Original Article

Institution of MECHANICAL ENGINEERS
Journal of POWER AND ENERGY

Proc IMechE Part A:
J Power and Energy
226(6) 738–750

DOI: 10.1177/0957650912453536
pia.sagepub.com
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Experimental study of friction damping of blade with loosely assembled dovetail attachment

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Abstract

In order to study the friction damping characteristics of blade with loosely assembled dovetail attachment, an experimental method and a test rig are presented in this article. In the experiment, a axial load is imposed at the bottom of blade attachment to simulate the blade centrifugal force. A harmonic force is imposed on the blade as airflow force. The forced responses of a group of plane blades and a compressor blade are measured at different centrifugal forces. The results show that the resonant frequency increases with the centrifugal force, and gradually reaches the maximum. When the centrifugal force increases, the amplitude of resonant response decreases at the beginning and then increases. There is an optimal centrifugal force under which the resonant amplitude is minimal and the friction damping effect is the best. There is softening spring effect when the friction forces worked on blade system. From the results of compressor blade, it is found that when the centrifugal force is 1.5 kN, the resonant amplitude reaches the least value of 3.42 g, which is only 22.4% of the biggest value 39.88 g. This means that 1.5 kN is the optimal centrifugal force.

Keywords

Blade, friction damping, dovetail attachment, experimental method

Date received: 1 August 2011; accepted: 31 May 2012

Introduction

Blades are core components of turbomachinery. Many turbomachinery accidents are caused by blades failure. The blades are vibrated by airflow force during the process of startup, shutdown or operation. High vibratory stress is one of major reasons of blade failure. To prevent blade failure, the vibratory response needs to be attenuated to an acceptable level. Some kinds of friction dampers are employed to suppress blade vibration for providing additional damping, for example, blade-to-ground, blade-to-blade and blade dovetail-to-disk groove.

The blade with loosely assembled dovetail attachment is usually used in gas turbine and aeroengine, as shown in Figure 1. This style of blade attachment is easy to manufacture and assemble. When the blade starts up, the friction contact joints are formed between the blade dovetail attachment and disk dovetail groove because of the centrifugal force of blade. When the blade is excited to vibrate by airflow force, the interfacial friction damping arises out from the two surfaces between the blade dovetail attachment and disk moving relative to each other. In other words, there is a dry friction force on the blade dovetail attachment, which dissipates blades' vibratory energy and increase blades' damping. There are some reasons to affect the friction damping of blade, such as the centrifugal force, the angle of dovetail attachment, the area of dovetail attachment interfaces and so on.

In the past, considerable research works focused on blade vibration control by friction damping and several kinds of blade friction dampers were investigated. Griffin[1,2] reviewed the strongpoint of friction damped blade in view of dynamics analysis method, friction force model, structural model and damper optimization. Srinivasan and Cutts[3] studied the dry friction

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damping of blade shroud and platform by analytical and experimental methods. Yang and Chu,[4] proposed a friction model to characterize the contact kinematics that imposed both friction nonlinearity and intermittent separation nonlinearity on the constrained structures. Ren and Zhu[5] used theoretical and experimental approaches to research the vibration characteristics and dry friction damping effect of shrouded blades. Ji et al.[6] experimentally investigated the dynamic characteristics of damping blade by using a lacing wire damper. Xu et al.[7] calculated the forced response of a five-blade group by a macroslip friction model and finite element method. Laxalde et al.[8,9] investigated a damping strategy for bladed disks of turbomachinery involving a friction ring by a lumped parameter model of the system, and the use of friction ring dampers for integrally bladed disks by numerical and experimental method. Yang and Menq[10,11] investigated the coupling contact kinematics, the analytical criteria for stick-slip transitions, the experimental verification for the dynamic response, and their applications in design of blade wedge dampers. Petrov and Ewins[12] developed a structural model for wedge and split underplatform dampers, whose models took into account inertia forces and the effects of normal force variation on stick-slip transitions at contact interfaces. The wedge damper works between two neighboring blades. It is used as an independent part. The friction contact surfaces of wedge damper are similar to one of dovetail attachment. The dovetail attachment belongs to blade, and it works between the blade and disk. There are some researches about contact stresses and dissipation energy on the dovetail attachments. Beisheim and Sinclair[13] investigated the contact stress of dovetail attachments by three-dimensional finite element method without considering friction. Alonso et al.[14] presented a detailed study of the sensitiveness of different blade attachments to normal stress distribution along the contact face and the influences of distribution on damping results. Asai et al.[15] evaluated the friction damping in dovetail attachment joints based on dissipation energy from the experimental results and simulation results. Shangguan et al.[16] predicted the forced responses of blades with loosely assembled dovetail attachment by using a lumped parameters model and harmonic balance method. Oloffson and Hagman[17] derived a model for micro-slip between flat surfaces based on modeling the asperities as ellipsoidal elastic bodies. Rao et al.[18] employed Oloffson–Hagman micro-slip damping model to determine analytically an effective friction coefficient, which is a function of the tangential and normal forces, contact stiffness and slip amplitude. In a word, many kinds of friction dampers are studied by designers for blade vibration control.

