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Optimal payment time for a retailer under permitted delay of payment by the wholesaler

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Abstract

The retailer (buyer) is usually allowed a permissible credit period to pay back the dues without paying any interest to the wholesaler (supplier). In this problem the retailer can pay the wholesaler either at the end of credit period or later incurring interest charges on the unpaid balance for the overdue period. This research develops a retailer's model for optimal cycle and payment times for a retailer in a deteriorating-item inventory situation where a wholesaler allows a specified credit period to the retailer for payment without penalty. Under these conditions, this wholesaler-and-retailer system is modeled as a cost minimization problem to determine the optimal payment time under various system parameters. The model is solved through an iterative search procedure and the overall findings indicate that the retailer has always an option to pay after the permissible credit period depending on interest rates, unit purchase and selling price, and the deterioration rate of the products. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Optimal payment time; Permissible delay of payment; Retailer and wholesaler

1. Introduction

In a typical buyer–seller situation, an inventory model considers a case in which depletion of inventory is caused by a constant demand rate, but in real-life situations there is inventory loss by deterioration also. This paper considers a retailer's model in which the deterioration rate is constant, and the retailer has an option to fix the payment period instead of settling the account with the wholesaler (supplier) at a particular allowable time frame.

In today's competitive business transactions, it is common to find that the retailers (buyers) are allowed some credit period before they settle the account with the wholesaler. This provides a very big advantage to the customers, due to the fact that they do not have to pay the wholesaler immediately after receiving the product, but instead, can delay their payment until the end of the allowed period. The customer pays no interest during the permissible time for payment, but interest will be charged if the payment is delayed beyond that period.

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A lot of work has been done on deteriorating inventory systems [1–7]. Heng et al. [5] integrated Misra's [1] and Shah's [2] models to consider a lot-size, order-level inventory system with finite replenishment rate, constant demand rate, and exponential decay. Su et al. [8] considered an inventory under inflation for stock dependent consumption rate and exponential decay while Hariga [6,9] developed models for deteriorating items with time-dependent demand.

Kim et al. [10] developed an optimal credit policy to increase wholesaler's profits with price-dependent demand functions. Goyal [11] developed an economic order quantity under the conditions of permissible delay in payments for an inventory system. Aggarwal and Jaggi [12] developed a model to determine the optimum order quantity for deteriorating items under a permissible delay in payment. Hwang and Shinn [13] modeled an inventory system for retailer's pricing and lot sizing policy for exponential deteriorating products under the condition of permissible delay in payment. Other researchers also considered similar issues relating to payment period or lot sizing [14–19].

1.1. *The wholesaler-and-retailer problem*

The retailer pays neither the interest nor the purchase price of the items to the wholesaler before the permissible credit period expires. The retailer is subjected to pay the interest on the purchase amount if the account is not settled before the permissible credit (delay) period expires. The reason for offering a credit period to the retailers is to stimulate the demand. The wholesaler usually expects that the interest loss incurred during the credit period can be compensated by the increase in profit due to stimulated sales.

From a wholesaler's point of view, an important question is how to set a credit period, and from the retailer's point of view, the question is how to take the advantage of the credit period as well as his payment time. An ordering policy for deteriorating items with allowable shortage, and permissible delay in payment is studied by Jamal et al. [20], but it does not seek for an optimal payment period. Assuming that the permissible credit period is already set by wholesalers on the basis of trade practice, with infinite replenishment rate and no shortages of products, the problem in this research is to ascertain the *optimal payment period* for the retailer to minimize the total cost of the inventory system. This paper finds an optimal payment time for an inventory system with deteriorating items under a condition that the wholesaler offers a permissible credit period for payment after the purchase of the goods.

2. The retailer and wholesaler's payment system

In this problem the retailer can pay the wholesaler either at time M to avoid the interest payment or afterwards with interest on the unpaid balance due at M. Typically, the retailer may not pay fully the wholesaler by time M for lack of cash. On the other hand, his cost will be higher the longer he waits beyond M. Therefore, the retailer will gradually pay the wholesaler until the payment is complete. Since the selling price is higher than the unit cost, and interest earned during the credit period M may also be used to pay off the wholesaler, the payment will be complete at time P before the end of each cycle T (i.e., $M \leqslant P \leqslant T$).

