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Analysis on an On-Line Iterative Correction Control Law for Visual Tracking

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Abstract

We present and analyze an iterative control algorithm that enables us to find a control input that generates the desired output asymptotically with parameter uncertainties. The proposed algorithm keeps the clearness and compactness of the control input update form of iterative learning control while being applicable to nonrepeatable tracking problems, specifically visual tracking. Uncertainties, such as misalignment of the optical axis of the camera with the motion platform, miscalibration, and other unknown parametric inaccuracies can be eliminated. Sufficient conditions for the convergence of the trajectories to the desired one are given. The performance and effectiveness are demonstrated through simulation.

1 Introduction

The proposed iterative correction control algorithm has its roots in iterative learning control (ILC) [1, 2, 3, 4]. ILC is a technique for improving the tracking performance of systems that execute the same task over and over. The errors of the output response in the current trial are recorded and used to compute a modified input signal that will be applied to the system during the next trial so that the error will be reduced. The role of ILC is to improve performance rather than to stabilize. The main features of the existing ILC algorithms include: (1) The control input is updated "iterationwise", thus requiring the whole task to be executed repeatedly until satisfactory performance is yielded. (2) The system is reinitialized to the same value at the beginning of each iteration (trial). (3) Little a priori knowledge of the system parameters is required. Features (1) and (2) form the barrier for control applications that are nonrepeatable, e.g. real-time visual tracking.

Direct learning control [5, 4] is another type of learning control algorithm with the feature that there is no need for repetitive learning because the ideal input can be calculated directly. However, prior control information, e.g. the exactly same output trajectory executed with a different time period, must be first available as a profile for direct calculation.

The objective of this paper is to propose another iterative control algorithm, which we call iterative correction control (ICC), that keeps the clearness and compactness of the control input update form of ILC while being applicable to nonrepeatable tracking problems. The sufficient conditions for the convergence of the trajectories to the desired one are given.

In section 2, we give the problem formulation. Section 3 details the main result of this paper. In section 4 we show the simulation results by comparing control with and without ICC. Finally, some concluding remarks are given in section 5.

2 Problem Formulation

The goal of visual tracking is to find and achieve a desired camera rotation such that the target point is kept at the origin of the image-plane. Suppose T is the finite sampling period of the trajectory planner, and $t_0 = kT$ is the current time instant. Assume the motor drive is in the velocity control mode, which responds to a conventional zero to ± 10 Volt DC signal. Most variable speed drives and servo amplifiers on the market receive commands via this type of signal [6]. The dynamics are modeled by:

$$\dot{\omega}(t) = -\alpha\omega(t) + \alpha f_{real}^{-1}(V(t)), \tag{1}$$

where $V(t)$ stands for input voltage, α is a positive constant, and $f_{real}(\cdot)$ is the exact but unknown steady-state characteristic function mapping ω to V. Given the desired motor angle at the next sampling time,

$$\theta_{\mathrm{d}}(t_0 + T) = \theta(t_0) + \Delta\theta_{\mathrm{d}}(t_0), \tag{2}$$

the *nominal control input* is found by looking up the nominal characteristic curve $f_{nom}(\cdot)$ for the control input

$$V_{nom}(t_0) = f_{nom}\left(\frac{\Delta\theta_d(t_0)}{T}\right), \tag{3}$$

where *nom* stands for nominal. The nominal characteristic curve function $f_{nom}(\cdot)$, as shown in Fig.1, is provided either by the product manual or obtained via curve fitting. Then,

$$\dot{\omega}(t) = -\alpha\omega(t) + \alpha f_{real}^{-1}(V_{nom}(t_0)). \quad (4)$$

Note that in general $f_{real}(\cdot) \neq f_{nom}(\cdot)$. Besides, there exist a variety of other uncertainties that also contribute to failure of perfect tracking, such as misalignment of the optical axis of the camera with the motion platform, miscalibration, and other unknown parametric inaccuracies.

3 Iterative Correction Control

We consider the following control strategy to fulfill the task of perfect tracking. We include the term $\overline{\beta}$ to represent all disturbances incurred by the problems aforementioned, i.e.,

$$\ddot{\theta}(t) = -\alpha\dot{\theta}(t) + \overline{\beta} + \alpha f_{nom}^{-1}(V_{nom}(t_0)). \quad (5)$$

