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On Semi Prime Ideals in Nearlattices

[1]Md Zaidur Rahman, [1]Md Bazlar Rahman and [2]A.S.A.Noor

[1]Department of Mathematics
Khulna University of Engineering & Technology
Khulna, Bangladesh
Email: mzrahman1968@gmail.com

[2]Department of ECE
East West University, Dhaka, Bangladesh.
Email: noor@ewubd.edu

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Abstract. Recently Yehuda Rav has given the concept of Semi prime ideals in a general lattice by generalizing the notion of 0-distributive lattices. In this paper we study several properties of these ideals in a general nearlattice and include some of their characterizations. We give some results regarding maximal filters and include a number of Separation properties in a general nearlattice with respect to the annihilator ideals. We also include a Separation property for a filter disjoint to the semi prime ideal $\{x\}^{\perp_J}$.

Keywords: 0-distributive nearlattice, prime ideal, semi-prime ideal, annihilator ideal, maximal filter

AMS Mathematics Subject Classification (2010): 06A12, 06A99, 06B10

1. Introduction

The concept of 0-distributive lattices was given by J.C.Varlet [6] in generalizing the concept of pseudocomplementation . In a bounded lattices L, for an element $a \in L$, a^* is called the pseudocomplement of a if $a \wedge a^* = 0$ and for $x \in L$, $a \wedge x = 0$ implies $x \leq a^*$. In other words , the set of all elements disjoint to the element a forms a principal ideal $(a^*]$. A lattice with 0 and 1 whose every element has a pseudocomplement, is called a pseudocomplemented lattice. By Varlet, a lattices L with 0 is called 0-distributive if the set of all elements disjoint to element a form an ideal (not necessarily principal ideal). Equivalently, L with 0 is called 0-distributive if for all $a,b,c \in L$, $a \wedge b = 0 = a \wedge c$ imply $a \wedge (b \vee c) = 0$. Of course , every distributive lattice is 0-distributive. Also every pseudocomplemented lattice is 0-distributive. Dually, we can study 1-distributive lattices if the lattices have 1.

It is easy to see that Pentagonal lattice (Figure1) is 0-distributive but the Diamond lattice (Figure2) is not.

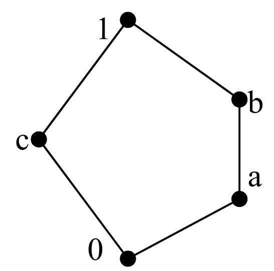

Figure 1

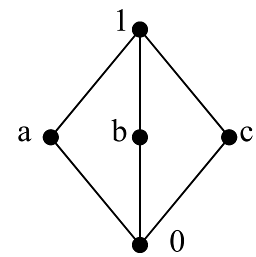

Figure 2

For detailed literature on this topic, see [1] and [4].

Recently, Y. Rav [5] has generalized this concept and gave the definition of semi prime ideals in a lattice. An ideal I of a lattice L is called a *semi prime ideal* if for all $x, y, z \in L$, $x \wedge y \in I$ and $x \wedge z \in I$ imply $x \wedge (y \vee z) \in I$. Thus, for lattice L with 0, L is called *0-distributive* if and only if $(0]$ is a semi prime ideal. In a distributive lattice L, every ideal is a semi prime ideal. Moreover, every prime ideal is semi prime. In a pentagonal lattice(Figure 1) $(0]$ is semi prime but not prime. Here (b] and (c] are prime, but (a] is not even semi prime. Again in Figure 2, $(0]$, $(a]$, $(b], (c]$ are not semi prime.

In this paper we extend this concept for nearlattices and include a number of separation properties in a general nearlattice with respect to the annihilator ideals. Moreover, by studying a congruence related to Glivenko congruence we give a separation theorem related to separation properties in distributive nearlattices given by [4].

2. Semi Prime Ideals in a Nearlattice

A *nearlattice* S is a meet semilattice with the property that any two elements possessing a common upper bound, have a supremum. This property is known as the *upper bound property.* S is called a distributive nearlattice if for all $x, y, z \in S$, $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$, provided $y \vee z$ exists. Here right hand expression exists by the upper bound property. For detailed literature on nearlattices we refer the reader to consult [2] and [3]. By [7], a nearlattice S with 0 is called *a 0-distributive nearlattice*, if for all $x, y, z \in S$, $x \wedge y = 0 = x \wedge z$ imply $x \wedge (y \vee z) = 0$, provided $y \vee z$ exists. Of course, every distributive nearlattice is 0-distributive. Since a nearlattice with 1 is a lattice (by the upper bound property), so we can not bring the idea of pseudocomplementation in a nearlattice. But [7] have proved that a nearlattices with 0 is 0-distributive if and only if the lattice of ideals I(S) is pseudocomplemented, which is also equivalent to I(S) is 0-distributive.

