be uncertain. With risk averse investors and savers, part of the return received would be an interest premium paid for the risk that is borne.

In our discussion of the discount rate, however, we first ignore uncertainty. We also assume that the returns to investment and saving are each taxed at a constant rate and that these returns are consumed. As is usually recommended in cost-benefit analysis, we also first remove the effects of inflation from all relevant variables.

For concreteness, we use the corporate income tax rate of one-third of profits as the tax on the returns from investment. The returns from saving are assumed to be taxed at an average rate of twenty-five percent--between the current income tax rates of fifteen and thirty-one percent. Figure 8 depicts the gross and net demands for investment, I_{GROSS} and I_{NET}, and the gross and net demands for saving, S_{GROSS} and S_{NET}, where the terms gross and net are being used from the vantage point of the gross and net interest payments.

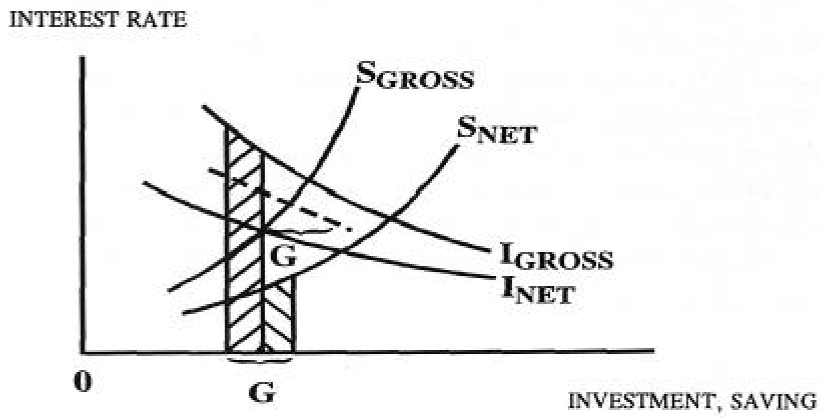

FIGURE 8
Computation of Social Discount Rate

Without government borrowing, the market interest rate, i, is determined at the intersection of the net demand curve for investment and gross supply curve of saving. Therefore, if the gross return on investment before taxes, ö, is nine percent, the return after corporate income taxes, t, would equal six percent.

This is the gross return to private savings and the market interest rate. The net return to saving, r, would equal four percent. At the margin individuals in the private sector are willing forego $1 of consumption today for $1.04 of consumption tomorrow. Foregoing a dollar of consumption, therefore, is equivalent to receiving returns from saving of four percent.

When the government adds its demand, G, to the demand for private investment, as indicated by the dashed line to the right of the net private demand, the market interest rate rises slightly. Some private investment is foregone, the value of which is indicated by the cross-hatched area under the gross demand for private investment curve. In addition, the higher interest rate increases saving, and the cross-hatched area under the net supply of saving curve equals the value of the current consumption foregone as a result of the government's actions in the capital market.

The opportunity cost of the funds obtained by the government, therefore, equals the sum of the value of investment and consumption foregone. For a small change in the interest rate, these values foregone can be expressed as a weighted sum of the gross return on investment and the net return on saving where the relevant weights are the (absolute values) of the interest elasticity of demand for investment, $ç_I$, and the interest elasticity of supply of saving, $ç_S$. So if the former is estimated to be 3, and the latter 1, an estimate of the social discount rate, è, would be calculated as follows:[3]

$$è = \frac{ç_I ö + ç_S r}{ç_I + ç_S} = \frac{3(9) + 1(4)}{3 + 1} = 7.75 \qquad (2)$$

In the above discussion, we have abstracted from risk, which is, of course, a key characteristic of the capital market. We also discussed the costs and benefits of the defense project as if these amounts were certain. Let us deal, first, with the risks faced by investors and savers.

If the returns from investment and saving are average returns, and the investors and savers are risk averse, there will be risk premia included in these percentages. Average returns either foregone by investors or received by savers would not, therefore, reflect opportunity cost. To determine opportunity cost, one must first subtract these risk premia from the average returns. This yields the certainty equivalent returns from the investment and consumption foregone.

Suppose for simplicity the government is risk neutral. The average taxes foregone and received as a result of the government borrowing (and the average costs and benefits of the defense project) would then equal the certainty equivalent. Also suppose that the risk averse investors and savers receive risk premia of two and one percent respectively. The recalculated social discount rate would then be reduced to six percent.

