Chapter 10

Conclusions & Future Directions

10.1 Conclusions

This thesis has attempted to develop a theoretical framework to design systems which recognize shapes, motions and identities of moving rigid objects. We have presented a random sampling based methodolgy for drawing inferences in situations with parameterizations on Cartesian products of Matrix Lie groups, the special Euclidean group $\mathbf{SE}(n)$ in particular. Tools have been developed for intrinsic optimization of Bayesian cost functions on these parametric manifolds. Stochastic flows superimposed with Markovian jumps with desired statistical properties are constructed to simulate the posterior distribution on the representation space. This paper extends the pattern theoretic techniques presented first in Grenander & Miller [38], for inferences on complex systems, to the representations on Matrix Lie groups. The algorithmic details are illustrated through specific applications in rigid object tracking and recognition.

Chapter 2 reviews the geometry of matrix Lie groups leading up to the construction of smooth invariant vector fields and the smooth flows they generate. Several basic tools for differential calculus such as the integral curves, their infinitesimal generators, and invariant volume forms for the special orthogonal group $\mathbf{SO}(n),\ n = 2, 3$ are established. The invariant volume forms provide the base or reference measures for probabilistic problem formulation. In Chapter 3 patterns are developed to represent the dynamics scenes containing multiple moving objects using the *deformable template* approach. Basic templates are designed from the surface descriptions of rigid targets and the variability in occurrence is introduced using the group operations- translation and rotation, on these rigid surface manifolds.

Chapter 4 derives the posterior probability distributions $\mu = \frac{e^{-H}}{\mathcal{Z}}$ on the representation space combining: (i) the prior on configuration space $\mathcal{C}(l, \alpha)$ based on the Newtonian description of target motion, and (ii) the likelihood of observed data based on the physical models on sensors for data generation.

Chapter 5 provides a general discussion on the jump and diffusion processes and the conditions for ergodic inferences on a given probability measure. In Chapter 6 we constructed a jump-diffusion Markov process to empirically generate statistics on the posterior μ. It was shown that the posterior μ is the unique stationary measure of the jump-diffusion Markov process resulting in an important ergodic theorem Theorem 6.4.

In Chapter 7 detailed algorithms are presented for specific situations: (i) estimation of orientation of a ground based object in $\mathbf{SO}(2)$, (ii) estimation of orientation of an airborne object in $\mathbf{SO}(3)$, (iii) tracking and identification of moving single target in $\mathcal{C} = \cup_{\alpha \in \mathcal{A}} \cup_{l=0}^{\infty} \mathcal{C}(l, \alpha)$, and (iv) detection, tracking and recognition of multiple dynamic targets using a multi-sensor system in $\mathcal{C}_{total} = \cup_{M=0}^{\infty} \mathcal{C}^M$. The results obtained from simulating these secanrios are presented in Chapter 8.

To analyze the performance of target recognition algorithms we have derived Hilbert-Schmidt bound on expected error of estimators taking values on Lie groups, $\mathbf{SO}(2)$ and $\mathbf{SO}(3)$ in particular, in Chapter 9. Results are presented from the derivation and evaluation of this bound for specific military scenarios. This bound quantifies the relative information contribution from individual sensors in a multi-sensor system.

10.2 Discussion & Future Directions

10.2.1 Application to Real Imagery

To have sufficient real observations to train the algorithms is hard to accomplish, therefore, the role of synthetic imagery in these applications is crucial. All the implementation results presented here, in Chapters 7 and 9, are based on observations simulated via the statistical models of sensors utilized. There are several widely accepted simulators in remote sensing such as XPATCH (from demaco) for high range resolution profiles and PRISM (from Michigan Tech.) for thermodynamic profile generation in FLIR imagery; both of these were utilized in the experiments described earlier. In case of visual imagery such as in robotics applications the actual sensor models are more complicated that what we have used. Variations in illuminations, shadowing, occlusions and random texture models are real issues and these factors have to be accounted for in the statistical description of the scene-sensor system.

But perhaps an important yardstick for any inference system is to test it in a real application environment. We believe that the Bayesian framework we have developed to organize the automated target recognition problem will be fruitful in practical situations although that remains to be shown.

10.2.2 Generation of Target Profile Archives

In the introduction we restricted to the situations where the complete profile of the target (not just feature points such as points, lines etc) under a sensor viewpoint can be accurately predicted. This prediction can be accomplished via sensor prediction tools (such as XPATCH, PRISM) or target signatures can be generated through actual sensors via controlled experiments. Mostly, these signatures can not be generated real time and hence generated and stored as dictionaries, before the scenes are observed for inference. The parameters determining the target signature such as its pose, illumination angle, thermodynamic profile, wavelengths of illumination signal, form a vast parameter space. The issue is how to sample this space efficiently to generate manageable archives on target profiles ?