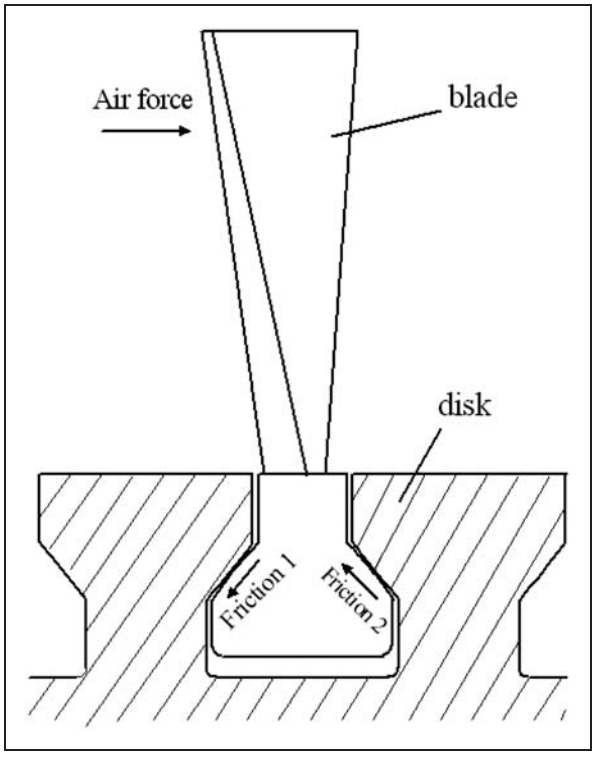

Figure 1. Schematic of a blade with loosely assembled dovetail attachment.

In this article, a test system is presented for studying the dynamic responses of blade with loosely assembled dovetail attachment, in which centrifugal force is simulated by an axial load. The normal loads can be applied on two friction interfaces by the simulation centrifugal force. The measurement method of the centrifugal force is introduced. The dynamic responses of a group of plane blades with different dovetail attachment angles are measured in consideration of the variable simulation centrifugal force. The influences of centrifugal force and dovetail angle on the friction damping of blade with dovetail attachment are investigated. The damping characteristics of a compressor blade with dovetail attachment are measured and the influences of centrifugal force on responses are investigated too.

Test systems

Testing facility

The test system for measuring the friction damping characteristics of blade with loosely assembled dovetail attachment is designed as shown in Figure 2. A loading arm is used to fix the dovetail attachment into dovetail groove, which is clamped at the base. A cushion block is placed between the loading arm and dovetail attachment for stress measurement by a strain meter. The loading arm imposes an external axial load, N, at the

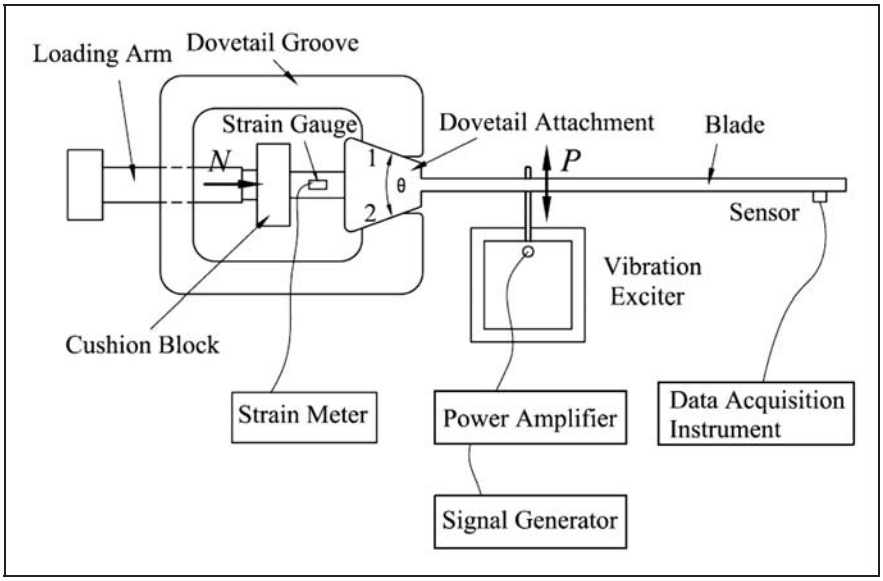

Figure 2. Test system.