For a non-empty subset I of S, I is called a *down set* if for $a \in I$ and $x \le a$ imply $x \in I$. Moreover I is an *ideal* if $a \vee b \in I$ for all $a, b \in S$, provided $a \vee b$ exists. Similarly, F is called a *filter* of S if for $a, b \in F$, $a \wedge b \in F$ and for $a \in F$ and $x \ge a$ imply $x \in F$. F is called a *maximal filter* if for any filter $M \supseteq F$ implies either $M = F$ or $M = L$. A proper ideal(down set) I is called a *prime ideal(down set)* if for $a, b \in S$, $a \wedge b \in I$ imply either $a \in I$ *or* $b \in I$. A prime ideal P is called a *minimal prime ideal* if it does not contain any other prime ideal. Similarly, a proper filter Q is called a *prime filter* if $a \vee b \in Q$ $(a, b \in S)$ when $a \vee b$ exists, implies either $a \in Q$ or $b \in Q$. It is very easy to check that F is a filter of S if and only if S-F is a prime down set. Moreover, F is a prime filter if and only if S-F is a prime ideal.

An ideal I of a nearlattice S is called a *semi prime ideal* if for all $x, y, z \in L$, $x \wedge y \in I$ and $x \wedge z \in I$ imply $x \wedge (y \vee z) \in I$ provided $y \vee z$ exists. Thus, for nearlattice S with 0, S is called *0-distributive* if and only if $(0]$ is a semi prime ideal. In a distributive nearlattice S, every ideal is a semi prime ideal. Moreover, every prime ideal is semi prime. In the nearlattice of figure 3,

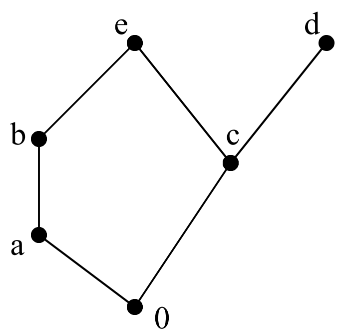

Figure 3

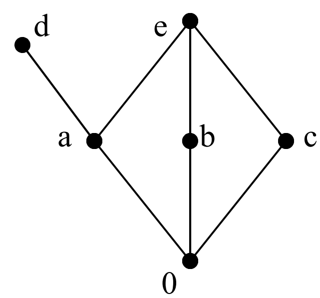

Figure 4

(b] and (d] are prime, (c] is not prime but semi prime and (a] is not even semi prime. Again in figure 4, $(0]$, $(a]$, $(b]$, $(c]$ and (d] are not semi prime.

Lemma 1. *Non empty intersection of all prime (semi prime) ideals of a nearlattice is a semi-prime ideal.*

Proof. Let $a, b, c \in S$ and $I = \bigcap \{P : P$ is a prime ideal $\}$ and I is nonempty. Let $a \wedge b \in I$ and $a \wedge c \in I$. Then $a \wedge b \in P$ and $a \wedge c \in P$ for all P. Since each P is prime (semi prime), so $a \wedge (b \vee c) \in P$ for all P. Hence $a \wedge (b \vee c) \in I$, and so I is semi-prime. ●

Corollary 2. *Intersection of two prime(semi prime) ideals is a semi-prime ideal.* ●

Lemma 3. *Every filter disjoint from an ideal I is contained in a maximal filter disjoint from I.*

Proof. Let F be a filter in L disjoint from I. Let $\mathcal{F}$ be the set of all filters containing F and disjoint from I. Then $\mathcal{F}$ is nonempty as $F \in \mathcal{F}$. Let C be a chain in $\mathcal{F}$ and let $M = \bigcup(X : X \in C)$. We claim that M is a filter. Let $x \in M$ and $y \geq x$. Then $x \in X$ for some $X \in C$. Hence $y \in X$ as X is a filter. Therefore, $y \in M$. Let $x, y \in M$. Then $x \in X$ and $y \in Y$ for some $X, Y \in C$. Since C is a chain, either $X \subseteq Y$ or $Y \subseteq X$. Suppose $X \subseteq Y$. So $x, y \in Y$. Then $x \wedge y \in Y$ and so $x \wedge y \in M$. Moreover, $M \supseteq F$. So M is a maximum element of C. Then by Zorn's Lemma, $\mathcal{F}$ has a maximal element, say $Q \supseteq F$. ●

