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Continuous Optimization

Solving fuzzy transportation problems based on extension principle

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Abstract

Transportation models play an important role in logistics and supply chain management for reducing cost and improving service. This paper develops a procedure to derive the fuzzy objective value of the fuzzy transportation problem, in that the cost coefficients and the supply and demand quantities are fuzzy numbers. The idea is based on the extension principle. A pair of mathematical programs is formulated to calculate the lower and upper bounds of the fuzzy total transportation cost at possibility level α. From different values of α, the membership function of the objective value is constructed. Two different types of the fuzzy transportation problem are discussed: one with inequality constraints and the other with equality constraints. It is found that the membership function of the objective value of the equality problem is contained in that of the inequality problem. Since the objective value is expressed by a membership function rather than by a crisp value, more information is provided for making decisions.

Keywords: Transportation problem; Membership function; Fuzzy sets; Nonlinear programming

1. Introduction

In today's highly competitive market the pressure on organizations to find better ways to create and deliver value to customers becomes stronger. How and when to send the products to the customers in the quantities they want in a cost-effective manner become more challenging. Transportation models provide a powerful framework to meet this challenge. They ensure the efficient movement and timely availability of raw materials and finished goods.

Transportation problem is a linear programming problem stemmed from a network structure consisting of a finite number of nodes and arcs attached to them. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. However, there are cases that these parameters may not be presented in a precise manner. For example, the unit shipping cost may vary in a time frame. The supplies and demands may be uncertain due

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to some uncontrollable factors. To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh [1] and Zadeh [16] introduce the notion of fuzziness.

Since the transportation problem is essentially a linear program, one straightforward idea is to apply the existing fuzzy linear programming techniques [2,3,6,7,9,10,12,13] to the fuzzy transportation problem. Unfortunately, most of the existing techniques [2,3,6,9,12,13] only provide crisp solutions. The method of Julien [7] and Parra et al. [10] is able to find the possibility distribution of the objective value provided all the inequality constraints are of " $\geqslant$ " type or " $\leqslant$ " type. However, due to the structure of the transportation problem, in some cases their method requires the refinement of the problem parameters to be able to derive the bounds of the objective value. There are also studies discussing the fuzzy transportation problem. Chanas et al. [5] investigate the transportation problem with fuzzy supplies and demands and solve them via the parametric programming technique in terms of the Bellman–Zadeh criterion. Their method is to derive the solution which simultaneously satisfies the constraints and the goal to a maximal degree. Chanas and Kuchta [4] discuss the type of transportation problems with fuzzy cost coefficients and transform the problem to a bicriterial transportation problem with crisp objective function. Their method is able to determine the efficient solutions of the transformed problem; nevertheless, only crisp solutions are provided. Verma et al. [14] apply the fuzzy programming technique with hyperbolic and exponential membership functions to solve a multi-objective transportation problem, the solution derived is a compromise solution. Similar to the method of Chanas and Kuchta [4], only crisp solutions are provided.

Obviously, when the cost coefficients or the supply and demand quantities are fuzzy numbers, the total transportation cost will be fuzzy as well. In this paper, we develop a solution procedure that is able to calculate the fuzzy objective value of the fuzzy transportation problem, where at least one of the parameters are fuzzy numbers. The idea is to apply Zadeh's extension principle [15–17]. A pair of two-level mathematical programs is formulated to calculate the lower and upper bounds of the α-level cut of the objective value. The membership function of the fuzzy objective value is derived numerically by enumerating different values of α.

In the sections that follow, firstly, the fuzzy transportation problem is briefly stated. Then a pair of mathematical programs for calculating the α-cuts of the fuzzy total transportation cost is formulated based on the extension principle. We use an example to illustrate the difference between inequality-constraint and equality-constraint problems. Finally, some conclusions are drawn from the discussions.

2. Fuzzy transportation problem

Consider a transportation problem with m supply nodes and n demand nodes, in that $s_j > 0$ units are supplied by supply node i and $d_j > 0$ units are required by demand node j. Associated with each link (i,j) from supply node i to demand node j, there is a unit shipping cost c_{ij} for transportation. The problem is to determine a feasible way of shipping the available amount to satisfy the demand that minimizes the total transportation cost.

Let x_{ij} denote the number of units to be transported from Supply i to Demand j. The mathematical description of the conventional transportation problem is:

$$\begin{aligned} Z = \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} \leqslant s_i, \quad i = 1,\ldots,m, \\ & \sum_{j=1}^{m} x_{ij} \geqslant d_j, \quad j = 1,\ldots,n, \\ & x_{ij} \geqslant 0, \quad \forall i,j. \end{aligned} \tag{1}$$

Intuitively, if any of the parameters c_{ij}, s_i, or d_j is fuzzy, the total transportation cost Z becomes fuzzy as well. The conventional transportation problem defined in (1) then turns into the fuzzy transportation problem.

Suppose the unit shipping cost c_{ij}, supply s_i, and demand d_j are approximately known and can be represented by the convex fuzzy sets, $\widetilde{C}_{ij}$, $\widetilde{S}_i$, and $\widetilde{D}_j$, respectively. Note that a fuzzy set $\widetilde{A}$ is convex if $\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geqslant \min\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\}$, $x_1, x_2 \in X$, $\lambda \in [0,1]$ [17]. Let $\mu_{\widetilde{C}_{ij}}$, $\mu_{\widetilde{S}_i}$, and $\mu_{\widetilde{D}_j}$ denote their membership functions. We have,

$$\begin{aligned}
&\widetilde{C}_{ij} = \{(c_{ij}, \mu_{\widetilde{C}_{ij}}(c_{ij}))|c_{ij}, \in S(\widetilde{C}_{ij})\},\\
&\widetilde{S}_i = \{(s_i, \mu_{\widetilde{S}_i}(s_i))|s_i \in S(\widetilde{S}_i)\},\\
&\widetilde{D}_j = \{(d_j, \mu_{\widetilde{D}_j}(d_j))|d_j \in S(\widetilde{D}_j)\},
\end{aligned} \tag{2}$$