Define

$$\Delta\theta(t) \equiv \theta(t) - \theta_d(t). \quad (6)$$

Then, with $\Delta\dot{\theta}(t) \equiv \frac{d}{dt}\Delta\theta(t) = \dot{\theta}(t) - \dot{\theta}_d(t)$ and $\Delta\ddot{\theta}(t) \equiv \frac{d}{dt}\Delta\dot{\theta}(t) = \ddot{\theta}(t) - \ddot{\theta}_d(t)$, we have

$$\Delta\ddot{\theta}(t) = -\alpha\,\Delta\dot{\theta}(t) - \ddot{\theta}_d(t) - \alpha\dot{\theta}_d(t) + \overline{\beta} + \alpha f_{nom}^{-1}(V_{nom}(t_0)). \quad (7)$$

As mentioned, since perfect tracking is never achieved, we propose a design of the Iterative Correction Control (ICC) as follows:

$$\Delta\ddot{\theta}(t) = -\alpha\,\Delta\dot{\theta}(t) + \beta + \alpha f_{nom}^{-1}(V_{cor}(t_0)), \quad (8)$$

where the correction control input $V_{cor}(t_0)$ is defined as

$$V_{cor}(t_0) = f_{nom}\left(f_{nom}^{-1}(V_{nom}(t_0)) + u(t) - \frac{\sigma}{\alpha}\Delta\theta(t) \right) \quad (9)$$

where $\sigma > 0$ is a design parameter satisfying $\alpha^2 > 4\cdot\sigma$, and $u(\cdot)$ is the correction control defined as:

$$u(t) = u_n,\ nT \le t < (n+1)T,\ n \in \mathrm{N} \cup \{0\} \quad (10)$$

and

$$u_{n+1} = u_n - \lambda\cdot\Delta\theta((n+1)T),\ u_0 \equiv 0, \quad (11)$$

where λ satisfies

$$\frac{1-\rho}{\eta\cdot(1+\rho)} < \lambda < \frac{1+\rho}{\eta\cdot(1-\rho)}, \quad (12)$$

with the constants η and ρ being defined as

$k = \frac{\alpha + \sqrt{\alpha^2 - 4\cdot\sigma}}{2}$, $\rho = \exp[-0.5\cdot(\alpha - k)\cdot T]$, and $\eta = \frac{\alpha}{\alpha - k}$.

The proposed ICC scheme guarantees

$$\Delta\theta(t) \to 0 \text{ as } t \to \infty \quad (13)$$

provided that

$$\beta = -\ddot{\theta}_d(t) - \alpha\dot{\theta}_d(t) + \overline{\beta} \text{ is a constant.} \quad (14)$$

In the following, we will present two lemmas, one theorem, and their corresponding proofs to formally establish the ICC design procedure and validate the controller effectiveness as well as performance.

Lemma 1. Given $\dot{x}(t) = -\alpha x(t) + \beta + \alpha\cdot\eta\cdot u(t)$, where α and η are positive constants, β is an unknown constant, and the control input has the form $u(t) = u_n$ for $nT_I \le t < (n+1)T_I$, with $T_I > 0$ being the sampling period, $n \in \mathrm{N} \cup \{0\}$, $u_{n+1} = u_n - \lambda\cdot x((n+1)T_I)$, $\lambda > 0$ is a constant design parameter, and $u_0 \equiv 0$.

If $\frac{1-\exp(-0.5\alpha T_I)}{\eta\cdot(1+\exp(-0.5\alpha T_I))} < \lambda < \frac{1+\exp(-0.5\alpha T_I)}{\eta\cdot(1-\exp(-0.5\alpha T_I))}$,

then $x(t) \to 0$ as $t \to \infty$.

Proof:

The general solution of $x(t)$ for $t \in [kT_I, (k+1)T_I]$ has the form

$$x(t) = x(kT_I)\cdot\exp[-\alpha(t - kT_I)] + \{1 - \exp[-\alpha(t - kT_I)]\}\cdot\left[\frac{\beta}{\alpha} - \lambda\cdot\eta\cdot\sum_{i=1}^{k} x(iT_I)\right] \quad (15)$$

from which we obtain the following recursive expression:

$$x((k+2)T_I) = \{1 + \exp(-\alpha T_I) - \lambda\cdot\eta\cdot[1 - \exp(-\alpha T_I)]\}\cdot x((k+1)T_I) - [\exp(-\alpha T_I)]\cdot x(kT_I) \quad (16)$$

This in turn is expressed in the state-space form

$$\begin{bmatrix} x_1((k+1)T_I) \\ x_2((k+1)T_I) \end{bmatrix} = \mathbf{M} \begin{bmatrix} x_1(kT_I) \\ x_2(kT_I) \end{bmatrix} \quad (17)$$

where

$$\mathbf{M} = \begin{bmatrix} 0 & 1 \\ -\exp(-\alpha T_I) & [1+\exp(-\alpha T_I)] - \lambda \cdot \eta \cdot [1-\exp(-\alpha T_I)] \end{bmatrix},$$