This wholesaler and retailer system is modeled as a cost minimization problem to determine the optimal payment time P^* under various system parameters. Both credit period and payment period in this system is depicted along with the inventory level in Fig. 1. The following assumptions and notation are used throughout the paper.

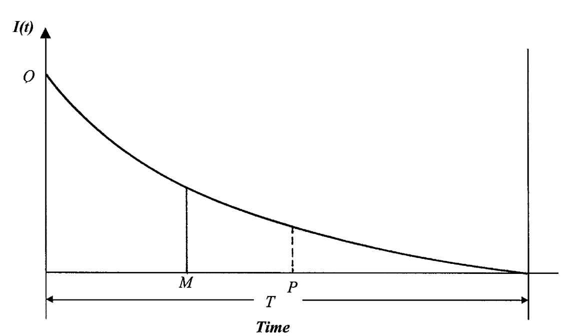

Fig. 1. The wholesaler and retailer's deteriorating inventory system.

Notation

- θ deterioration rate, a fraction of the on-hand inventory
- A the ordering cost of inventory (dollars/order)
- c the unit cost per item (dollars/unit)
- C_{D} total cost of deterioration per cycle
- C_{H} total holding cost per cycle
- D the demand rate (units per unit time)
- D_T amount of materials deteriorated during a cycle time, T
- i the inventory carrying cost rate
- I_{e} the interest earned per dollar per unit time,
- I_{p} the interest paid per dollar per unit time dollars/dollar-year
- I_{T} total interest earned per cycle
- M Permissible delay fixed by the wholesaler in settling the account
- P payment time of the retailer
- P_T interest payable per cycle
- Q the order quantity (units/order)
- S selling price (dollars/unit)
- T the length of the inventory cycle (time units)

3. The retailer's inventory payment model

The inventory level varies with time due to decaying or loss of materials. Inventory depletes due to the simultaneous demand and deterioration or loss of materials. The deterioration can occur when the materials are physically existing in the inventory at time t $(0 \leqslant t \leqslant T)$.

3.1. General decaying function

The wholesaler and retailer model works on a number of system parameters of which decaying is an important factor. Let $I(t)$ be the inventory level at time t. For a deterioration rate θ, the inventory level at time t, $I(t)$, during the time period $(0 \leqslant t \leqslant T)$ is given by

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \theta I(t) = -D, \quad 0 \leqslant t \leqslant T, \tag{1}$$

which yields

$$I(t) = I_0 \mathrm{e}^{-\theta t} + \frac{D}{\theta}(\mathrm{e}^{-\theta t} - 1), \quad 0 \leqslant t \leqslant T, \tag{2}$$

where $I(t) = I_0$ at time $t = 0$. It is obvious that at $t = T$, $I(T) = 0$. So Eq. (2) yields

$$Q = I_0 = D(\mathrm{e}^{\theta T} - 1)/\theta. \tag{3}$$

Replacing I_0 in Eq. (2) with its value from Eq. (3), we get

$$I(t) = \frac{D}{\theta}(\mathrm{e}^{\theta(T-t)} - 1), \quad 0 \leqslant t \leqslant T. \tag{4}$$

Since the total demand during T is DT, the amount of materials which deteriorates during one cycle is written as

$$D_T = I_0 - DT = \frac{D}{\theta}(\mathrm{e}^{\theta T} - 1) - DT. \tag{5}$$

3.2. Evaluating the cost functions

The inventory level at time t, $I(t)$, is known in terms of known parameters. The total cost function may now be evaluated under different situations. The variable cost is a function of ordering cost, carrying cost, cost due to deterioration of materials, interest payable and the interest earned. Individual costs are now evaluated before they are grouped together.

(a) For most inventory system the ordering cost of raw materials is fixed at A dollars/order.