where $S(\widetilde{C}_{ij})$, $S(\widetilde{S}_i)$, and $S(\widetilde{D}_j)$ are the supports of $\widetilde{C}_{ij}$, $\widetilde{S}_i$, and $\widetilde{D}_j$, which denote the universe sets of the unit shipping cost, the quantity supplied by ith supplier, and the quantity required by jth customer, respectively. The fuzzy transportation problem is of the following form:

$$\begin{aligned}
\widetilde{Z} = \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} \widetilde{C}_{ij}x_{ij}\\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} \leqslant \widetilde{S}_i, \quad i = 1,\ldots,m,\\
& \sum_{i=1}^{m} x_{ij} \geqslant \widetilde{D}_j, \quad j = 1,\ldots,n,\\
& x_{ij} \geqslant 0, \quad \forall i,j.
\end{aligned} \tag{3}$$

Without loss of generality, all the unit shipping costs, supply quantities and demand quantities are assumed to be fuzzy numbers in this model, as crisp values can be represented by degenerated membership functions which only have one value in their domains.

Theoretically, the solution procedure proposed by Julien [7] and Parra et al. [10] can be applied to produce the possibility distribution of the objective value. However, in some cases their methods are not able to obtain correct solutions. For example, consider the transportation problem of two supply nodes $\widetilde{S}_1 = (2,3,5)$, $s_2 = 5$ and two demand nodes $d_1 = 4$, $\widetilde{D}_2 = (1,3,6)$, where $\widetilde{S}_1$ and $\widetilde{D}_2$ are triangular fuzzy numbers. This fuzzy transportation problem can be formulated as:

$$\begin{aligned}
\widetilde{Z} = \min \quad & x_{11} + 3x_{12} + 7x_{21} + 2x_{22}\\
\text{s.t.} \quad & x_{11} + x_{12} \leqslant (2,3,5)\\
& x_{21} + x_{22} \leqslant 5\\
& x_{11} + x_{21} \geqslant 4\\
& x_{12} + x_{22} \geqslant (1,3,6)\\
& x_{11}, x_{12}, x_{21}, x_{22} \geqslant 0.
\end{aligned}$$

According to the methods of Julien [7] and Parra et al. [10], the lower bound of the objective value Z_α^{L} and the upper bound of the objective value Z_α^{U} at $\alpha = 0$ are obtained by setting the right-hand side value of the constraints with fuzzy numbers to the lower bound and upper bound, respectively. Specifically, we have,

$$
\begin{aligned}
Z^{\mathrm{L}}_{\alpha=0} = \min \quad & x_{11} + 3x_{12} + 7x_{21} + 2x_{22} \\
\text{s.t.} \quad & x_{11} + x_{12} \leqslant 2 \\
& x_{21} + x_{22} \leqslant 5 \\
& x_{11} + x_{21} \geqslant 4 \\
& x_{12} + x_{22} \geqslant 1 \\
& x_{11}, x_{12}, x_{21}, x_{22} \geqslant 0.
\end{aligned}
$$

$$
\begin{aligned}
Z^{\mathrm{U}}_{\alpha=0} = \min \quad & x_{11} + 3x_{12} + 7x_{21} + 2x_{22} \\
\text{s.t.} \quad & x_{11} + x_{12} \leqslant 5 \\
& x_{21} + x_{22} \leqslant 5 \\
& x_{11} + x_{21} \geqslant 4 \\
& x_{12} + x_{22} \geqslant 6 \\
& x_{11}, x_{12}, x_{21}, x_{22} \geqslant 0.
\end{aligned}
$$

These two transportation problems are feasible, and the optimal objective values derived are $Z^{\mathrm{L}}_{\alpha=0} = 18$ and $Z^{\mathrm{U}}_{\alpha=0} = 17$. The lower bound is contradictorily greater than the upper bound. As a matter of fact, the correct answer of this example is $Z^{\mathrm{L}}_{\alpha} = 6$, which occurs at $s_1 = 4$ or 5 and $d_2 = 1$; and $Z^{\mathrm{U}}_{\alpha=0} = 22$, which occurs at $s_1 = 2$ and $d_2 = 3$.

In the next two sections, we shall develop the solution procedures for inequality-constraint type and equality-constraint type of the fuzzy transportation problem.

3. The solution procedure

Denote the α-cuts of $\widetilde{C}_{ij}$, $\widetilde{S}_i$ and $\widetilde{D}_j$ as:

$$(C_{ij})_{\alpha} = [(C_{ij})^{\mathrm{L}}_{\alpha}, (C_{ij})^{\mathrm{U}}_{\alpha}] = [\min_{c_{ij}} \{c_{ij} \in S(\widetilde{C}_{ij}) | \mu_{\tilde{C}_{ij}}(c_{ij}) \geqslant \alpha\}, \quad \max_{c_{ij}} \{c_{ij} \in S(\widetilde{C}_{ij}) | \mu_{\tilde{C}_{ij}}(c_{ij}) \geqslant \alpha\}] \tag{4.1}$$

$$(S_i)_{\alpha} = [(S_i)^{\mathrm{L}}_{\alpha}, (S_i)^{\mathrm{U}}_{\alpha}] = [\min_{s_i} \{s_i \in S(\widetilde{S}_i) | \mu_{\tilde{S}_i}(s_i) \geqslant \alpha\}, \quad \max_{s_i} \{s_i \in S(\widetilde{S}_i) | \mu_{\tilde{S}_i}(s_i) \geqslant \alpha\}] \tag{4.2}$$

$$(D_j)_{\alpha} = [(D_j)^{\mathrm{L}}_{\alpha}, (D_j)^{\mathrm{U}}_{\alpha}] = [\min_{d_j} \{d_j \in S(\widetilde{D}_j) | \mu_{\tilde{D}_j}(d_j) \geqslant \alpha\}, \quad \max_{d_j} \{d_j \in S(\widetilde{D}_j) | \mu_{\tilde{D}_j}(d_j) \geqslant \alpha\}] \tag{4.3}$$

These intervals indicate where the unit shipping cost, supply, and demand lie at possibility level α. Suppose we are interested in deriving the membership function of the total transportation cost $\widetilde{Z}$. The major difficulty lies on how to deal with the varying ranges of the unit shipping costs, the supply quantities, and the demand quantities. One idea is to apply Zadeh's extension principle [15–17].