Lemma 4. *Let I be an ideal of a nearlattice S. A filter M disjoint from I is a maximal filter disjoint from I if and only if for all $a \notin M$, there exists $b \in M$ such that $a \wedge b \in I$.*

Proof. Let M be maximal and disjoint from I and $a \notin M$. Let $a \wedge b \notin I$ for $b \in M$. Consider $M_1 = \{y \in L : y \geq a \wedge b, \quad b \in M \}$. Clearly M_1 is a filter. For any $b \in M$, $b \geq a \wedge b$ implies $b \in M_1$. So $M_1 \supseteq M$. Also $M_1 \cap I = \phi$. For if not, let $x \in M_1 \cap I$. This implies $x \in I$ and $x \geq a \wedge b$ for some $b \in M$. Hence $a \wedge b \in I$, which is a contradiction. Hence $M_1 \cap I \neq \phi$. Now $M \subset M_1$ because $a \notin M$ but $a \in M_1$. This contradicts the maximality of M. Hence there exists $b \in M$ such that $a \wedge b \in I$.

Conversely, if M is not maximal disjoint from I, then there exists a filter $N \supset M$ and disjoint with I. For any $a \in N - M$, there exists $b \in M$ such that $a \wedge b \in I$. Hence, $a,b \in N$ implies $a \wedge b \in I \cap N$, which is a contradiction. Hence M must be a maximal filter disjoint with I. ●

Let S be a nearlattice with 0. For $A \subseteq S$, We define $A^{\perp} = \{x \in L : x \wedge a = 0 \text{ for all } a \in A\}$. $A^{\perp}$ is always down set of S. Moreover, it is convex but it is not necessarily an ideal.

Theorem 5. *Let S be a 0-distributive nearlattice. Then for $A \subseteq S$, $A^{\perp} = \{x \in L : x \wedge a = 0 \text{ for all } a \in A\}$ is a semi-prime ideal.*

Proof. We have already mentioned that $A^{\perp}$ is a down set of S. Let $x, y \in A^{\perp}$ and $x \vee y$ exists. Then $x \wedge a = 0 = y \wedge a$ for all $a \in L$. Since S is 0-distributive, so $a \wedge (x \vee y) = 0$ for all $a \in A$. This implies $x \vee y \in A^{\perp}$ and so $A^{\perp}$ is an ideal.

Now let $x \wedge y \in A^{\perp}$ and $x \wedge z \in A^{\perp}$ and $y \vee z$ exists. Then $x \wedge y \wedge a = 0 = x \wedge z \wedge a$ for all $a \in A$. This implies
$(x \wedge a) \wedge y = 0 = (x \wedge a) \wedge z$ and so by 0-distributivity again,
$x \wedge a \wedge (y \vee z) = 0$ for all $a \in L$. Hence $x \wedge (y \vee z) \in A^{\perp}$ and so $A^{\perp}$ is a semi prime ideal. ●

Let $A \subseteq S$ and J be an ideal of S. We define $A^{\perp_J} = \{x \in L : x \wedge a \in J \textit{ for all } a \in A\}$. This is clearly a down set containing J. In presence of distributivity, this is an ideal. $A^{\perp_J}$ is called an annihilator of A relative to J. We denote $I_J(S)$, by the set of all ideals containing J. Of course, $I_J(S)$ is a bounded lattice with J and S as the smallest and the largest elements. If $A \in I_J(S)$, and $A^{\perp_J}$ is an ideal, then $A^{\perp_J}$ is called an annihilator ideal and it is the pseudo complement of A in $I_J(S)$.

Theorem 6. *Let A be a non-empty subset of a nearlattice S and J be an ideal of S. Then*
$A^{\perp_J} = \bigcap(P : P \textit{ is minimal prime down set containing } J \textit{ but not containing } A)$.