There can be several approaches to handle this problem. One is to model the target signature as a Gaussian process, determine its covariance kernel and utilize a Karhunen-Loeve type eigen expansion to reduce the parameter space to finite dimensions. Then one can express the variability in signatures through its finite eigen functions. Another approach is to build the target libraries through the information theoretic rate distortion analysis. The basic philosophy is to sample the parameter space according to the posterior distribution in such a way that the inference is reduced to a finite sample set and the expected distortion due to this finite sampling is less than some pre-determined value, Rimoldi et al. [68].

10.2.3 Discrete Approximations to Diffusions

In chapter 5 we presented diffusions as a continuous time method of sampling from a given distribution μ_{total}, generated as solutions of the SDEs. For this discussion consider only the Euclidean component of the parameterization so the diffusions are solutions of the Langevin's SDE. In practice, for a computer implementation the discretized approximations to the SDE are made, see Section 7.5. While the continuous time processes may perform well on these distributions, the situation may be different for their discrete approximations. In particular, according to Roberts et al. [69] simple discretizations of the continuous time models may lose their convergence properties, even for simple distributions. Roberts et. al. Roberts et al. [69] have proposed two remedies to the problem: (i) a Metropolis adjusted Langevin's algorithm (MALA), and (ii) a Metropolis adjusted Langevin's truncated algorithm (MALTA). The drawback here is that these two algorithms may lose the geometric convergence properties even if present in the original Langevin's algorithm.

In our work, we present algorithms based on simple discretizations of the original SDEs basing the convergence of discretized versions on the empirical arguments. For a precise strategy any one of the above two algorithms can be used through a simple modifications of our algorithms presented in Chapter 7.

10.2.4 Rates of Convergence Issues

This thesis presents a Markov chain Monte Carlo (MCMC) type random sampling algorithm to generate statistics under a complicated probability distribution. The theoretical basis for such a construction comes from the ergodic results in Section 5.1 which are primarily asymptotic results. In certain situations, specially the military scenarios, the real time target tracking and recognition inferencing is an important requirement. Therefore, a natural question to ask is: what is the speed of convergence of the sampling algorithm, i.e. for a given $\epsilon > 0$ what is k such that

$$||P^k(x, \cdot) - \mu_{total}|| < \epsilon ,$$

where P^k is the transition density after k steps of the skeleton chain? For an arbitrary initial condition, the MCMC algorithms spend some *burn-in* period before they start sampling from the target distributions. An important question is: given a target distribution and a sampling technique (such as Langevin, Gibbs, Metropolis-Hastings etc) can one determine the burn-in period before-hand?

Currently, the answer is no unless the distribution is Gaussian or a sequence of distributions converging to Gaussian (δ-perturbations of Gaussian). In the case of this Gaussian family, one can relate the rate of convergence to the spectral radius of the covariance under the target distribution Amit [2] and also determine what sampling strategies are optimal in the sense of exponential convergence Amit [3].

In this thesis, we have completely left out the issues arising out of optimal sampling strategies depending upon the topology of the parameter space and the complexity of the target distributions. For recent results on convergence rates and related issues please refer to Roberts et. al. [70] and the references therein.

10.2.5 Cramer-Rao Type Analysis on Lie Groups

In this thesis we have presented a Hilbert-Schmidt bound on the expected error of the estimator taking values on matrix Lie groups. One can try to relate this bound to a more classical performance analysis in statistics through Cramer-Rao bounds. Hendricks [40] has derived a Cramer-Rao type information inequalities for the estimators on differentiable manifolds. Due to the curvature of underlying state space, the notion of Hessian and covariance functions has to be modified as described in Hendricks [40].

10.2.6 Jump-Diffusion Processes on Differentiable Manifolds

There are an interesting set of engineering problems which can be posed on differentiable manifolds which are not Lie groups. For example, a popular way of tracking of multiple signal sources is via subspace tracking algorithms. In this situation, the goal is estimate the moving subspace spanned by the eigen vectors corresponding to signal eigen values in the sample covariance matrix. This problem can be naturally posed on the Grassmanian manifolds, the space of r-dimensional subspace in $n \times n$ space, see e.g. Bucy [15].