Based on the extension principle, the membership function $\mu_{\tilde{Z}}$ can be defined as:

$$\mu_{\tilde{Z}}(z) = \sup_{c,s,d} \min\{\mu_{\tilde{C}_{ij}}(c_{ij}), \mu_{\tilde{S}_i}(s_i), \mu_{\tilde{D}_j}(d_j), \forall i, j | \boldsymbol{z} = Z(\boldsymbol{c}, \boldsymbol{s}, \boldsymbol{d})\} \tag{5}$$

where $Z(\boldsymbol{c}, \boldsymbol{s}, \boldsymbol{d})$ is defined in Model (1). If the α-cuts of $\widetilde{Z}$ at all α values degenerate to the same point, then the total transportation cost is a crisp number. Otherwise, it is a fuzzy number. In Eq. (5), several membership functions are involved. To derive $\mu_{\tilde{Z}}$ in closed form is hardly possible. According to (5), $\mu_{\tilde{Z}}$ is the minimum of $\mu_{\tilde{C}_{ij}}$, $\mu_{\tilde{S}_i}$, and $\mu_{\tilde{D}_j}$, $\forall i, j$. We need $\mu_{\tilde{C}_{ij}}(c_{ij}) \geqslant \alpha$, $\mu_{\tilde{S}_i}(s_i) \geqslant \alpha$, $\mu_{\tilde{D}_j}(d_j) \geqslant \alpha$, and at least one $\mu_{\tilde{C}_{ij}}(c_{ij})$, $\mu_{\tilde{S}_i}(s_i)$, or $\mu_{\tilde{D}_j}(d_j)$, $\forall i, j$, equal to α such that $z = Z(\boldsymbol{c}, \boldsymbol{s}, \boldsymbol{d})$ to satisfy $\mu_{\tilde{Z}}(z) = \alpha$. To find the membership function $\mu_{\tilde{Z}}$, it suffices to find the left shape function and right shape function of $\mu_{\tilde{Z}}$, which is equivalent to finding the lower bound Z^{L}_{α} and upper bound Z^{U}_{α} of the α-cuts of $\widetilde{Z}$. Since Z^{L}_{α} is the minimum of $Z(\boldsymbol{c}, \boldsymbol{s}, \boldsymbol{d})$ and Z^{U}_{α} is the maximum of $Z(\boldsymbol{c}, \boldsymbol{s}, \boldsymbol{d})$, they can be expressed as:

$$Z_\alpha^{\mathrm{L}} = \min\{Z(\boldsymbol{c},\boldsymbol{s},\boldsymbol{d}) | (C_{ij})_\alpha^{\mathrm{L}} \leqslant c_{ij} \leqslant (C_{ij})_\alpha^{\mathrm{U}}, (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}}, (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}}, \forall i,j\}$$

$$Z_\alpha^{\mathrm{U}} = \max\{Z(\boldsymbol{c},\boldsymbol{s},\boldsymbol{d}) | (C_{ij})_\alpha^{\mathrm{L}} \leqslant c_{ij} \leqslant (C_{ij})_\alpha^{\mathrm{U}}, (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}}, (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}}, \forall i,j\}$$

which can be reformulated as the following pair of two-level mathematical programs:

$$Z_\alpha^{\mathrm{L}} = \min_{\substack{(C_{ij})_\alpha^{\mathrm{L}} \leqslant c_{ij} \leqslant (C_{ij})_\alpha^{\mathrm{U}} \\ (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}} \\ (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}} \\ \forall i,j}} \left\{ \begin{array}{lll} \min & \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} & \\ \text{s.t.} & \displaystyle\sum_{j=1}^{n} x_{ij} \geqslant s_i, & i = 1,\ldots,m, \\ & \displaystyle\sum_{i=1}^{m} x_{ij} \leqslant d_j, & j = 1,\ldots,n \\ & x_{ij} \geqslant 0, & \forall i,j. \end{array} \right. \tag{6a}$$

$$Z_\alpha^{\mathrm{U}} = \max_{\substack{(C_{ij})_\alpha^{\mathrm{L}} \leqslant c_{ij} \leqslant (C_{ij})_\alpha^{\mathrm{U}} \\ (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}} \\ (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}} \\ \forall i,j}} \left\{ \begin{array}{lll} \min & \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} & \\ \text{s.t.} & \displaystyle\sum_{j=1}^{n} x_{ij} \leqslant s_i, & i = 1,\ldots,m, \\ & \displaystyle\sum_{i=1}^{m} x_{ij} \geqslant d_j, & j = 1,\ldots,n \\ & x_{ij} \geqslant 0, & \forall i,j. \end{array} \right. \tag{6b}$$

At least one c_{ij}, s_i, or d_j must hit the boundary of their α-cuts to satisfy $\mu_{\tilde{Z}}(z) = \alpha$.