If the government is not risk neutral, then it is necessary to first convert the benefits, costs, and tax implications of the project to certainty equivalents before applying the discount rate. The adjustment from expected returns to certainty equivalents would take account of not only the degree of government risk aversity, but also the specific character of the uncertainty associated with the project under consideration. This latter issue must be addressed when the returns from the project under consideration are related to the returns from other projects.

As a result of these considerations, as the risk aversity of the government increases, the certainty equivalent of the expected costs and benefits would decrease. Also, for a given level of risk aversity, the more this project reduces the variation of the government's entire portfolio of projects, the greater is the certainty equivalent.

It may be helpful to summarize the argument for using the social discount rate for defense projects. At the margin, every dollar obtained from the capital market entails a cost equal to the social discount rate calculated in the recommended manner. When the government evaluates projects using this discount rate, the present value of the project will be positive only if the benefits gained from the project exceed the costs, namely the value of both the private investment projects and consumption foregone. Even if costs and benefits are not commensurable, costs must still be discounted in order to properly compare these costs with the subjective value of the benefits obtained.

FINAL OBSERVATION

The opportunity cost calculations for both a material input used in the defense sector and for military personnel suggest that the budgetary expense for defense activities is higher than the opportunity cost. By systematically applying these techniques it should be possible to estimate the true burden of defense.

NOTES

1. These principles were taught by Arnold Harberger at Princeton University during a project evaluation course taken by the author in 1973 and are also contained in Jenkins and Harberger (1994). Also see Harberger (1974) for an application of these principles to the calculation of the social discount rate. The incomplete measure of defense cost is contained Neu (1990). Neu's analysis identifies the cost of defense activities as the change in consumer surplus. As discussed in the

Appendix to this paper, the change in consumer income is also part of defense cost.

2. Quoted in *Early Austrian Economics* (1988). As indicated in the audio discussion, the ideas of Viser are partly present in Karl Menger's earlier work.

3. This equation is more convenient for determining the social discount rate than one in the same form as Equation 1 because both investment and saving are taxed using *ad valorem* taxes. For a discussion of this equation, see Harberger, "On Measuring the Social Opportunity Cost of Public Funds" (1974). An interesting issues concerns the relationship between the Harberger result and the analysis contained in Bradford (1975). Bradford's two period analysis is equivalent to Harberger's if the social rate of time preference equals the consumer rate of time preference. The Bradford multiperiod analysis can be extended to show that if the if the returns on the government investment project are consumed, as would typically be the case for defense projects, the return on the government project must be no less than the rate of return calculated with Equation 2. For values of the shadow price of capital greater than its minimum, (1+ö)/(1+r), which occurs when the returns on the private investment project are consumed, the calculated social discount rate would be somewhat higher than the Equation 2 calculation. For another perspective on the discount rate that measures cost from the perspective of the Department of Defense, see Quirk and Terasawa (1987).

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APPENDIX

In this Appendix we derive the opportunity cost or negative monetary benefits imposed on a representative individual when the government purchases some quantity of a good X and there are no tax distortions in the economy. We consider an individual consumer with utility function, U(X,Y), and budget constraint, $I = P_X X + P_Y Y$. where X and Y are private goods with prices, P_X and P_Y, respectively.

When the individual maximizes utility subject to the budget constraint, the following first order conditions are obtained:

MU/MX = ëP_X
MU/MY = ëP_Y
$I = P_X X + P_Y Y$,

where ë, the Lagrangian multiplier of the budget constraint, equals the marginal utility of income.

Totally differentiating the budget constraint yields

$$dI = P_X dX + P_Y dY + X dP_X + Y dP_Y.$$

Letting MU/MX and MU/MY, each divided ë equal the marginal monetary benefits of X and Y, MB/MX and MB/MY respectively, and setting dU/ë = dB, the change in monetary benefits resulting from dX and dY, we have