$x_1(kT_I) \equiv x(kT_I)$, and $x_2(kT_I) \equiv x_1((k+1)T_I)$. It can be shown that there exists a pair of complex eigenvalues r_1 and r_2 satisfying $0 \le |r_1| < 1$ and $0 \le |r_2| < 1$, where $|\cdot|$ denotes the modulus of complex numbers, provided the following bounds on the design parameter λ are satisfied:

$$\frac{1-\exp(-0.5\alpha T_I)}{\eta \cdot (1+\exp(-0.5\alpha T_I))} < \lambda < \frac{1+\exp(-0.5\alpha T_I)}{\eta \cdot (1-\exp(-0.5\alpha T_I))} \quad (18)$$

This in turn implies that

$$x(kT_I) \to 0 \text{ as } k \to \infty. \quad (19)$$

To guarantee the convergence to zero of $x(t)$, we construct its upper bound as follows:

$$\|x(t)\| \le \|x(kT_I)\| \cdot \exp[-\alpha(t-kT_I)] + \{1-\exp[-\alpha(t-kT_I)]\} \cdot \left\| \frac{\beta}{\alpha} - \lambda \cdot \eta \cdot \sum_{i=1}^{k} x(iT_I) \right\| \quad (20)$$

for $t \in [kT_I, (k+1)T_I]$, which should be clear from the general form (15) of the solution $x(t)$. Besides, from the general form of the solution $x(t)$, the following expression is valid:

$$x((k+1)T_I) = x(kT_I) \cdot \exp(-\alpha T_I) + [1-\exp(-\alpha T_I)] \cdot \left[\frac{\beta}{\alpha} - \lambda \cdot \eta \cdot \sum_{i=1}^{k} x(iT_I) \right]. \quad (21)$$

Taking limits on both sides, we have

$$\lim_{k\to\infty} x((k+1)T_I) = \lim_{k\to\infty} \{x(kT_I) \cdot \exp(-\alpha T_I)\} + \{1-\exp(-\alpha T_I)\} \cdot \lim_{k\to\infty} \left\{ \frac{\beta}{\alpha} - \lambda \cdot \eta \cdot \sum_{i=1}^{k} x(iT_I) \right\} \quad (22)$$

From (19), (22) yields

$$\lim_{k\to\infty} \sum_{i=1}^{k} x(iT_I) = \frac{\beta}{\alpha \cdot \lambda \cdot \eta}. \quad (23)$$

By noting that $t \to \infty$ implies $k \to \infty$ for $t \in [kT_I, (k+1)T_I]$, we take the limits of (20) on both sides, and substitute (23) into its appropriate position.

$$\lim_{t\to\infty} \|x(t)\| \le \lim_{k\to\infty} \{ \|x(kT_I)\| \cdot \exp[-\alpha(t-kT_I)] \} + \lim_{k\to\infty} \left\{ \{1-\exp[-\alpha(t-kT_I)]\} \cdot \left\| \frac{\beta}{\alpha} - \lambda \cdot \eta \cdot \sum_{i=1}^{k} x(iT_I) \right\| \right\} = 0 + 0 = 0 \quad (24)$$

Then

where the convergence of the discrete sequence (19) is used again. This finishes the proof of $x(t) \to 0$ as $t \to \infty$.

■

Lemma 2. Given $s(t) = \dot{x}(t) + k\,x(t)$, where k is a positive constant, and $s(t) \to 0$ as $t \to \infty$. Then, $x(t) \to 0$ and $\dot{x}(t) \to 0$ as $t \to \infty$.

Proof:

The proof can be found in [7]. ■

Now, we present our main result in terms of the following theorem.

Theorem. Given $\ddot{x}(t) = -\alpha_1 \dot{x}(t) + \beta + \alpha_1 \cdot v(t)$, where α_1 is a positive constant, β is an unknown constant, and the control input has the form $v(t) = u(t) - \frac{\alpha_2}{\alpha_1} x(t)$, where $\alpha_2 > 0$ is a constant design parameter satisfying $\alpha_1^{\,2} > 4 \cdot \alpha_2$, $u(t) = u_n$, for $nT_I \le t < (n+1)T_I$, $T_I > 0$ is the sampling period, $n \in \mathrm{N} \cup \{0\}$, $u_{n+1} = u_n - \lambda \cdot x((n+1)T_I)$, $\lambda > 0$ is a constant design parameter, with $u_0 \equiv 0$. Define constants $k \equiv \frac{\alpha_1 + \sqrt{\alpha_1^{\,2} - 4\cdot\alpha_2}}{2}$, $\alpha_3 \equiv \alpha_1 - k$, $w \equiv \frac{\alpha_1}{\alpha_1 - k}$, and $\rho = \exp(-0.5\alpha_3 T_I)$.