A necessary and sufficient condition for Model (6) to have feasible solutions is $\sum_{i=1}^{m} s_i \geqslant \sum_{j=1}^{n} d_j$. In the first level of Model (6), s_i and d_j are allowed to vary in the range of $[(S_i)_\alpha^{\mathrm{L}}, (S_i)_\alpha^{\mathrm{U}}]$ and $[(D_j)_\alpha^{\mathrm{L}}, (D_j)_\alpha^{\mathrm{U}}]$, respectively. However, to ensure the transportation problem of the second level to be feasible, it is necessary that the constraint $\sum_{i=1}^{m} s_i \geqslant \sum_{j=1}^{n} d_j$ be imposed in the first level. Hence, Model (6) becomes:

$$Z_\alpha^{\mathrm{L}} = \min_{\substack{(C_{ij})_\alpha^{\mathrm{L}} \leqslant c_{ij} \leqslant (C_{ij})_\alpha^{\mathrm{U}} \\ (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}} \\ (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}} \\ \sum_{i=1}^{m} s_i \geqslant \sum_{j=1}^{n} d_j \\ \forall i,j}} \left\{ \begin{array}{lll} \min & \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} & \\ \text{s.t.} & \displaystyle\sum_{j=1}^{n} x_{ij} \geqslant s_i, & i = 1,\ldots,m, \\ & \displaystyle\sum_{i=1}^{m} x_{ij} \leqslant d_j, & j = 1,\ldots,n \\ & x_{ij} \geqslant 0, & \forall i,j. \end{array} \right. \tag{7a}$$

$$Z_\alpha^{\mathrm{U}} = \max_{\substack{(C_{ij})_\alpha^{\mathrm{L}} \leqslant c_{ij} \leqslant (C_{ij})_\alpha^{\mathrm{U}} \\ (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}} \\ (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}} \\ \sum_{i=1}^{m} s_i \geqslant \sum_{j=1}^{n} d_j \\ \forall i,j}} \left\{ \begin{array}{lll} \min & \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} & \\ \text{s.t.} & \displaystyle\sum_{j=1}^{n} x_{ij} \leqslant s_i, & i = 1,\ldots,m, \\ & \displaystyle\sum_{i=1}^{m} x_{ij} \leqslant d_j, & j = 1,\ldots,n \\ & x_{ij} \leqslant 0, & \forall i,j. \end{array} \right. \tag{7b}$$

Model (7) will be infeasible for any α level if $\sum_{i=1}^{m} (S_i)_{\alpha=0}^{\mathrm{U}} \leqslant \sum_{j=1}^{n} (D_j)_{\alpha=0}^{\mathrm{L}}$. In other words, a fuzzy transportation problem is feasible if the upper bound of the total fuzzy supply is greater than or equal to the lower bound of the total fuzzy demand.

To derive the lower bound of the objective value in Model (7a), we can directly set c_{ij} to its lower bound $(C_{ij})_\alpha^{\mathrm{L}}$, $\forall i,j$, to find the minimum objective value. Hence, Model (7a) can be reformulated as:

$$Z_\alpha^{\mathrm{L}} = \min_{\substack{(S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}} \\ (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}} \\ \sum_{i=1}^m s_i \geqslant \sum_{j=1}^n d_j \\ \forall i,j}} \begin{cases} \min & \sum_{i=1}^m \sum_{j=1}^n (C_{ij})_\alpha^{\mathrm{L}} x_{ij} \\ \text{s.t.} & \sum_{j=1}^n x_{ij} \geqslant s_i, \qquad i = 1,\ldots,m, \\ & \sum_{i=1}^m x_{ij} \leqslant d_j, \qquad j = 1,\ldots,n \\ & x_{ij} \geqslant 0, \qquad \forall i,j. \end{cases} \tag{8}$$

Since Model (8) is to find the minimum of all the minimum objective values, one can insert the constraints of level 1 into level 2 and simplify the two-level mathematical program to the conventional one-level program as follows:

$$\begin{aligned} Z_\alpha^{\mathrm{L}} = \min \quad & \sum_{i=1}^m \sum_{j=1}^n (C_{ij})_\alpha^{\mathrm{L}} x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^n x_{ij} \leqslant s_i, \quad i = 1,\ldots,m, \\ & \sum_{i=1}^m x_{ij} \geqslant d_j, \quad j = 1,\ldots,n, \\ & \sum_{i=1}^m s_i \geqslant \sum_{j=1}^n d_j, \\ & (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}}, \quad i = 1,\ldots,m, \\ & (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}}, \quad j = 1,\ldots,n, \\ & x_{ij} \geqslant 0, \quad \forall i,j. \end{aligned} \tag{9}$$

This model is a linear program which can be solved easily. In this model, since all c_{ij} have been set to the lower bounds of their α-cuts, that is, $\mu_{\tilde{C}_{ij}}(c_{ij}) = \alpha$, this assures $\mu_{\tilde{Z}}(z) = \alpha$ as required by (5).

To solve Model (7b), the dual of the level 2 problem is formulated to become a maximization problem to be consistent with the maximization operation of level 1. It is well-known from the duality theorem of linear programming that the primal model and the dual model have the same objective value. Thus, Model (7b) becomes:

$$Z_\alpha^{\mathrm{U}} = \max_{\substack{(C_{ij})_\alpha^{\mathrm{L}} \leqslant c_{ij} \leqslant (C_{ij})_\alpha^{\mathrm{U}} \\ (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}} \\ (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}} \\ \sum_{i=1}^m s_i \geqslant \sum_{j=1}^n d_j \\ \forall i,j}} \begin{cases} \max & -\sum_{i=1}^m s_i u_i + \sum_{j=1}^n d_j v_j \\ \text{s.t.} & -u_i + v_j \leqslant c_{ij}, \qquad i = 1,\ldots,m, \quad j = 1,\ldots,n, \\ & u_i, v_j \geqslant 0, \qquad \forall i,j. \end{cases} \tag{10}$$

Since $(C_{ij})_\alpha^{\mathrm{L}} \leqslant c_{ij} \leqslant (C_{ij})_\alpha^{\mathrm{U}}$, $\forall i,j$, in Model (10), one can derive the upper bound of the objective value by setting c_{ij} to its upper bound $(C_{ij})_\alpha^{\mathrm{U}}$, $\forall i,j$, because this gives the largest feasible region. Thus, we can re-formulate Model (10) as:

$$Z_\alpha^{\mathrm{U}} = \max_{\substack{(S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}} \\ (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}} \\ \sum_{i=1}^m s_i \geqslant \sum_{j=1}^n d_j \\ \forall i,j}} \begin{cases} \max & -\sum_{i=1}^m s_i u_i + \sum_{j=1}^n d_j v_j \\ \text{s.t.} & -u_i + v_j \leqslant (C_{ij})_\alpha^{\mathrm{U}}, \quad i = 1,\ldots,m, \quad j = 1,\ldots,n, \\ & u_i, v_j \geqslant 0, \qquad \forall i,j. \end{cases} \tag{11}$$