Proof. Suppose $X = \bigcap(P : A \not\subset P, P \textit{ is a minimal prime down set})$. Let $x \in A^{\perp_J}$. Then $x \wedge a \in J$ for all $a \in A$. Choose any P of right hand expression. Since $A \not\subset P$, there exists $z \in A$ but $z \notin P$. Then $x \wedge z \in J \subseteq P$. So $x \in P$, as P is prime. Hence $x \in X$.

Conversely, let $x \in X$. If $x \notin A^{\perp_J}$, then $x \wedge b \notin J$ for some $b \in A$. Let D = $[x \wedge b)$.

Hence D is a filter disjoint from J. Then by Lemma 3, there is a maximal filter $M \supseteq D$ but disjoint from J. Then $S - M$ is a minimal prime down set containing J. Now $x \notin S - M$ as $x \in D$ implies $x \in M$. Moreover, $A \not\subseteq S - M$ as $b \in A$, but $b \in M$ implies $b \notin S - M$, which is a contradiction to $x \in X$. Hence $x \in A^{\perp_J}$. ●

Following Theorem gives some nice characterizations semi prime ideals.

Theorem 7. *Let S be a nearlattice and J be an ideal of S. The following conditions are equivalent.*

(i) J *is semi prime.*

(ii) $\{a\}^{\perp_J} = \{x \in L : x \wedge a \in J\}$ *is a semi prime ideal containing* J.

(iii) $A^{\perp_J} = \{x \in L : x \wedge a \in J \textit{ for all } a \in A\}$ *is a semi prime ideal containing* J.

(iv) $I_J(S)$ *is pseudo complemented*

(v) $I_J(S)$ *is a 0- distributive lattice.*

(vi) Every maximal filter disjoint from J *is prime.*

Proof. (i)$\Rightarrow$(ii). $\{a\}^{\perp_J}$ is clearly a down set containing J. Now let $x, y \in \{a\}^{\perp_J}$ and $x \vee y$ exists. Then $x \wedge a \in J, y \wedge a \in J$. Since J is semi prime, so $a \wedge (x \vee y) \in J$. This implies $x \vee y \in \{a\}^{\perp_J}$, and so it is an ideal containing J. Now let $x \wedge y \in \{a\}^{\perp_J}$ and $x \wedge z \in \{a\}^{\perp_J}$ with $y \vee z$ exists. Then $x \wedge y \wedge a \in J$

and $x \wedge z \wedge a \in J$. Thus, $(x \wedge a) \wedge y \in J$ and $(x \wedge a) \wedge z \in J$. Then $(x \wedge a) \wedge (y \vee z) \in J$, as J is semi prime. This implies $x \wedge (y \vee z) \in \{a\}^{\perp_J}$, and so $\{a\}^{\perp_J}$ is semi prime.

(ii)$\Rightarrow$(iii). This is trivial by Lemma 1, as $A^{\perp_J} = \bigcap(\{a\}^{\perp_J}; a \in A)$.

(iii)$\Rightarrow$(iv). Since for any $A \in I_J(S)$, $A^{\perp_J}$ is an ideal, it is the pseudo complement of A in $I_J(S)$, so $I_J(S)$ is pseudo complemented.

(iv)$\Rightarrow$(v). This is trivial as every pseudo complemented lattice is 0-distributive.

(v) $\Rightarrow$(vi). Let $I_J(S)$ is 0-distributive. Suppose F is a maximal filter disjoint from J. Suppose $f, g \notin F$ and $f \vee g$ exists. By Lemma 5, there exist $a, b \in F$ such that $a \wedge f \in J, b \wedge g \in J$. Then $f \wedge a \wedge b \in J, g \wedge a \wedge b \in J$. Hence $(f] \wedge (a \wedge b] \subseteq J$ *and* $(g] \wedge (a \wedge b] \subseteq J$. Then
$(f \vee g] \wedge (a \wedge b] = ((f] \vee (g]) \wedge (a \wedge b] \subseteq J$, by the 0-distributive property of $I_J(S)$. Hence, $(f \vee g) \wedge a \wedge b \in J$. This implies $f \vee g \notin F$ as $F \cap J = \varphi$, and so F is prime.
(vi)$\Rightarrow$(i) Let (vi) holds. Suppose $a, b, c \in S$ with $a \wedge b \in J, a \wedge c \in J$ with $b \vee c$ exists. If $a \wedge (b \vee c) \notin J$, then $[a \wedge (b \vee c)) \cap J = \varphi$. Then by Lemma 3, there exists a maximal filter $F \supseteq [a \wedge (b \vee c))$ and disjoint from J. Then $a \in F, b \vee c \in F$. By (vi) F is prime. Hence either $a \wedge b \in F$ or $a \wedge c \in F$. In any case $J \cap F \neq \phi$, which gives a contradiction. Hence $a \wedge (b \wedge c) \in J$, and so J is semi prime. ●