(b) The cost of deterioration is directly related to demand during the time. Hence, for an inventory with exponential decaying rate of θ, the cost of deterioration incurred to D_T units of materials per cycle time T, C_{D}, is given by

$$C_{\mathrm{D}} = cD_T = \frac{cD}{\theta}(\mathrm{e}^{\theta T} - 1) - cDT. \tag{6}$$

(c) The carrying or holding cost is a function of average inventory and it is given by

$$C_{\mathrm{H}} = ic\int_0^T I(t)\,\mathrm{d}t = ic\int_0^T \frac{D}{\theta}(\mathrm{e}^{\theta(T-t)} - 1)\,\mathrm{d}t$$

which, upon simplification, yields

$$C_{\mathrm{H}} = \frac{icD}{\theta^2}(\mathrm{e}^{\theta T} - \theta T - 1). \tag{7}$$

(d) The net cost of the unpaid inventory at time t is the cost of the current inventory at any time t, *minus* the profit on the amount sold during time M, *minus* the interest earned from the sales revenue during time M. The extra amount that can be paid off is determined by profit on the amount sold after the permissible delay time M. Therefore, the interest payable per cycle for the inventory not being sold after the due date is given by

$$\begin{aligned} P_T &= I_p \int_M^P (cI(t) - (S-c)DM - SI_e M^2/2)\,dt \\ &\quad - (S-c)I_p \int_0^{P-M} Dt\,dt \\ &= I_p \int_M^P \left(\frac{cD}{\theta}(e^{\theta(T-t)} - 1) - (S-c)DM \right. \\ &\quad \left. - SI_e M^2/2\right) dt - (S-c)I_p \int_0^{P-M} Dt\,dt \\ &= -\frac{cI_p D}{\theta^2}(e^{\theta(T-P)} - e^{\theta(T-M)}) \\ &\quad - \frac{cI_p D}{\theta}(P-M) \\ &\quad - I_p(S-c)DM(P-M) \\ &\quad - I_p SI_e DM^2(P-M)/2 \\ &\quad - I_p(S-c)D(P-M)^2/2. \end{aligned} \tag{8}$$

(e) Interest earned per cycle, I_T, is the interest earned during the positive inventory, and it is given by

$$\begin{aligned} I_T &= SI_e \int_0^M Dt\,dt + SI_e \int_0^{T-P} Dt\,dt \\ &= SI_e D(M^2 + (T-P)^2)/2. \end{aligned} \tag{9}$$

3.3. Total cost function

The variable cost is aggregately comprised of ordering cost, carrying cost, cost due to deterioration of materials, and the interest payable minus the interest earned. Thus, the total variable cost per cycle, TVC, is defined as

$$\mathrm{TVC} = A + C_D + C_H + P_T - I_T. \tag{10}$$

Collecting the values of the individual terms from Eqs. (6)–(9), the TVC in terms of P and T can be written as

$$\begin{aligned} \mathrm{TVC}(P,T) &= A + \frac{cD}{\theta}(e^{\theta T} - 1) - cDT \\ &\quad + \frac{icD}{\theta^2}(e^{\theta T} - \theta T - 1) \\ &\quad - \frac{cI_p D}{\theta^2}(e^{\theta(T-P)} - e^{\theta(T-M)}) \\ &\quad - \frac{cI_p D}{\theta}(P-M) \\ &\quad - I_p(S-c)D(P^2 - M^2)/2 \\ &\quad - I_p SI_e DM^2(P-M)/2 \\ &\quad - SI_e D(M^2 + (T-P)^2)/2. \end{aligned} \tag{11}$$

The variable cost per unit time, TC, is simply given by

$$\begin{aligned} \mathrm{TC}(P,T) &= \mathrm{TVC}(P,T)/T \\ &= A/T + \frac{cD}{\theta^2 T}(e^{\theta T} - 1)(\theta + i) - cD \\ &\quad - \frac{icD}{\theta} - \frac{cI_p D}{\theta^2 T}(e^{\theta(T-P)} - e^{\theta(T-M)}) \\ &\quad - \frac{cI_p D}{\theta T}(P-M) - I_p(S-c)D(P^2 - M^2)/2T \\ &\quad - I_p SI_e DM^2(P-M)/2T \\ &\quad - SI_e D(M^2 + (T-P)^2)/2T. \end{aligned} \tag{12}$$

The appropriate values of the decision variables that minimize the total cost function lead to the solution of the problem.