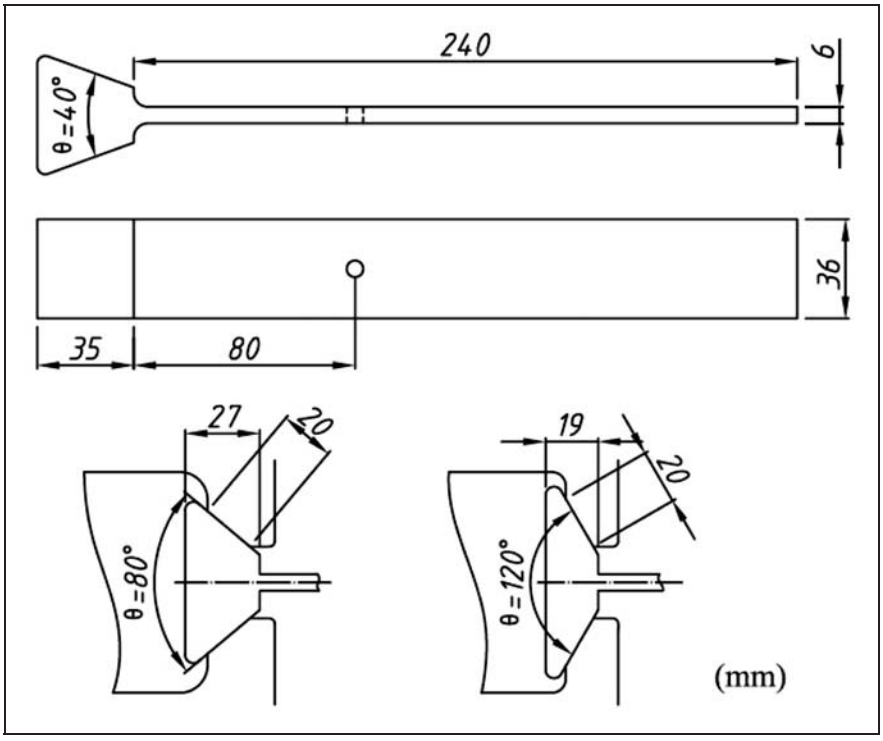

Figure 3. Dimensions of plane blades.

bottom of dovetail attachment to simulate the blade centrifugal force. The external axial load N is increased gradually to imitate the diversification of centrifugal force during the start-up process.

A vibration exciter is placed at the base. A power amplifier which connects the vibration exciter and signal generator is used to enhance harmonic excitation signal. The excitation point is distanced 80 mm to dovetail attachment. The excitation force is given by $P = P_a \sin(2\pi f \cdot t)$, where P_a denotes the amplitude of harmonic excitation force, and f denotes the harmonic frequency. The amplitude of excitation force P_a is 10 N and is invariable in this test. The direction of excitation force is perpendicular to blade. A acceleration sensor is

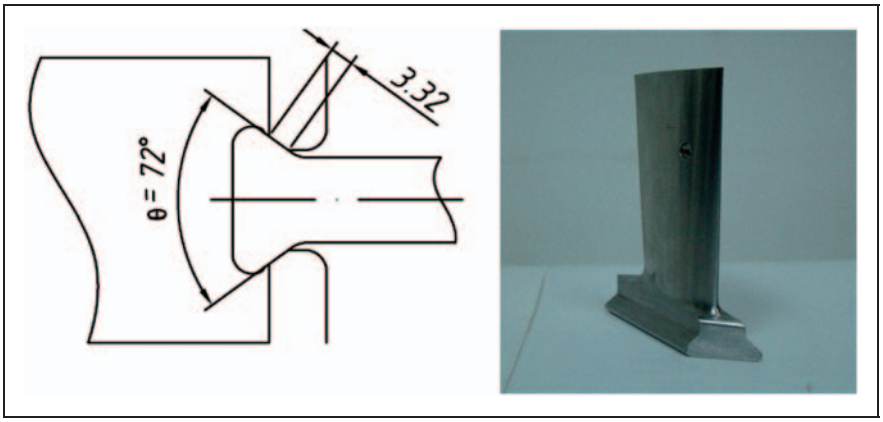

Figure 4. The last stage blade of a compressor.

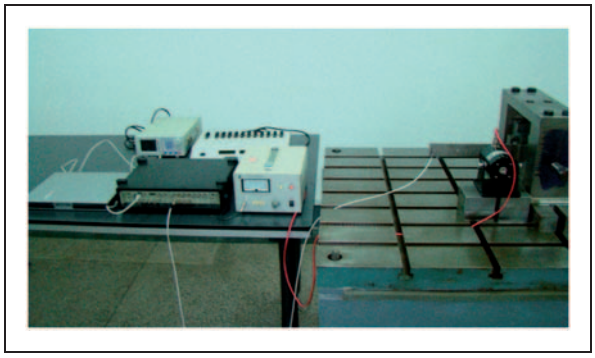

Figure 5. Test rig.

placed at the end of blade for acquiring the vibration data.

Testing blades

Three plane blades have the same dimensions, but the different dovetail angles as shown in Figure 3. The blade dimensions are 240 mm long, 36 mm wide and 6 mm thick. The dovetail angles are 40°, 80° and 120°, respectively. The blade material is 2Cr13, and its density is 7750 kg/m^3. The first natural frequency is 88.758 Hz. The total contact area between dovetail attachment and groove is 1440 mm^2, which is not changed with the different dovetail angle θ.