Corollary 8. *In a nearlattice S, every filter disjoint to a semi-prime ideal J is contained in a prime filter.*
Proof. This immediately follows from Lemma 3 and Theorem 7. ●

Theorem 9. *If J is a semi-prime ideal of a nearlattice S and $J \neq A = \bigcap\{J_\lambda : J_\lambda$ is an ideal containing $J\}$, Then $A^{\perp_J} = \{x \in L : \{x\}^{\perp_J} \neq J\}$.*
Proof. Let $x \in A^{\perp_J}$. Then $x \wedge a \in J$ for all $a \in A$. So $a \in \{x\}^{\perp_J}$ for all $a \in A$. Then $A \subseteq \{x\}^{\perp_J}$ and so $\{x\}^{\perp_J} \neq J$ .Conversely, let $x \in S$ such that $\{x\}^{\perp_J} \neq J$. Since J is semi-prime, so $\{x\}^{\perp_j}$ is an ideal containing J. Then $A \subseteq \{x\}^{\perp_J}$, and so $A^{\perp_J} \supseteq \{x\}^{\perp_J \perp_J}$. This implies $x \in A^{\perp_J}$, which completes the proof. ●

In [1], the authors have provided a series of characterizations of 0-distributive lattices. Here we give some results on semi prime ideals related to their results for nearlattices.

Theorem 10. *Let S be a nearlattice and J be an ideal. Then the following conditions are equivalent.*
(i) J is semi-prime.
(ii) Every maximal filter of S disjoint with J is prime
(iii) Every minimal prime down set containing J is a minimal prime ideal containing J
(iv) Every filter disjoint with J is disjoint from a minimal prime ideal containing J.
(v) For each element $a \notin J$, there is a minimal prime ideal containing J but not containing a.
(vi) Each $a \notin J$ is contained in a prime filter disjoint to J.

Proof. (i)$\Leftrightarrow$(ii) follows from Theorem 7.

$(ii) \Rightarrow (iii)$. Let A be a minimal prime down set containing J. Then S-A is a maximal filter disjoint with J . Then by (ii) S-A is prime and so A is a minimal prime ideal.

$(iii) \Rightarrow (ii)$. Let F be a maximal filter disjoint with J. Then S-F is a minimal prime down set containing J. Thus by (iii), S-F is a minimal prime ideal and so F is a prime filter.

$(i) \Rightarrow (iv)$. Let F a filter of S disjoint from J. Then by Corollary 8, there is a prime filter $Q \supseteq F$ and disjoint from F.

$(iv) \Rightarrow (v)$. Let $a \in S$, $a \notin J$. Then $[a) \cap J = \varphi$. Then by (iv) there exists a minimal prime ideal A disjoint from $[a)$. Thus $a \notin A$.

$(v) \Rightarrow (vi)$. Let $a \in L$, $a \notin J$. Then by (v) there exists a minimal prime ideal P such that $a \notin P$. Implies $a \in S - P$ and S-P is a prime filter.

$(vi) \Rightarrow (i)$. Suppose J is not semi-prime . Then there exists $a, b, c \in L$ such that $a \wedge b \in J$, $a \wedge c \in J$ and $b \vee c$ exists, but $a \wedge (b \vee c) \notin J$. Then by (vi) there exists a prime filter Q disjoint from J and $a \wedge (b \vee c) \in Q$. Let $F = [a \wedge (b \vee c))$. Then $J \cap F = \varphi$ and $F \subseteq Q$. Now $a \wedge (b \vee c) \in Q$ implies $a \in Q$, $b \vee c \in Q$. Since Q is prime so either $a \wedge b \in Q$ or $a \wedge c \in Q$. This gives a contradiction to the fact that $Q \cap J = \varphi$. Therefore, $a \wedge (b \vee c) \in J$ and so J is semi-prime. ●

Now we give another characterization of semi-prime ideals with the help of Prime Separation Theorem using annihilator ideals.