3.4. Search procedure for optimal solution

The total cost function $\mathrm{TC}(P,T)$ in Eq. (12) is a higher-order exponential function. So it is not easy to evaluate the Hessians in closed-form to conclude about its positive definiteness directly, and thus it is not trivial to ascertain if the total cost function is convex. An indirect approach to check the convexity of the function TC(P, T) is employed here by evaluating the response surface of the total cost function over a possible range of the parametric values. Computational results indicates that the response surface of the total cost function TC(P, T) in Eq. (12) is convex in P and T within the feasible range of P and T. Therefore, the values of P and T which minimize TC(P, T) can be obtained by simultaneously solving $\partial \mathrm{TC}(P,T)/\partial P = 0$ and $\partial \mathrm{TC}(P,T)/\partial T = 0$ within the stated ranges. The two partial differential equations lead to the equations

$$\frac{\partial \mathrm{TC}(P,T)}{\partial P} = \frac{1}{\theta}(\mathrm{e}^{\theta(T-P)} - 1) - (S/c - 1)P - SI_\mathrm{e}M^2/2c + S(T-P)I_\mathrm{e}/cI_\mathrm{p} = 0 \quad (13)$$

and

$$\frac{\partial \mathrm{TC}(P,T)}{\partial T} = -\frac{A}{cD} + \frac{\theta+i}{\theta^2}(\mathrm{e}^{\theta T}(T\theta - 1) + 1) - \frac{I_\mathrm{p}}{\theta^2}[(\mathrm{e}^{\theta(T-P)} - \mathrm{e}^{\theta(T-M)})(T\theta - 1) - \theta(P-M)] + I_\mathrm{p}(S/c-1)(P^2-M^2)/2 + I_\mathrm{e}S(I_\mathrm{p}M^2(P-M) - T^2 + M^2 + P^2)/2c = 0. \quad (14)$$

An iterative search approach is employed simultaneously to obtain solutions for P and T. The optimal ordering quantity is calculated easily once P^* and T^* are obtained.

4. Computational results

The optimal seeking procedure finds the optimal payment period (P) and the inventory cycle time (T). In this model we assumed $M \leqslant P^* \leqslant T^*$. Aggarwal and Jaggi [12] provided an illustrative example. An example with a partial set of data from their problem is devised here to illustrate the optimal payment strategy of the retailer to the wholesaler of deteriorating items. The data assumed from Aggarwal and Jaggi [12] are $D = 1{,}000$ units/year, $A = 200$ dollars/order, $i = 0.12$/year, and $I_\mathrm{e} = 0.13$/year.

Table 1
Optimal delay payment with fixed I_e ($I_\mathrm{p} = 0.15$)

		$I_\mathrm{p} = 0.15$ and $I_\mathrm{e} = 0.13$											
	θ	0.05			0.1			0.15			0.2		
c	M	T^*	P^*	TC	T^*	P^*	TC	T^*	P^*	TC	T^*	P^*	TC
20	0	54	49	1785	54	49	1861	54	49	1938	54	49	2014
	15	71	64	1478	71	64	1575	71	64	1677	71	64	1777
	30	87	79	1293	87	79	1419	80	72	1540	77	70	1652
	45	96	87	1170	90	81	1302	83	75	1425	77	70	1538
60	0	54	49	2652	52	47	2837	48	43	3077	46	41	3272
	15	57	51	2280	52	47	2509	48	43	2714	46	41	2908
	30	57	51	1927	52	47	2164	48	43	2369	47	42	2566
	45	58	52	1609	52	47	1838	50	45	2052	50	45	2261
100	0	40	36	3379	39	35	3696	36	32	3968	36	32	4220
	15	42	36	2776	40	35	3102	36	32	3368	36	32	3619
	30	45	38	2215	42	38	2541	38	34	2812	36	32	3065
	45	51	46	1757	50	45	2100	50	45	2446	50	45	2795

Notes: $D = 1000$ units/yr, $A = 200$ dollars/order, $i = 0.12$/yr, $S = 1.2c$ and $Q^* = D(\mathrm{e}^{\theta T^*} - 1)/\theta$.