A last stage blade of compressor is shown in Figure 4. Its dovetail angle θ is 72°. The total contact area between the dovetail attachment and groove is 478.08 mm^2. The dimensions of the each side contact area are 72 mm long and 3.32 mm wide.

Measuring methods

In this experiment, the forced responses of three plane blades and the compressor blade are measured at six different centrifugal forces. The test rig is shown in Figure 5. A group of time-domain vibration response curves are acquired when the excitation frequency is changed from 70 to 100 Hz at each centrifugal force for plane blades. The response amplitudes at different excitation frequency are collected to draw the amplitude–frequency curve. Once the centrifugal force is changed, this testing process can be repeated to get a group of amplitude–frequency curves at variant centrifugal force.

The semi-power points method[6] is applied approximately to measure the damping ratio of the friction damping blade in this article.

The strain of cushion block between the loading arm and dovetail attachment is measured by strain gage. The load value at the bottom of dovetail attachment is obtained according to Hooke's law. In order to reduce the measuring error, 4 strain gages are put on the 4 sides of the cushion block, and the strains of cushion block 4 sides are measured at one time. And then the average of 4 strains is used to compute the axial load.

Results and discussion

Influence of friction damping on steady response

In order to investigate the influence of friction damping of steady response, the time-domain steady-state response of 40° angle dovetail attachment blade are measured at two assembled ways, tightly and loosely. The results are shown in Figure 6. The acceleration is used as ordinate and its unit is gravity acceleration marked as 'g'. The time is used as abscissa and its unit is second marked as 's'. It is known that hysteresis damping is through in this test and friction damping depends on the relation movement of contact surfaces. In this section, the comparison of steady responses of two assembled ways is presented.

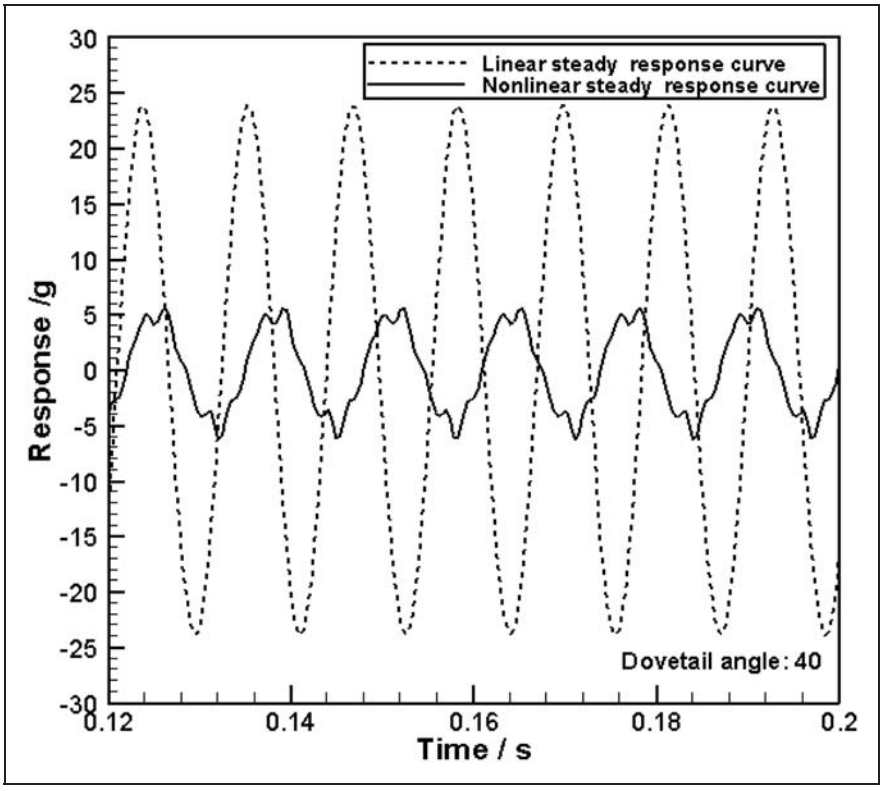

Figure 6. Time domain steady-state response.

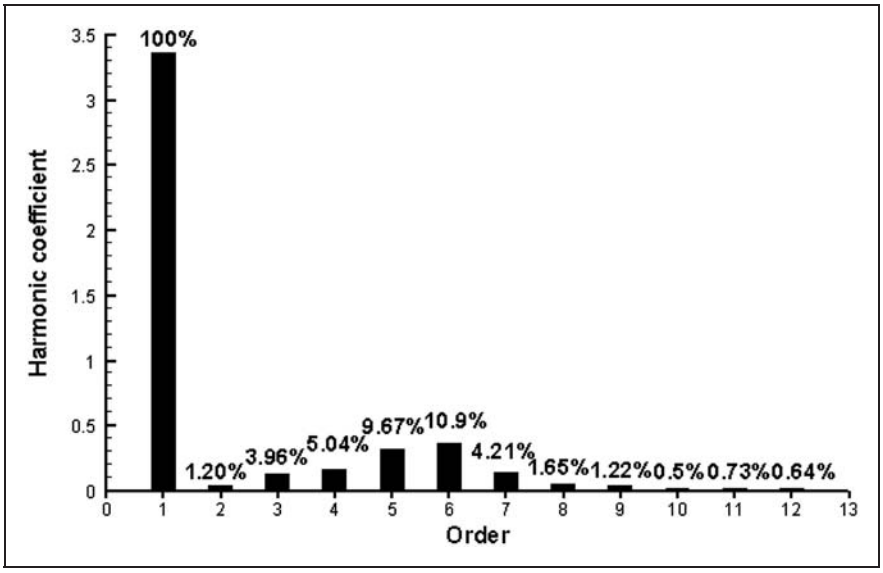

Figure 7. Spectrum of time domain steady-state response.

