Bit Error Rate Estimation for Turbo Decoding

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Abstract — **We propose a method for on-line estimation of the Bit Error Rate (BER) during turbo decoding.**

I. Summary

The Gaussian assumption has been widely used in analyzing the convergence behavior of Turbo codes. In [1], this assumption is used to characterize convergence behavior in terms of the extrinsic Signal-to-Noise ratio. Density Evolution [2] found that a *symmetry* assumption in addition to the Gaussian assumption gave more accurate convergence results. Using EXIT charts [3], one can predict the BER at a given iteration, by imposing a Gaussian assumption.

Denote the log likelihood ratio (LLR) for bit k as $\lambda(k) = \log\left(\Pr\left(b_k = 1\right) / \Pr\left(b_k = 0\right)\right)$. Let $\Lambda = \{\lambda(0), \ldots, \lambda(k), \ldots \lambda(N-1)\}$ be a sequence of N LLRs. We model the LLRs unconditioned on the data bit as a mixture of two Gaussian random variables, $f_{\lambda(k)}(x) = \frac{1}{2}\mathcal{N}\left(\mu, \sigma^2\right) + \frac{1}{2}\mathcal{N}\left(-\mu, \sigma^2\right)$. By estimating μ and σ the BER can be estimated by calculating the cross over area of the conditional densities of the LLRs (from the decoder output). We considered two approaches, first assuming no relationship between μ and σ, then we impose a symmetry assumption $\sigma^2 = 2\mu$.

The log likelihood function of the sequence Λ is,

$$\ln f_\Lambda(\lambda; \mu; \sigma^2) = -N \ln\left(2\sqrt{2\pi\sigma^2}\right) - \frac{N\mu^2}{2\sigma^2} - \sum_{k=0}^{N-1} \frac{\lambda^2(k)}{2\sigma^2} + \sum_{k=0}^{N-1} \ln\left(2\cosh\left(\frac{\lambda(k)\mu}{\sigma^2}\right)\right) \quad (1)$$

Partial differentiating(1) and using the large x approximation $x\tanh(x) \approx |x|$ we derived estimates for μ and σ:

$$\hat{\mu} = \hat{E}\left[|\Lambda|\right] \quad (2)$$

$$\hat{\sigma}^2 = \hat{E}\left[|\Lambda|^2\right] - \hat{\mu}^2 \quad (3)$$

where $\hat{E}\left[|\Lambda|^2\right] = \frac{1}{N}\sum_{k=0}^{N-1}|\lambda(k)|^2$ and $\hat{E}\left[|\Lambda|\right] = \frac{1}{N}\sum_{k=0}^{N-1}|\lambda(k)|$. Note (2) has already been used (somewhat arbitrarily) for convergence analysis without the above motivation or derivations.

Using the assumption $\sigma^2 = 2\mu$ (also used in [2, 3]), from (1) the ML estimator can be determined:

$$\hat{\mu} = -1 + \sqrt{1 + \hat{E}\left[|\Lambda|^2\right]} \quad (4)$$

We found that the assumption $\sigma^2 = 2\mu$ gave a better approximation for the error tails which is more important for BER estimation. Thus using (4) to estimate μ, and $\sigma = \sqrt{2\mu}$ we find the following "Gaussian Assumption Model Maximum Likelihood" (GAMML) estimate,

$$\hat{P}_b = \frac{1}{2}\operatorname{erfc}\left(\sqrt{|\hat{\mu}|}/2\right) \quad (5)$$

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Since $\hat{P}_b$ is a function of $\hat{\mu}$ its Cramer-Rao bound (CRB) is

$$\operatorname{var}[\hat{P}_b] \geq \frac{\mu}{1+\mu}\frac{\exp(-\mu/2)}{8\pi N} \quad (6)$$

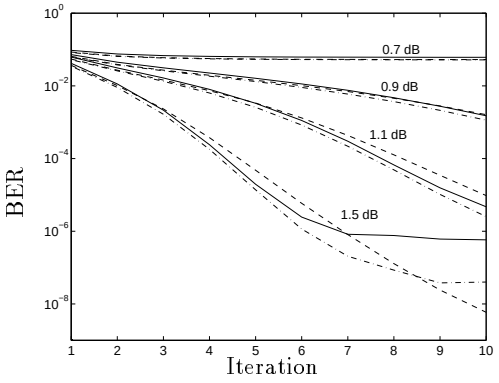

Fig. 1: Measured BER (solid) compared to GAMML (dashed) and HLS (dot-dashed) estimates for a turbo code $(G_r, G) = (7, 5)$.

We compared the GAMML estimator to that of Method Two of [4] (which we label HLS). The performance of GAMML depends on how well the LLRs fit the Gaussian model. For LLRs that perfectly fit the Gaussian mixture model (with symmetry) the GAMML estimator achieves the CRB (it is the ML estimator in this case). When APP decoding of a convolutional code, the LLRs become less Gaussian and the GAMML estimator becomes biased and strays away from the CRB but still out-performs HLS. For turbo decoding (Fig. 1), the Gaussianity of the LLRs varies between iterations and it becomes difficult to compare the performance of the two estimators. The GAMML estimator tends to have estimates closer to the average BER but suffers from a larger bias at the error floor region. HLS tends to have a higher variance but less average bias at the error floor region.

References

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