Now, since both level 1 and level 2 perform the same maximization operation, their constraints can be combined to form the following one-level mathematical program:

$$\begin{aligned} Z_\alpha^{\mathrm{U}} = \max \quad & -\sum_{i=1}^{m} s_i u_i + \sum_{j=1}^{n} d_j v_j \\ \text{s.t} \quad & -u_i + v_j \leqslant (C_{ij})_\alpha^{\mathrm{U}}, \quad i = 1,\ldots,m, \quad j = 1,\ldots,n, \\ & \sum_{i=1}^{m} s_i \geqslant \sum_{j=1}^{n} d_j \\ & (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}}, \quad i = 1,\ldots,m, \\ & (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}}, \quad j = 1,\ldots,n, \\ & u_i, v_j \geqslant 0, \quad \forall i,j. \end{aligned} \tag{12}$$

This model is a linearly constrained nonlinear program. There are several effective and efficient methods for solving this problem [11]. Similar to Model (9) since all c_{ij} have been set to the upper bounds of their α-cuts, that is, $\mu_{\tilde{C}_{ij}}(c_{ij}) = \alpha$, this assures $\mu_{\tilde{Z}}(z) = \alpha$ as required by (5).

Problems (7a) and (7b) are assured to be feasible if the lower bound of the total fuzzy demand is smaller than the upper bound of the total fuzzy supply, i.e., $\sum_{j=1}^{n} (D_j)_{\alpha=0}^{\mathrm{L}} \leqslant \sum_{i=1}^{m} (S_i)_{\alpha=0}^{\mathrm{U}}$. If this condition is not satisfied, then the problem will be infeasible. In this case, a fictitious supply point $m+1$ with an amount of $s_{m+1} \geqslant \sum_{j=1}^{m} (D_j)_{\alpha=0}^{\mathrm{L}} - \sum_{i=1}^{m} (S_i)_{\alpha=0}^{\mathrm{U}}$ just like the conventional transportation problem can be assumed to make the problem feasible. The amount to be shipped from the fictitious supply point is the shortage of that demand point.

For two possibility levels α_1 and α_2 such that $0 < \alpha_2 < \alpha_1 \leqslant 1$, the feasible regions defined by α_1 in Models (9) and (12) are smaller than those defined by α_2. Consequently, $(Z)_{\alpha_1}^{\mathrm{L}} \geqslant (Z)_{\alpha_2}^{\mathrm{L}}$ and $(Z)_{\alpha_1}^{\mathrm{U}} \leqslant (Z)_{\alpha_2}^{\mathrm{U}}$; in other words, the left shape function is nondecreasing and the right shape function is nonincreasing. This property, based on the definition of "convex fuzzy set" [17], assures the convexity of $\widetilde{Z}$. If both Z_α^{L} and Z_α^{U} are invertible with respect to α, then a left shape function $L(z) = (Z_\alpha^{\mathrm{L}})^{-1}$ and a right shape function $R(z) = (Z_\alpha^{\mathrm{U}})^{-1}$ can be obtained. From $L(z)$ and $R(z)$, the membership function $\mu_{\tilde{Z}}$ is constructed as:

$$\mu_{\tilde{z}} = \begin{cases} L(z), & (Z)_{\alpha=0}^{\mathrm{L}} \leqslant z \leqslant (Z)_{\alpha=1}^{\mathrm{L}} \\ 1, & (Z)_{\alpha=1}^{\mathrm{L}} \leqslant z \leqslant (Z)_{\alpha=1}^{\mathrm{U}} \\ R(z), & (Z)_{\alpha=1}^{\mathrm{U}} \leqslant z \leqslant (Z)_{\alpha=0}^{\mathrm{U}}. \end{cases} \tag{13}$$

In most cases, the values of Z_α^{L} and Z_α^{U} may not be solved analytically. However, the numerical solutions for Z_α^{L} and Z_α^{U} at different possibility level α can be collected to approximate the shapes of $L(z)$ and $R(z)$.

Example 1. To illustrate the proposed approach, consider a transportation problem with one fuzzy shipping cost, two fuzzy supplies, and three fuzzy demands. Supply 1 and Demand 3 are triangular fuzzy numbers and the remainders are trapezoidal fuzzy numbers. The problem has the following form:

$$\begin{aligned} \widetilde{Z} = \min \quad & 10x_{11} + 50x_{12} + 80x_{13} + (60,70,80,90)x_{21} + 60x_{22} + 20x_{23} \\ \text{s.t.} \quad & x_{11} + x_{12} + x_{13} \leqslant (70,90,100) \\ & x_{21} + x_{22} + x_{23} \leqslant (40,60,70,80) \\ & x_{11} + x_{21} \geqslant (30,40,50,70) \\ & x_{12} + x_{22} \geqslant (20,30,40,50) \\ & x_{13} + x_{23} \geqslant (40,50,80) \\ & x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geqslant 0. \end{aligned}$$

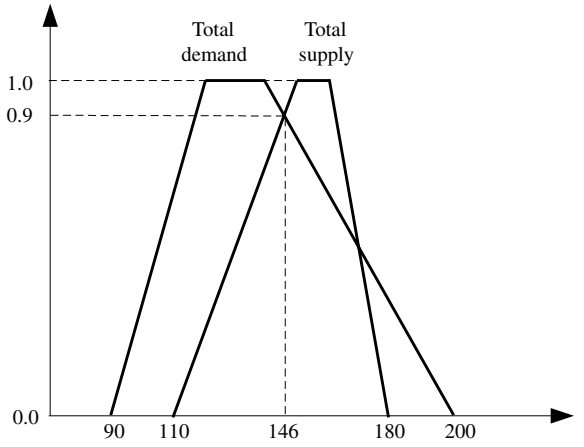

Fig. 1. Total supply and total demand of Examples 1 and 2.