In Figure 6, the dotted line denotes the steady response of blade system in which the dovetail attachment is tightly assembled into groove. There is no macro-slip and only micro-slip or fretting damping left, and it is slightly for blade vibration in macro view. The frequency is 88 Hz, which is about the first natural frequency. It can be seen that the response of linear blade system is a regular sine curve, and has larger oscillation amplitude which is about 25 g. The real line denotes the resonant response of blade system in which the dovetail attachment is loosely assembled into groove, and there exists relative

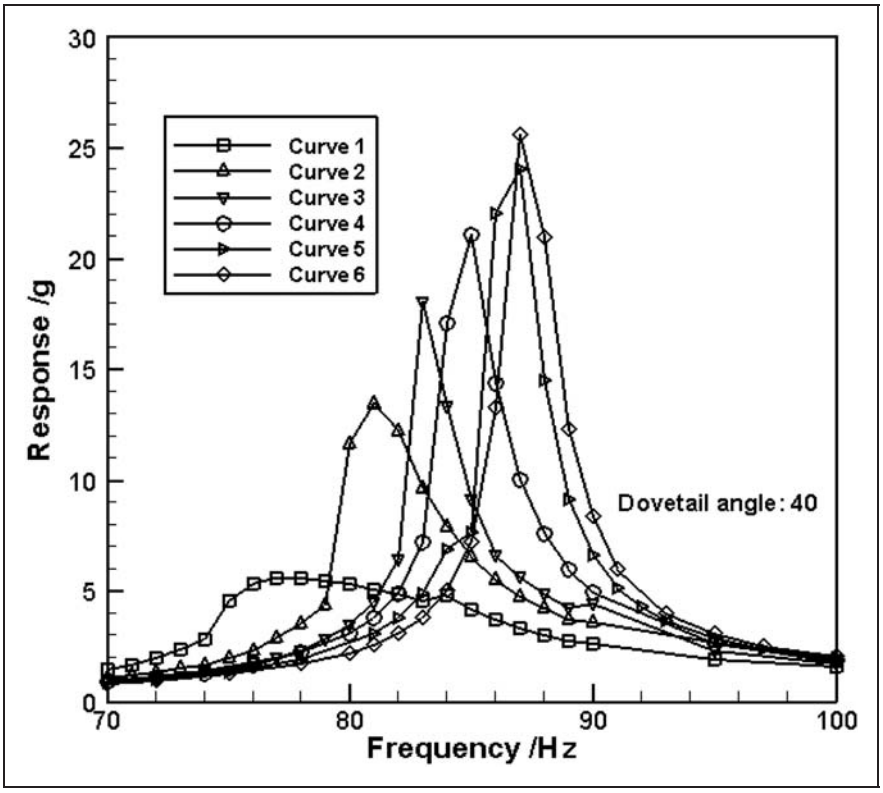

Figure 8. Amplitude–frequency curves at different centrifugal forces.

Table 1. Resonant frequency and damping ratio at different centrifugal forces.

Curve no.	Strain value	Centrifugal force (kN)	Resonant frequency (Hz)	Resonant amplitude (g)	Damping ratio
1	6.6	0.54	77.28	5.64	0.070
2	9.9	0.82	80.81	13.53	0.021
3	11.55	0.95	83.14	18.50	0.009
4	18.15	1.50	84.87	21.15	0.013
5	23.1	1.90	86.59	25.25	0.011
6	45.375	3.74	87.20	25.90	0.010

movement and friction damping when the blade is vibrating. The response of loosely assembled blade system is an irregular periodic curve and the oscillation amplitude becomes smaller and is about 5 g, which is 20% of linear blade system. The resonant frequency is 77 Hz, which is 11 Hz less than 88 Hz of linear blade system.

The spectrum bar chart of steady-state response of loosely assembled blade is given in Figure 7. The first 12-order harmonics are presented. And the percentages of each order harmonic coefficient in the first order are shown at the tip of bars. From Figure 7, the first-order harmonic coefficient is about 3.4 g, which is the biggest of all. The contributions of the 3rd to 7th orders are 3.96%, 5.04%, 9.67%, 10.9% and 4.21%, respectively, which are more than other higher order harmonics. The contributions of these order harmonics are not neglected and the first 10-order harmonics should be considered at least for calculating the responses of friction damping blade by numerical methods, such as the multi-harmonic balance method.