We believe that the jump-diffusion methodology can be generalized even to those differentiable manifolds which may not have any group structure. We briefly illustrate this in the context of subspace tracking problem applicable in mobile communication or passive array tracking scenarios. The goal is to determine adaptively the signal subspace of a data covariance matrix which changes in time due to changing locations of the signal sources. In an N dimensional euclidean space $\mathbb{R}^N$ the moving signals generate an M-dimensional subspace ($M \leq N$) which is evolving in time. There are several ways to parameterize a subspace: any set of M orthogonal N-vectors or a unique $N \times N$ real valued projection matrix of rank M as suggested in Fuhrmann et al. [29]. Of these, we choose the second representation for reasons described in Fuhrmann et al. [29]. The space of all $N \times N$ projection matrices of rank M form a connected differentiable manifold denoted by $\mathbb{P}_{N,M}$. By definition, the projection matrices are symmetric and idempotent, i.e. for any $P \in \mathbb{P}_{N,M}$, $P = P^\dagger$ and $P = P^2$.

To allow for th variability in M the complete space is extended to be

$$\mathbb{P} = \cup_{M=0}^{N} \mathbb{P}_{N,M} \ .$$

It can be shown Fuhrmann [28] that $I\!P$ is the disjoint union of subspaces each being a connected set.

Basic Geometry

According to Fuhrmann [28], for any $N \times N$ skew-symmetric matrix A the element $Y = A \cdot P - P \cdot A$ is tangent to $I\!P^{N,M}$ at $p \in I\!P_{N,M}$, or $Y \in T_p(I\!P_{N,M})$. Furthermore, the relationship between the tangent vectors and the skew-symmetric matrices is not one-to-one, i.e. many skew-symmetric A can result in the same tangent element Y. Given a tangent element Y, one such matrix is given by $A = Y - 2P \cdot Y$. Let $A_1, A_2, \ldots, A_{N(N-1)/2}$ be the basis elements the space of skew-symmetric matrices. For any point $P \in I\!P_{N,M}$ the tangent space $T_P(I\!P_{N,M})$ is an M-dimensional vector space with the basis elements given by $Y_{i,P} = A_i \cdot P - P \cdot A_i$. Through Graam-Schmidt or a similar analysis we can derive the M orthonormal basis elements of $T_p(I\!P_{N,M})$ and name them $Y'_{i,P}$, $i = 1, 2, \ldots, M$. The vectors $Y'_{i,P}$ form smooth vector fields in neighborhood of P, but perhaps not over the entire manifold. The integral curve generated by the vector field Y'_i is given by $\xi_i(t)P = \xi_i(t, P) = e^{tA_i} \cdot P \cdot e^{-tA_i}$ (it can be verified that $\frac{d\xi_i(t)}{dt} = Y'_{i,\xi_i(t)}$). Due to lack of a group structure it may not be possible to write an SDE over the whole space, but the diffusions can be generated using these local integral curves.

Posterior

To develop a prior on the process $P(t) \in I\!P^{N,M}$ we consider a subspace as a fixed center rigid object undergoing only the rotational motion. As shown in Fuhrmann et al. [29] the system dynamics can be modeled by the equation

$$\dot{P}(t) = P(t)A(t) - A(t)P(t) \; , \tag{10.1}$$

for an $N \times N$ skew-symmetric matrix $A(t)$. The elements of $A(t)$ provide the rotational velocities of moving subspace in $I\!R^{N \times N}$. Assuming that the targets are observed at some fundamental sampling intervals, Δ, and assuming piecewise constant angular velocities between sample times a solution is given by

$$P(t + i\Delta) = e^{A_i \Delta} P(t) e^{-A_i \Delta} \; . \tag{10.2}$$

Assuming a statistical model on $A_i, i = 1, 2, \ldots, l$ (perhaps from an equation of the type $A_{i+1} = F(A_i) + w_i$, w_i being additive noise with skew-symmetric components) results in a prior probability model on the space of l-projection matrices, $I\!P^l_{N,M}$ (similar to the steps in Chapter 4. For an example of the data generation model please refer to Fuhrmann et al. [29]. Taking a Bayesian approach derive a posterior energy function $H : I\!P^l_{N,M} \to I\!R_+$ conditioned on the data collected at l-sample times.

There two types of discrete moves appropriate for this problem:

1. Jump within the subspace $I\!P_{N,M}$:

$$T^{(5)} : I\!P_{N,M} \to I\!P_{N,M}$$

2. Jump across subspaces,

$$T^{(6)} : \mathbb{P}_{N,M} \rightarrow \mathbb{P}_{N,M'}$$

   where $M \neq M'$.

Similar to the construction in Chapter 5 we can derive a jump-diffusion Markov process which samples the given posterior distribution. The effectiveness of such a stochastic gradient type procedure compared to existing eigen techniques remains to be seen.

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