The total supply is $\widetilde{S}_1 + \widetilde{S}_2 = (110, 150, 160, 180)$ and the total demand is $\widetilde{D}_1 + \widetilde{D}_2 + \widetilde{D}_3 = (90, 120, 140, 200)$ as shown in Fig. 1. The spread of the total fuzzy supply overlaps with the spread of the total fuzzy demand. In other words, the upper bound of the total fuzzy supply is greater than the lower bound of the total fuzzy demand, implying that the problem is feasible. According to Models (9) and (12), the lower and upper bounds of $\widetilde{Z}$ at possibility level α can be solved as:

$$
\begin{aligned}
Z_\alpha^{\mathrm{L}} = \min \quad & 10x_{11} + 50x_{12} + 80x_{13} + (60 + 10\alpha)x_{21} + 60x_{22} + 20x_{23} \\
\text{s.t.} \quad & x_{11} + x_{12} + x_{13} \leqslant s_1 \\
& x_{21} + x_{22} + x_{23} \leqslant s_2 \\
& x_{11} + x_{21} \geqslant d_1 \\
& x_{12} + x_{22} \geqslant d_2 \\
& x_{13} + x_{23} \geqslant d_3 \\
& s_1 + s_2 \geqslant d_1 + d_2 + d_3 \\
& 70 + 20\alpha \leqslant s_1 \leqslant 100 - 10\alpha, \quad 40 + 20\alpha \leqslant s_2 \leqslant 80 - 10\alpha, \\
& 30 + 10\alpha \leqslant d_1 \leqslant 70 - 20\alpha, \quad 20 + 10\alpha \leqslant d_2 \leqslant 50 - 10\alpha, \\
& 40 + 10\alpha \leqslant d_3 \leqslant 80 - 30\alpha, \\
& x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geqslant 0.
\end{aligned}
$$

$$
\begin{aligned}
Z_\alpha^{\mathrm{U}} = \max \quad & -s_1u_1 - s_2u_2 + d_1v_1 + d_2v_2 + d_3v_3 \\
\text{s.t.} \quad & -u_1 + v_1 \leqslant 10 \\
& -u_1 + v_2 \leqslant 50 \\
& -u_1 + v_3 \leqslant 80 \\
& -u_2 + v_1 \leqslant (90 - 10\alpha) \\
& -u_2 + v_2 \leqslant 60 \\
& -u_2 + v_3 \leqslant 20 \\
& s_1 + s_2 \geqslant d_1 + d_2 + d_3 \\
& 70 + 20\alpha \leqslant s_1 \leqslant 100 - 10\alpha, \quad 40 + 20\alpha \leqslant s_2 \leqslant 80 - 10\alpha, \\
& 30 + 10\alpha \leqslant d_1 \leqslant 70 - 20\alpha, \quad 20 + 10\alpha \leqslant d_2 \leqslant 50 - 10\alpha, \\
& 40 + 10\alpha \leqslant d_3 \leqslant 80 - 30\alpha, \\
& u_1, u_2, v_1, v_2, v_3 \geqslant 0.
\end{aligned}
$$

Table 1
The α-cuts of the total transportation cost at 11 α values for Example 1

α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Z_α^L	2100	2180	2260	2340	2420	2500	2580	2660	2740	2820	2900
Z_α^U	5800	5600	5400	5200	5000	4800	4440	4080	3860	3680	3500

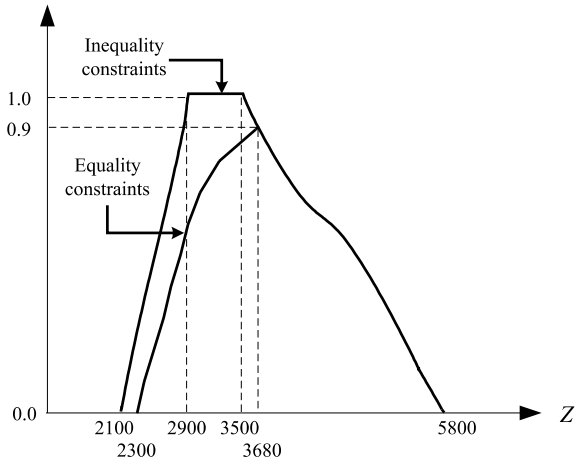

Fig. 2. The membership functions of $\widetilde{Z}$ of Examples 1 and 2.

A mathematical programming solver Lingo [8] is used to solve the above mathematical programs. Table 1 lists the α-cuts of the total transportation cost at eleven distinct α values: $0, 0.1, 0.2, \ldots, 1.0$. The α-cut of $\widetilde{Z}$ represents the possibility that the transportation cost will appear in the associated range. Specifically, the $\alpha = 1.0$ cut shows the total transportation cost that is most likely to be and the $\alpha = 0$ cut shows the range that the total transportation cost could appear. In this example, while the total transportation cost is fuzzy, its most likely value falls between 2900 and 3500, and its value is impossible to fall outside the range of 2100 and 5800. The curve labeled "Inequality-constraints" in Fig. 2 is the membership function $\mu_{\widetilde{Z}}$ of this example.