Influence of centrifugal force on responses

The forced responses of plane blade with 40° angle dovetail attachment are measured at different centrifugal forces. The amplitude–frequency curves at different centrifugal forces are shown in Figure 8. The correlative parameters including damping ratio are listed in Table 1. It can be seen that, when the centrifugal force increases from 0.54 to 3.74 kN, the resonant amplitude increases from 5.64 to 25.9 g and the resonant frequency increases from 77.28 to 87.2 Hz. When the centrifugal force equals 3.74 kN, the resonant amplitude and the resonant frequency get the biggest values, 25.9 g and 87.2 Hz, respectively. When the centrifugal force is 0.54 kN, the resonant amplitude is the least, 5.64 g, which is 21.7% of the biggest amplitude.

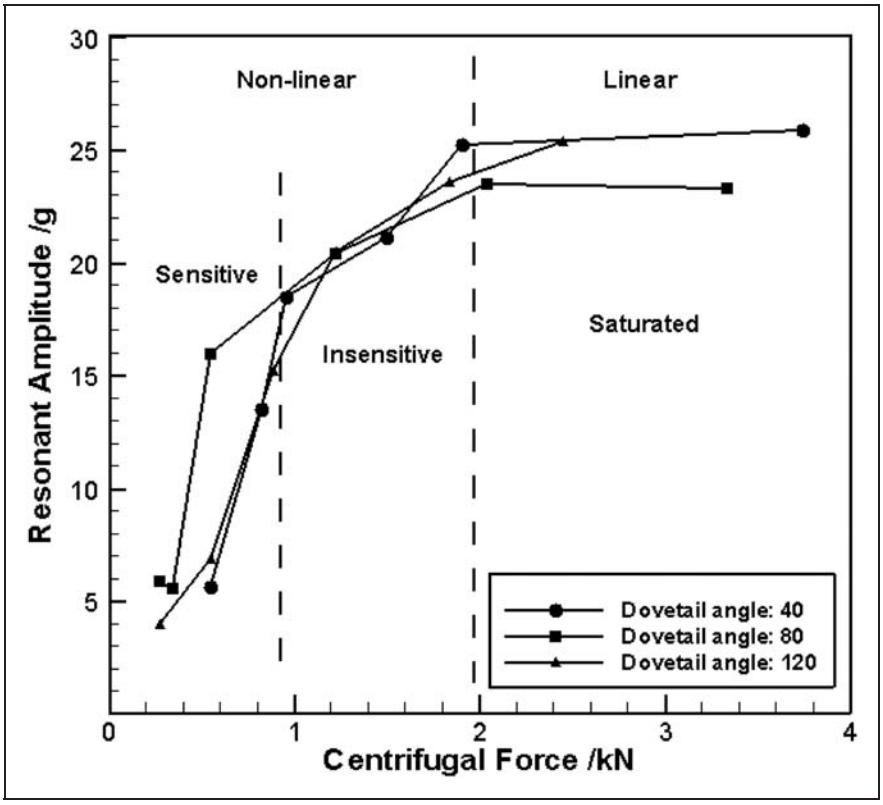

Figure 9. Resonant amplitude versus centrifugal force.

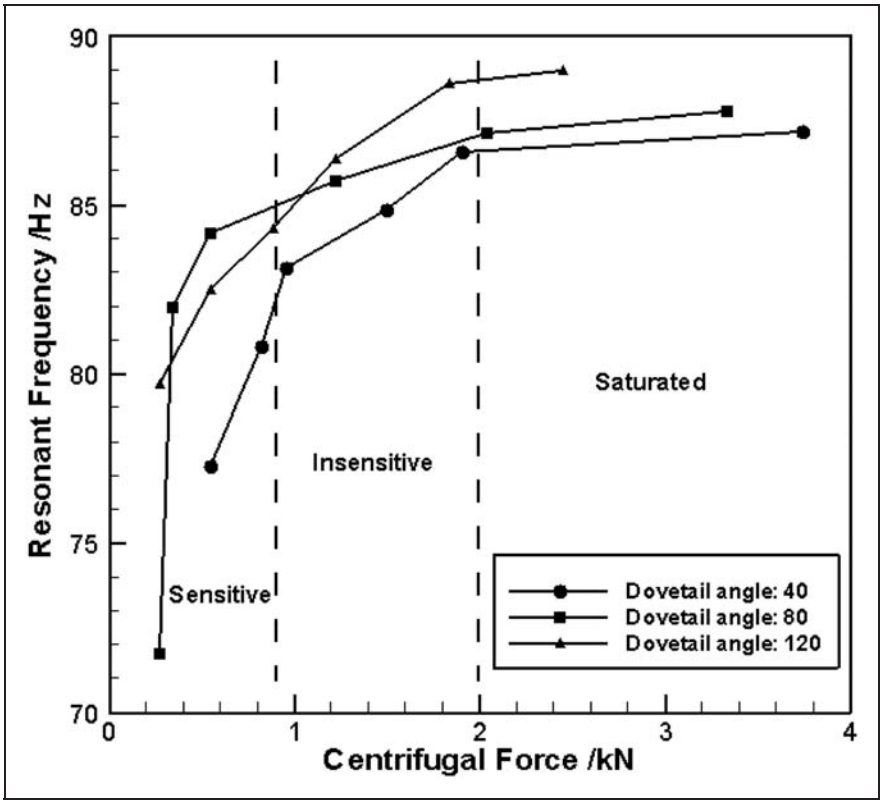

Figure 10. Resonant frequency versus centrifugal force.