$$dB = (MB/MX)dX + (MB/MY)dY = P_X dX + P_Y dY \qquad \text{(A-1)},$$

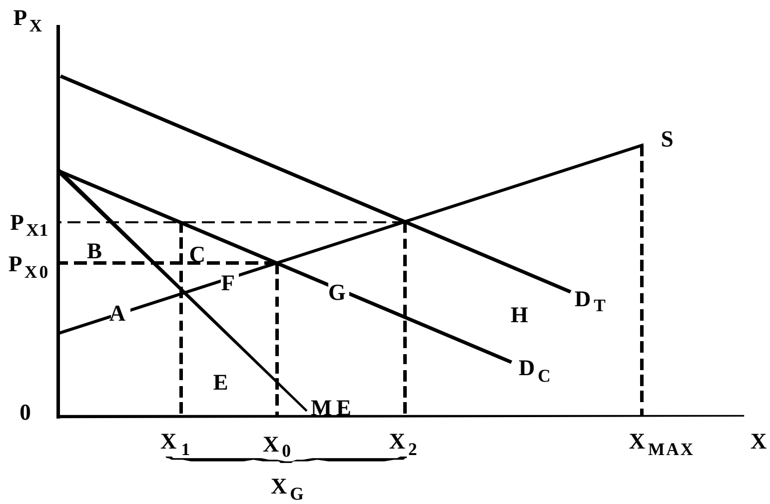

FIG (A-1)

Two Equivalent Methods of Computing Opportunity Costs

where the latter equality holds because of the first order conditions.

In the light of the budget constraint, it must be also true that

$$dB = dI - XdP_X - YdP_Y \qquad \text{(A-2).}$$

As a result, there are two ways of measuring opportunity cost. One can either use the perspective of the "vertical" axis and employ Equation (A-1) or the perspective of the "horizontal" axis and employ Equation (A-2). Either expression can be integrated to obtain equivalent measures of cost to the individual. Combining the two resulting measures, however, double counts the effect of the government activities on the individual.

An advantage of integrating Equation (A-1) is that the change in consumer income which necessarily occurs when the government obtains inputs from the private sector need not be directly calculated. As indicated by Equation (A-2), when this expression is integrated, the change in income must be added to the change in consumer surplus to obtain the correct measure of cost.

To illustrate this, consider the case in which $dP_Y = 0$, so that Y equals the numeraire good. We also assume that there are no income effects in the consumption of X. Let the government enter the marketplace and purchase quantity X_G of good X. As a result of these purchases, the shaded area in Figure 3 measures the cost to the individual of the government purchases, with X now replacing Q and X_G replacing Q_M.

As indicated in the text, the shaded area under the demand curve equals the value of the individual's reduced consumption of X. As the supply curve for X is, in effect, a demand curve for Y, the shaded area under the supply curve equals the value of Y foregone as a result of the government's purchases.

Consider now the measure derived from Equation (A-2). Figure (A-1) depicts both the consumer demand curve, D_C, and total demand curve for X, D_T, including the government purchases.

Areas B + C equal the traditional area to the left of the consumer's demand curve between the old price, P_{X0}, without government purchases and the new price, P_{X1}, with the purchases. This is the area that has been incorrectly identified in RAND analysis as the cost of the DoD purchases. However, this area can be correctly identified as the loss in consumer surplus resulting from the government purchases. We have

Loss in consumer surplus = Areas B + C.

There are a number of other areas identified on the diagram that are helpful for understanding the equivalencies of the vertical and horizontal perspectives. In addition to B, area A is part of the expenditure on X when the price equals P_{X1}. A marginal expenditure curve, ME, that is marginal to the individual's demand curve is also provided on the diagram, and the area under this curve between X_1 and X_0 equals E. The identified area F under the demand curve necessarily equals B. This is because the reduction in expenditure on X, area E, equals areas E + F - B.

Area G is the area under the supply curve between X_0 and X_2, where the latter is the total production of X including the government purchases of $X_G = X_2 - X_1$. Area H, which is under the supply curve between X_2 and X_{MAX}, equals the individual's remaining consumption of Y when the government purchases X_G.

The reason why areas B + C, the change in consumer surplus, is incomplete is that it fails to take account of the consumer's loss in income. Without government purchases, the income I_0 of the consumer will be:

$$I_0 = P_{X0}X_0 + Y_0 = \text{Areas } A + E + F + G + H.$$

With government purchases, however, the income, I_1, will be:

$$I_1 = P_{X1}X_1 + Y_1 = \text{Areas } A + B + H.$$

Taking into account the equality between B and F discussed above, the change in consumer income will be:

$$I_1 - I_0 = -\text{ Areas } E + G.$$

The total opportunity cost imposed on the consumer derived from Equation (A-2), therefore, will be:

$$\text{Total Opportunity Cost} = B + C + E + G.$$

However, as B = F [from Equation (A-1)], the above equation becomes:

$$\text{Total Opportunity Cost} = C + E + F + G.$$

Therefore, the two equivalent ways of measuring opportunity cost, derived from Equations (A-1) and (A-2), are respectively: 1) the value of the foregone consumption resulting from the government purchases and 2) the change in consumer surplus plus the change in consumer income.