For the $\alpha = 0$ cut of $\widetilde{Z}$, the lower bound of $Z^* = 2100$ occurs at $x_{11}^* = 30$, $x_{12}^* = 20$, $x_{23}^* = 40$, $x_{13}^* = x_{21}^* = x_{22}^* = 0$ with $d_1 = 30$, $d_2 = 20$, $d_3 = 40$ and $s_1 = 70$, $s_2 = 40$. The upper bound of $Z^* = 5800$ occurs at $x_{11}^* = 30$, $x_{12}^* = 30$, $x_{13}^* = 40$, $x_{23}^* = 40$, $x_{21}^* = x_{22}^* = 0$ with $d_1 = 30$, $d_2 = 30$, $d_3 = 80$ and $s_1 = 100$, $s_2 = 40$. At the other extreme end of $\alpha = 1$, the lower bound of $Z^* = 2900$ occurs at $x_{11}^* = 40$, $x_{12}^* = 30$, $x_{23}^* = 50$, $x_{13}^* = x_{21}^* = x_{22}^* = 0$ with $d_1 = 40$, $d_2 = 30$, $d_3 = 50$ and $s_1 = 70$, $s_2 = 50$. The upper bound of $Z^* = 3500$ occurs at $x_{11}^* = 50$, $x_{12}^* = 40$, $x_{23}^* = 50$, $x_{13}^* = x_{21}^* = x_{22}^* = 0$ with $d_1 = 50$, $d_2 = 40$, $d_3 = 50$ and $s_1 = 90$, $s_2 = 50$. Notably, the total transportation cost associated with the largest total quantity being shipped need not be the highest. In this example, the largest possible amount to be shipped is 180, which is the largest total supply. The total transportation cost for this amount is $Z^* = 4600$. However, as noted previously, the highest cost is $Z^* = 5800$, occurring at the total amount of 140. This may explain why the methods of Julien [7] and Parra et al. [10] are not suitable for the fuzzy transportation problem.

4. Equality constraints

In the preceding section, the transportation model being considered has inequality constraints. The total supply must be greater than or equal to the total demand to assure feasibility. In this section, we discuss the transportation model with equality constraints:

$$\begin{aligned} Z = \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1,\ldots,m, \\ & \sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1,\ldots,n, \\ & x_{ij} \geqslant 0, \quad \forall i,j. \end{aligned} \tag{14}$$

This model is feasible if and only if $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$.

When the shipping costs, supplies, and demands are not known exactly, we have the following fuzzy transportation problem:

$$\begin{aligned} \widetilde{Z} = \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} \widetilde{C}_{ij}x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} = \widetilde{S}_i, \quad i = 1,\ldots,m, \\ & \sum_{i=1}^{m} x_{ij} = \widetilde{D}_j, \quad j = 1,\ldots,n, \\ & x_{ij} \geqslant 0, \quad \forall i,j. \end{aligned} \tag{15}$$

Similar to the discussion of the inequality-constraint case, the lower and upper bounds of $\widetilde{Z}$ at possibility level α can be solved from the following pair of two-level mathematical programs:

$$Z_\alpha^{\mathrm{L}} = \min_{\substack{(C_{ij})_\alpha^{\mathrm{L}} \leqslant c_{ij} \leqslant (C_{ij})_\alpha^{\mathrm{U}} \\ (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}} \\ (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}} \\ \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j \\ \forall i,j}} \left\{ \begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1,\ldots,m, \\ & \sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1,\ldots,n \\ & x_{ij} \geqslant 0, \quad \forall i,j. \end{aligned} \right. \tag{16a}$$

$$Z_\alpha^{\mathrm{U}} = \max_{\substack{(C_{ij})_\alpha^{\mathrm{L}} \leqslant c_{ij} \leqslant (C_{ij})_\alpha^{\mathrm{U}} \\ (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}} \\ (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}} \\ \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j \\ \forall i,j}} \left\{ \begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1,\ldots,m, \\ & \sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1,\ldots,n \\ & x_{ij} \geqslant 0, \quad \forall i,j. \end{aligned} \right. \tag{16b}$$

The corresponding pair of one-level mathematical programs is:

$$
\begin{aligned}
Z_\alpha^{\mathrm{L}} = \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} (C_{ij})_\alpha^{\mathrm{L}} x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1,\ldots,m, \\
& \sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1,\ldots,n, \\
& \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j, \\
& (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}}, \quad i = 1,\ldots,m, \\
& (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}}, \quad j = 1,\ldots,n, \\
& x_{ij} \geqslant 0, \quad \forall i,j.
\end{aligned} \tag{17a}
$$

$$
\begin{aligned}
Z_\alpha^{\mathrm{U}} = \max \quad & \sum_{i=1}^{m} s_i u_i + \sum_{j=1}^{n} d_j v_j \\
\text{s.t.} \quad & u_i + v_j \leqslant (C_{ij})_\alpha^{\mathrm{U}} \\
& \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j, \\
& (S_i)_\alpha^{\mathrm{L}} \leqslant s_i \leqslant (S_i)_\alpha^{\mathrm{U}}, \quad i = 1,\ldots,m, \\
& (D_j)_\alpha^{\mathrm{L}} \leqslant d_j \leqslant (D_j)_\alpha^{\mathrm{U}}, \quad j = 1,\ldots,n, \\
& u_i, v_j \text{ unrestricted in sign}, \quad \forall i,j.
\end{aligned} \tag{17b}
$$

The lower and upper bounds of the total transportation cost at α level can be obtained by solving Model (17). The α-level sets $[Z_\alpha^{\mathrm{L}}, Z_\alpha^{\mathrm{U}}]$ of $\widetilde{Z}$ at different possibility levels constitute the membership function $\mu_{\widetilde{Z}}$.