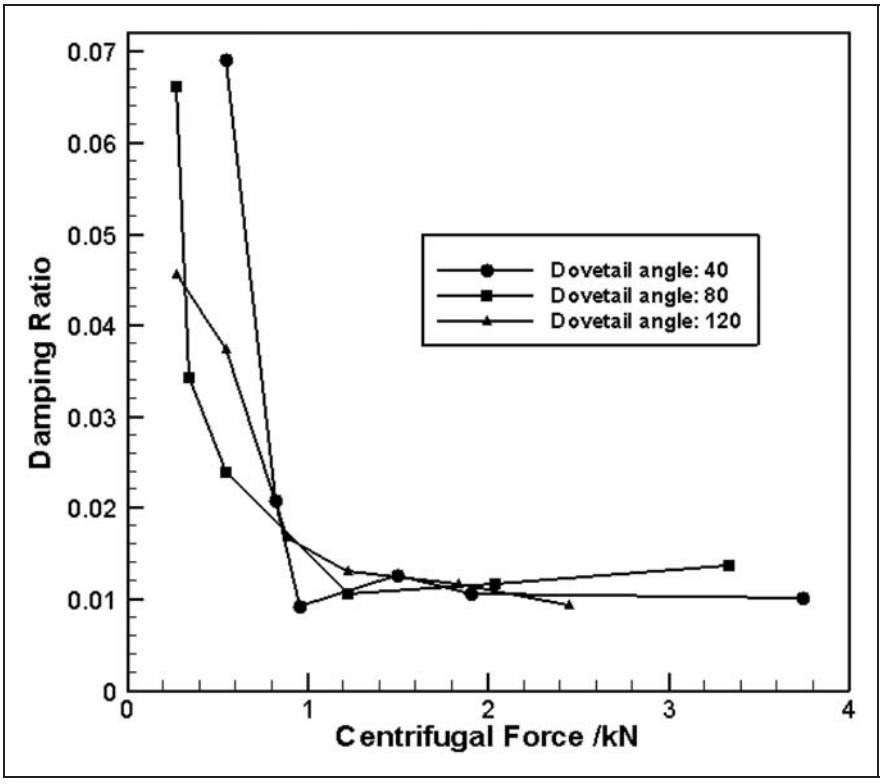

Figure 11. The damping ratio versus centrifugal force.

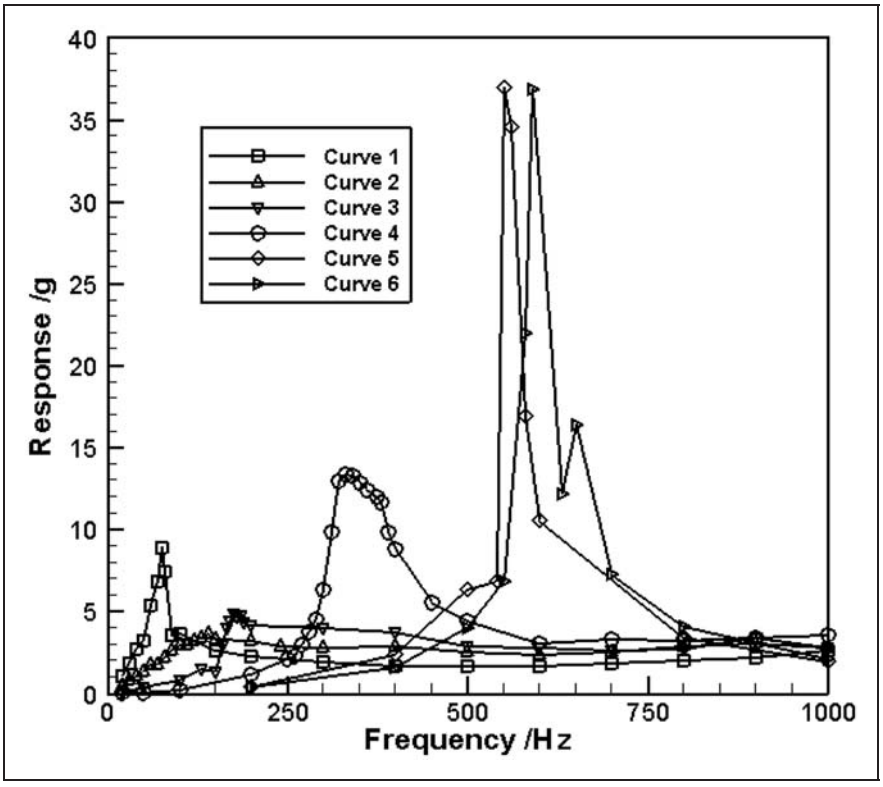

Figure 12. Amplitude–frequency curves at different centrifugal forces.