We found earlier that the payable interest rate, deteriorating rate, product unit cost, and permissible delay time have a significant decisive effects on payment delay time and inventory cycle time. Tables 1 and 2 are constructed to study these effects of payable interest rate I_p, deteriorating rate θ, variable unit cost c, and permissible delay time M on payment delay time P, inventory cycle time T, and total cost $TC(P, T)$. Different parametric values used in constructing these values are given in vector forms as $I_p = (0.15, 0.20)$ dollar/dollar/year, $\theta = (0.05,\ 0.10,\ 0.15,\ 0.20)$, $c = (20,\ 60,\ 100)$ dollars/unit, and $M = (0, 15, 30, 45)$ days. It should be noted here that the time units used for P, M, and T in the model are in 'year' while, for ease of convenience, the units exhibited in the example are in 'day'.

It is observed that the payment period P and the inventory cycle time T increase with the increase of permissible delay time M but the optimal total cost $TC(P^*, T^*)$ varies inversely. The payment period P and the cycle time T become shorter but TC becomes larger with increasing deterioration rate θ. It is also clear from both Tables 1 and 2 that the payment delay period has an inverse relationship and the total cost has a direct relationship with the payable interest rate I_p.

Table 3 shows that both cycle time T and payment period P tend to decrease as the earned interest rate I_e increases. It also indicates that there is a moderate reduction in inventory cost with the increase of earned interest rate. Table 4 is constructed to show the effect of selling price with respect to unit price of the item on decision variables P and T, and the total cost TC. The same data were used in Table 4 except that $S/c = 1.0$, 1.2, and 1.6. In general, results indicate that the optimal payment period P decreases and the cycle time T increase as S/c increases. It is always true that the total cost decreases as the ratio of selling price to unit cost increases.

Empirical results in Table 4 show that when $S/c = 1$ and no delay in payment is allowed (i.e., $M = 0$), the optimal payment time is at the end of the cycle (i.e., $P^* = T^*$), which confirms the theoretical model in Eq. (12). Once the optimal payment time P and cycle time T are obtained, the optimal order quantity may be determined from $Q^* = D(e^{\theta T^*} - 1)/\theta$.

Table 2
Optimal delay payment with fixed I_e ($I_p = 0.20$)

		$I_p = 0.20$ and $I_e = 0.13$											
	θ	0.05			0.1			0.15			0.2		
c	M	T^*	P^*	TC	T^*	P^*	TC	T^*	P^*	TC	T^*	P^*	TC
20	0	55	49	1834	55	49	1910	55	49	1986	55	49	2064
	15	72	64	1511	72	64	1616	72	64	1719	72	64	1822
	30	87	78	1324	82	73	1451	80	71	1567	75	67	1676
	45	96	86	1193	85	76	1316	80	71	1434	76	68	1544
60	0	51	45	2829	48	43	3039	43	38	3239	43	38	3420
	15	57	51	2370	48	43	2577	45	40	2775	42	37	2959
	30	57	51	1951	52	46	2177	48	43	2378	45	40	2571
	45	58	52	1606	52	46	1836	51	45	2053	51	45	2268
100	0	35	31	3601	37	33	3907	36	32	4173	31	27	4414
	15	39	35	2886	37	33	3155	36	32	3418	33	29	3663
	30	39	35	2250	37	33	2548	38	34	2811	36	32	3064
	45	51	45	1745	51	45	2110	51	45	2468	51	45	2825

Notes: $D = 1000$ units/yr, $A = 200$ dollars/order, $i = 0.12$/yr, $S = 1.2c$ and $Q^* = D(e^{\theta T^*} - 1)/\theta$.