Example 2. Consider Example 1. Suppose the inequality constraints are replaced by equality constraints:

$$
\begin{aligned}
\widetilde{Z} = \min \quad & 10x_{11} + 50x_{12} + 80x_{13} + (60,70,80,90)x_{21} + 60x_{22} + 20x_{23} \\
\text{s.t.} \quad & x_{11} + x_{12} + x_{13} = (70,90,100) \\
& x_{21} + x_{22} + x_{23} = (40,60,70,80) \\
& x_{11} + x_{21} = (30,40,50,70) \\
& x_{12} + x_{22} = (20,30,40,50) \\
& x_{13} + x_{23} = (40,50,80) \\
& x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geqslant 0.
\end{aligned}
$$

Based on Model (17), the lower and upper bounds of the α-cut of $\widetilde{Z}$ can be derived by solving the following pair of mathematical programs:

$$
\begin{aligned}
Z_\alpha^{\mathrm{L}} = \min \quad & 10x_{11} + 50x_{12} + 80x_{13} + (60 + 10\alpha)x_{21} + 60x_{22} + 20x_{23} \\
\text{s.t.} \quad & x_{11} + x_{12} + x_{13} = s_1 \\
& x_{21} + x_{22} + x_{23} = s_2 \\
& x_{11} + x_{21} = d_1 \\
& x_{12} + x_{22} = d_2 \\
& x_{13} + x_{23} = d_3 \\
& s_1 + s_2 = d_1 + d_2 + d_3 \\
& 70 + 20\alpha \leqslant s_1 \leqslant 100 - 10\alpha, \quad 40 + 20\alpha \leqslant s_2 \leqslant 80 - 10\alpha, \\
& 30 + 10\alpha \leqslant d_1 \leqslant 70 - 20\alpha, \quad 20 + 10\alpha \leqslant d_2 \leqslant 50 - 10\alpha, \\
& 40 + 10\alpha \leqslant d_3 \leqslant 80 - 30\alpha, \\
& x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geqslant 0.
\end{aligned}
$$

$$
\begin{aligned}
Z_\alpha^{\mathrm{U}} = \max \quad & s_1u_1 + s_2u_2 + d_1v_1 + d_2v_2 + d_3v_3 \\
\text{s.t.} \quad & u_1 + v_1 \leqslant 10 \\
& u_1 + v_2 \leqslant 50 \\
& u_1 + v_3 \leqslant 80 \\
& u_2 + v_1 \leqslant (90 - 10\alpha) \\
& u_2 + v_2 \leqslant 60 \\
& u_2 + v_3 \leqslant 20 \\
& s_1 + s_2 = d_1 + d_2 + d_3 \\
& 70 + 20\alpha \leqslant s_1 \leqslant 100 - 10\alpha, \quad 40 + 20\alpha \leqslant s_2 \leqslant 80 - 10\alpha, \\
& 30 + 10\alpha \leqslant d_1 \leqslant 70 - 20\alpha, \quad 20 + 10\alpha \leqslant d_2 \leqslant 50 - 10\alpha, \\
& 40 + 10\alpha \leqslant d_3 \leqslant 80 - 30\alpha, \\
& u_1, u_2, v_1, v_2, v_3 \text{ unrestricted in sign.}
\end{aligned}
$$

Table 2 lists the bounds of the total transportation cost at eleven α-cuts. The curve labeled "Equality-constraints" in Fig. 2 is the membership function $\mu_{\tilde{Z}}$ of this example. For α greater than 0.9, the problem is infeasible. In other words, the maximum degree to which the constraints could be satisfied is equal to 0.9. It is also worthwhile to note that the membership function of the objective value of this example is contained in that of Example 1. The reason is simply because equality constraints are more restrictive than inequality constraints.

At $\alpha = 0$, the lower bound of the objective value is 2300, occurring at $x_{11}^* = 50$, $x_{12}^* = 20$, $x_{23}^* = 40$, $x_{13}^* = x_{21}^* = x_{22}^* = 0$ with $d_1 = 50$, $d_2 = 20$, $d_3 = 40$ and $s_1 = 70$, $s_2 = 40$. The upper bound is 5800, which occurs at $x_{11}^* = 30$, $x_{12}^* = 30$, $x_{13}^* = 40$, $x_{23}^* = 40$, $x_{21}^* = x_{22}^* = 0$ with $d_1 = 30$, $d_2 = 30$, $d_3 = 80$ and $s_1 = 100$, $s_2 = 40$. At $\alpha = 0.9$, the α-cut is a single point 3680. The associated optimal solution is $x_{11}^* = 52$, $x_{12}^* = 36$, $x_{22}^* = 5$, $x_{23}^* = 53$, $x_{13}^* = x_{21}^* = 0$ with $d_1 = 52$, $d_2 = 41$, $d_3 = 53$ and $s_1 = 88$, $s_2 = 58$. Note that this point

Table 2
The α-cuts of the total transportation cost at 11 α values for Example 2

α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Z_α^{L}	2300	2400	2500	2600	2700	2800	2900	3040	3260	3680	Infeasible
Z_α^{U}	5800	5600	5400	5200	5000	4800	4440	4080	3860	3680	Infeasible

corresponds to the intersection of the left shape function of the total supply and the right shape function of the total demand, which also has a membership degree of 0.9.

5. Conclusion

Transportation models have wide applications in logistics and supply chain for reducing the cost. Some previous studies have devised solution procedures for fuzzy transportation problems. The objective values derived from those studies are crisp values, rather than fuzzy numbers. This paper develops a method to find the membership function of the fuzzy total transportation cost when the unit shipping costs, the supply quantities, and the demand quantities are fuzzy numbers. The idea is based on Zadeh's extension principle to transform the fuzzy transportation problem to a pair of mathematical programs. By enumerating different α values, the lower and upper bounds of the α-cuts of the fuzzy objective value are calculated to approximate the membership function.

Two different types of the fuzzy transportation problem are discussed: one with inequality constraints and the other with equality constraints. Since equality constraints are more restrictive than inequality constraints, the membership function of the objective value of equality problem is contained in that of the inequality problem. Interestingly, due to the structure of the transportation problem, the highest total transportation cost may not occur at the largest total quantity transported.

In real world applications, the parameters in the transportation problem may not be known precisely due to uncontrollable factors. If the obtained results are crisp values, then it might lose some helpful information. Since the objective value is expressed by the membership function rather than by a crisp value, more information is provided for making decisions.

In this study the membership function is derived numerically. No mathematical form is provided. Consequently, the membership degree of a specific transportation cost must be approximated from the α-cuts. A challenging task is to derive the mathematical form of the membership function so that the membership degree can be calculated directly.

Acknowledgements

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