On inscribing of big closed or G_δ sets into big Suslin sets

Luděk Zajíček, Prague

There exists a number of versions of the notion of porosity (of a set at a point). Most of them are special cases of a simple abstract notion, called "abstract porosity" or "porosity-like point-set relation". The recent articles [6], [7], [8] consider the question, which simple properties of an abstract porosity ensure that some theorem, which is known for the ordinary porosity, holds. In particular, [7] and [8] concern the possibility of inscribing of a compact (or closed) non-σ-porous set into any Suslin non-σ-porous set (i.e., the validity of the result which was proved by J. Pelant and M. Zelený for the ordinary porosity). In [8], the proof is based on a non-separable version of a theorem of Solecki [4] (which deal with inscribing of a big G_δ set into a big Suslin set). This version was proved in [2], where it was shown that Solecki's theorem (without extra assumptions) does not hold in the non-separable case.

Several results, examples, and open questions will be presented. In particular, we show that an old Kantorovich's result [3] on Dini derivatives implies (for continuous functions), via a "porosity" inscribing theorem, a result proved in [1] and [5].

References

[1] C.L. Belna, G.T. Cargo, M.J. Evans, P.D. Humke, *Analogues of the Denjoy-Young-Saks theorem*, Trans. Amer. Math. Soc. 271 (1982), 253–260.

[2] P. Holický, L. Zajíček, M. Zelený, *A remark on Solecki's theorem*, Comment. Math. Univ. Carolin. 46(2005), 43–54.

[3] L. Kantorovitch, *Sur les nombres dérivés des fonctions continues* (in Russian), Mat. Sb. 39 (1932), 153–170.

[4] S. Solecki, *Covering analytic sets by families of closed sets*, J. Symbolic Logic 59 (1994), 1022–1031.

[5] L. Zajíček, *On the symmetry of Dini derivates of arbitrary functions*, Comment. Math. Univ. Carolin. 22 (1981), 195–209.

[6] L. Zajíček, M. Zelený, *On complexity of a σ-ideal of σ-P-porous sets*, Comment. Math. Univ. Carolin. 44 (2003), 531-554.

[7] M. Zelený, L. Zajíček, *Inscribing compact non-σ-porous sets into analytic non-σ-porous sets*, Fund. Math. 185 (2005), 19–39.

[8] L. Zajíček, M. Zelený, *Inscribing closed non-σ-lower porous sets into Suslin non-σ-lower porous sets*, Abstr. Appl. Anal. 2005 (2005), 221–227.