Theorem 11. *Let J be an ideal in a nearlattice S. J is semi- prime if and only if for all filter F disjoint to $\{x\}^{\perp_J}$, there is a prime filter containing F disjoint to $\{x\}^{\perp_J}$.*

Proof. Using Zorn's Lemma we can easily find a maximal filter Q containing F and disjoint to $\{x\}^{\perp_J}$. We claim that $x \in Q$. If not, then $Q \vee [x) \supset Q$. By maximality of Q, $(Q \vee [x)) \cap \{x^{\perp_J}\} \neq \varphi$. If $t \in (Q \vee [x)) \cap \{x\}^{\perp_J}$, then $t \geq q \wedge x$ for some $q \in Q$ and $t \wedge x \in J$. This implies $q \wedge x \in J$ and so $q \in \{x\}^{\perp_J}$ gives a contradiction. Hence $x \in Q$.

Now, let $z \notin Q$. Then $(Q \vee [z)) \cap \{x\}^{\perp_J} \neq \varphi$. Suppose $y \in (Q \vee [z)) \cap \{x\}^{\perp_J}$ then $y \geq q_1 \wedge z$ & $y \wedge z \in J$ for some $q_1 \in Q$. This implies $q_1 \wedge x \wedge z \in J$ and $q_1 \wedge x \in Q$. Hence by Lemma 4, Q is a maximal filter disjoint to $\{x\}^{\perp_J}$. Then by Theorem 7, Q is prime.

Conversely, let $x \wedge y \in J, x \wedge z \in J$ and $y \vee z$ exists. If $x \wedge (y \vee z) \notin J$, then $y \vee z \notin \{x\}^{\perp_J}$. Thus $[y \vee z) \cap \{x\}^{\perp_J} = \varphi$. So there exists a prime filter Q containing $[y \vee z)$ and disjoint from $\{x\}^{\perp_J}$. As $y, z \in \{x\}^{\perp_J}$, *so* $y, z \notin Q$. Thus $y \vee z \notin Q$, as Q is prime. This implies $[y \vee z) \not\subset Q$, a contradiction. Hence $x \wedge (y \vee z) \in J$, and so J is semi-prime. ●

Here is another characterization of semi- prime ideals.

Theorem 12. *Let J be a semi-prime ideal of a nearlattice S and $x \in S$. Then a prime ideal P containing $\{x\}^{\perp_J}$ is a minimal prime ideal containing $\{x\}^{\perp_J}$ if and only if for $p \in P$, there exists $q \in S - P$ such that $p \wedge q \in \{x\}^{\perp_J}$.*

Proof. Let P be a prime ideal containing $\{x\}^{\perp_J}$ such that the given condition holds. Let K be a prime ideal containing $\{x\}^{\perp_J}$ such that $K \subseteq P$. Let $p \in P$. Then there is $q \in S - P$ such that $p \wedge q \in \{x\}^{\perp_J}$. Hence $p \wedge q \in K$. Since K is prime and $q \notin K$, so $p \in K$. Thus, $P \subseteq K$ and so $K = P$. Therefore, P must be a minimal prime ideal containing $\{x\}^{\perp_J}$.

Conversely, let P be a minimal prime ideal containing $\{x\}^{\perp_J}$. Let $p \in P$. Suppose for all $q \in S - P$, $p \wedge q \notin \{x\}^{\perp_J}$. Let $D = (S - P) \vee [p)$. We claim that $\{x\}^{\perp_J} \cap D = \varphi$. If not, let $y \in \{x\}^{\perp_J} \cap D$. Then $p \wedge q \leq y \in \{x\}^{\perp_J}$, which is a contradiction to the assumption. Then by Theorem 11, there exists a maximal (prime) filter $Q \supseteq D$ and disjoint to $\{x\}^{\perp_J}$. By the proof of Theorem 11, $x \in Q$. Let $M = S - Q$. Then M is a prime ideal. Since $x \in Q$, so $t \wedge x \in J \subseteq M$ implies $t \in M$ as M is prime. Thus $\{x\}^{\perp_J} \subseteq M$. Now $M \cap D = \varphi$. This implies $M \cap (S - P) = \varphi$ and hence $M \subseteq P$. Also $M \neq P$, because $p \in D$ implies $p \notin M$ but $p \in P$. Hence M is a prime ideal containing $\{x\}^{\perp_J}$ which is properly contained in P. This gives a contradiction to the minimal property of P. Therefore the given condition holds. ●

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