If $\frac{1-\rho}{w \cdot (1+\rho)} < \lambda < \frac{1+\rho}{w \cdot (1-\rho)}$, then $x(t) \to 0$ as $t \to \infty$.

Proof:

Let

$$s \equiv \dot{x} + k\,x. \quad (25)$$

Then $$\dot{s} = -(\alpha_1 - k)s + \beta + \alpha_1 \cdot u(t). \quad (26)$$

Since $(\alpha_1 - k)$ is a positive constant, by Lemma 1 we have $s = \dot{x} + kx \to 0$ as $t \to \infty$. (27)

Noting that k is positive, the following is true by Lemma 2:

$$x(t) \to 0 \text{ and } \dot{x}(t) \to 0 \text{ as } t \to \infty. \quad (28)$$

This completes the proof. ■

4 Simulation Results

Comparison of control with and without ICC is shown in Fig. 2 (a)(b). Parameter settings are $\alpha_1 = 50$, $\alpha_2 = 400$, $T_I = 0.1\text{sec}$, $\beta = 20$ $x(0) = \pi/6$, $\dot{x}(0) = -\pi/3$. Control with ICC decreases the steady state error to approximately zero within reasonable time.

5 Conclusion

In this study, we presented and analyzed an iterative control algorithm that enables us to find a control input that generates the desired output asymptotically with parameter uncertainties. Sufficient conditions for the convergence of the trajectories to the desired one are given. The performance and effectiveness are demonstrated through simulation.

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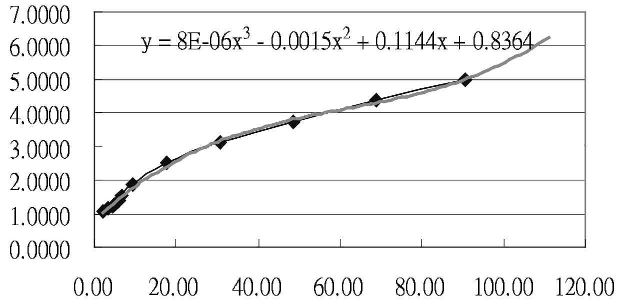

Fig. 1: A typical rotational velocity vs. input voltage characteristic curve. The X axis represents angular velocity (motor count displacement per program count). The Y axis represents input voltage (Volt). Data near the origin are not shown.

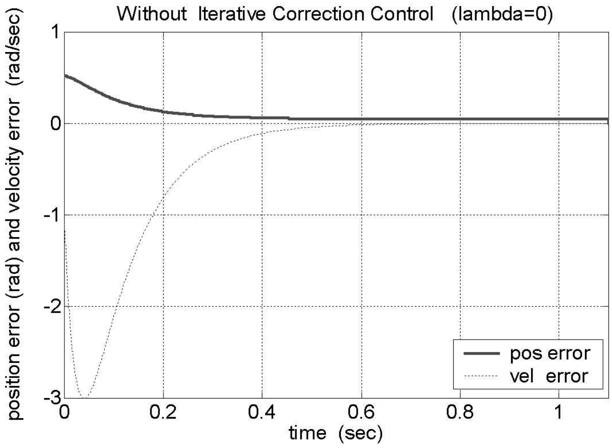

Fig. 2 (a)

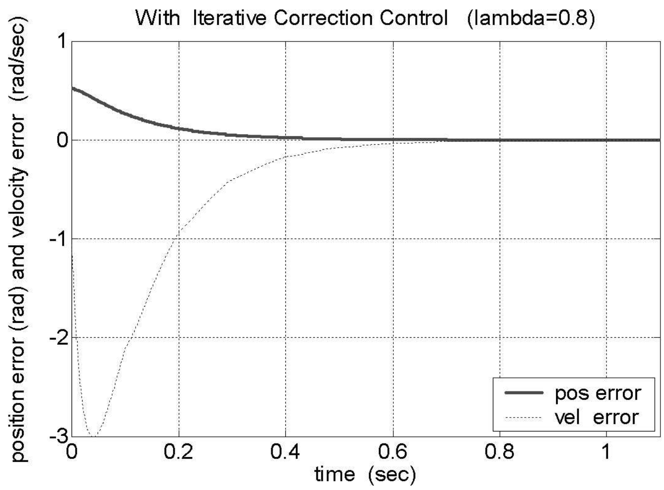

Fig. 2 (b)

Fig. 2 (a)(b): Comparison of control with and without ICC.