Table 3
Optimal delay payment with fixed I_p and varying I_e

		$I_p = 0.20$ and $\theta = 0.10$											
	I_e	0.08	0.10			0.12			0.13				
c	M	T^*	P^*	TC	T^*	P^*	TC	T^*	P^*	TC	T^*	P^*	TC
20	0	56	49	1899	56	49	1899	55	49	1910	55	49	1910
	15	73	64	1620	73	64	1617	72	64	1618	72	64	1616
	30	86	75	1471	86	76	1463	84	75	1455	82	73	1451
	45	87	76	1357	87	77	1340	87	77	1324	85	76	1316
60	0	48	42	3040	48	42	3038	48	43	3040	48	43	3039
	15	48	42	2601	48	42	2591	47	42	2583	48	43	2577
	30	52	45	2265	52	46	2231	52	46	2195	52	46	2177
	45	57	50	2024	54	47	1950	52	46	1875	52	46	1836
100	0	37	32	3923	36	32	3917	37	33	3908	37	33	3907
	15	37	32	3221	37	32	3198	37	33	3166	37	33	3155
	30	43	37	2733	42	37	2661	42	37	2588	37	33	2548
	45	52	45	2447	51	45	2310	51	45	2177	51	45	2110

Notes: $D = 1000$ units/yr, $A = 200$ dollars/order, $i = 0.12$/yr, $S = 1.2c$ and $Q^* = D(e^{\theta T^*} - 1)/\theta$.

Table 4
Optimal delay payment for varying S/c

		$I_p = 0.20, I_e = 0.13$ and $\theta = 0.10$															
	S/c	1.0				1.2				1.4				1.6			
c	M	T^*	P^*	P^*/T^*	TC	T^*	P^*	P^*/T^*	TC	T^*	P^*	P^*/T^*	TC	T^*	P^*	P^*/T^*	TC
20	0	49	49	1.0	2040	55	49	0.89	1910	60	49	0.82	1801	64	49	0.77	1719
	15	65	64	0.98	1717	72	64	0.89	1616	78	64	0.82	1539	83	64	0.77	1474
	30	79	78	0.99	1529	82	73	0.89	1451	86	71	0.83	1385	92	71	0.77	1324
	45	82	81	0.99	1389	85	76	0.89	1316	90	74	0.82	1254	93	71	0.76	1196
60	0	45	45	1.0	3176	48	43	0.90	3039	48	39	0.81	2914	52	40	0.77	2805
	15	46	45	0.98	2707	48	43	0.90	2578	48	39	0.81	2458	52	40	0.77	2349
	30	46	45	0.98	2297	52	46	0.88	2177	54	44	0.81	2066	54	41	0.76	1971
	45	50	49	0.98	1944	52	46	0.88	1836	58	47	0.81	1746	60	46	0.77	1663
100	0	36	36	1.0	4100	37	33	0.89	3907	37	30	0.81	3756	42	32	0.76	3612
	15	37	36	0.97	3335	37	33	0.89	3155	42	34	0.81	3012	42	32	0.76	2870
	30	37	36	0.97	2693	37	33	0.89	2548	42	34	0.81	2410	46	35	0.76	2292
	45	46	45	0.98	2202	51	45	0.88	2110	55	45	0.82	2030	59	45	0.76	1950

Notes: $D = 1000$ units/yr, $A = 200$ dollars/order, $i = 0.12$/yr, $I_e = 0.13$/yr, and $I_p = 0.20$/yr and $Q^* = D(e^{\theta T^*} - 1)/\theta$.

5. Conclusions

This research addresses a retailer's model for optimal strategy for payment time. Here we developed a model for optimal cycle and payment times for a retailer in an inventory situation with deteriorating products, where a wholesaler allows a specified credit period to the retailer (buyer) for payment without penalty. Order quantity and other schedules for the inventory system are easily obtained when the optimal cycle time is known.

Test results show that the total inventory cost decreases and the optimal payment period becomes shorter as the unit selling price increases relative to the unit cost — which means that the retailer should settle his account relatively sooner. It may be also noted that the payment time reduces in general as the difference between payable and earned interest rates increases. However, the total inventory cost in this case increases. It is further noted that both cycle time and payment period become shorter as the product deteriorates faster, resulting in higher total inventory cost.

It is to the advantage of the retailer that he should be prepared, in this case, to pay for his inventories sooner as well. Overall, the retailer has the freedom to adjust his payment depending on interest rates, unit purchase and selling price of the products, and the product deterioration rate. This retailer's model has wide range of applications in wholesale-retail business where the competition is stiff, especially in storage and warehousing since many competitive products are pushed to the market by offering greater advantage and profit margin to the retailers.

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