The least resonant frequency is 77.28 Hz which is about 10 Hz less then the biggest resonant frequency. The damping ratio of Curve 1, 0.07, is 7 times bigger than Curve 6. In Curve 1, it is found that softening spring effect gives rise to nonlinear response with the resonance peak bending towards lower frequencies. The resonant amplitude and frequency increase gradually, and the damping ratio decreases in the test centrifugal force range. The resonant response remains when the centrifugal force reaches a certain big value, 2 kN, which is called a saturated centrifugal force N_s. When the centrifugal force exceeds this big value, there is big normal pressure on contact surfaces. In this situation, the contact state between the blade dovetail attachment and disk groove is close to rigidness, the support stiffness achieves the saturated situation, and there is no relative movement and no dry friction damping.

Influence of dovetail angle on resonant response

In order to investigate the damping effect of blades with different angle dovetail attachments, the resonant amplitudes of three plane blades with different dovetail angles at variable centrifugal forces are measured. The relation curves of resonant amplitude, resonant frequency and damping ratio versus centrifugal force are shown in Figures 9, 10 and 11, respectively. In the three figures, the line with circle symbols is the result of 40° dovetail attachment blade, the line with square symbols is the result of 80° dovetail attachment blade, and the line with delta symbols is the result of 120° dovetail attachment blade.

In Figure 9, when the centrifugal force increases from 0.3 to 3.7 kN, three resonant amplitudes increase from about 5 to 25 g. They saturate finally when the centrifugal force exceeds the saturated centrifugal force N_s. The saturated centrifugal force, N_s, is about 2 kN for each plane blade. The least resonant amplitudes are almost the same. The range of centrifugal force can be divided into two parts, one part is from 0.3 to 2 kN, and the other is from 2 kN to the infinite. When centrifugal force is in the first part, the blade vibration system is nonlinear. In this situation, there is a relative movement occurring on contact surfaces when blade is vibrating. When the centrifugal force increases in this range, the resonant amplitude rises because the relative movement becomes difficult and the damping effect decreases. When the centrifugal force increases from 0.3 to 1 kN, the change rate of the resonant amplitude of 80° dovetail blade is biggest, and the change rate of 120° dovetail blade is least. When the centrifugal force increases from 1 to 2 kN, the change rate of the resonant amplitude of 80° dovetail blade is least. In this situation, the resonant amplitude of 80° dovetail blade is insensitive to centrifugal force. If centrifugal force is in the second part, the blade vibration system is almost a linear vibration system. In this situation, there is no relative movement on the contact surfaces, and the damping effect disappears. The resonant amplitude retains a constant value.

Table 2. Resonant frequency and damping ratio at different centrifugal forces.

Curve no.	Strain value	Centrifugal force (kN)	Resonant frequency (Hz)	Resonant amplitude (g)	Damping ratio
1	6.6	0.54	76	8.94	0.085
2	18.15	1.50	137	3.42	0.84
3	50.325	4.15	177	5.03	0.51
4	54.45	4.49	326	13.59	0.13
5	71.5	5.89	555	38.69	0.020
6	92.0	7.58	593	39.88	0.028

In Figure 10, the resonant frequency increases with centrifugal force, then saturates when centrifugal force exceeds N_s which is same as the saturation force in Figure 9. The least resonant frequency is 72 Hz for the blade dovetail angle 80°, which is 16 Hz less than the natural frequency 88 Hz. When the centrifugal force increases from 0.3 to 2 kN, the resonant amplitude of blade with dovetail angle 120° increases slowly, but the resonant frequency increases fast. The effect of dovetail angle on resonant frequency is obvious.

In Figure 11, the damping ratio of blade decreases with the centrifugal force increasing, then reaches a certain small value at the centrifugal force $N = N_s$. When the centrifugal force increases from 0.3 to 2 kN, the damping ratio of blade with dovetail angle 120° decreases slowly, and the damping ratio of blade with dovetail angle 80° decreases fast. The test results of blades with different dovetail angles show that the dovetail angle can influence the dynamic response of blade system, especially on the change rate of dynamic response. Therefore, the blade designers should choose an appropriate dovetail angle to make the blade dynamic response insensitive to centrifugal force.

Responses of the compressor blade

The amplitude–frequency curves at six different centrifugal forces are depicted in Figure 12, and the correlative parameters including damping ratio are listed in Table 2.

In Figure 12, the curves 3 and 4 show the softening spring effect on the amplitude frequency responses. The resonant frequency 76 Hz is the lowest resonance frequency in experimental frequency range in Table 2, which has 500 Hz difference when compared with the highest resonance frequency 593 Hz. The least peak

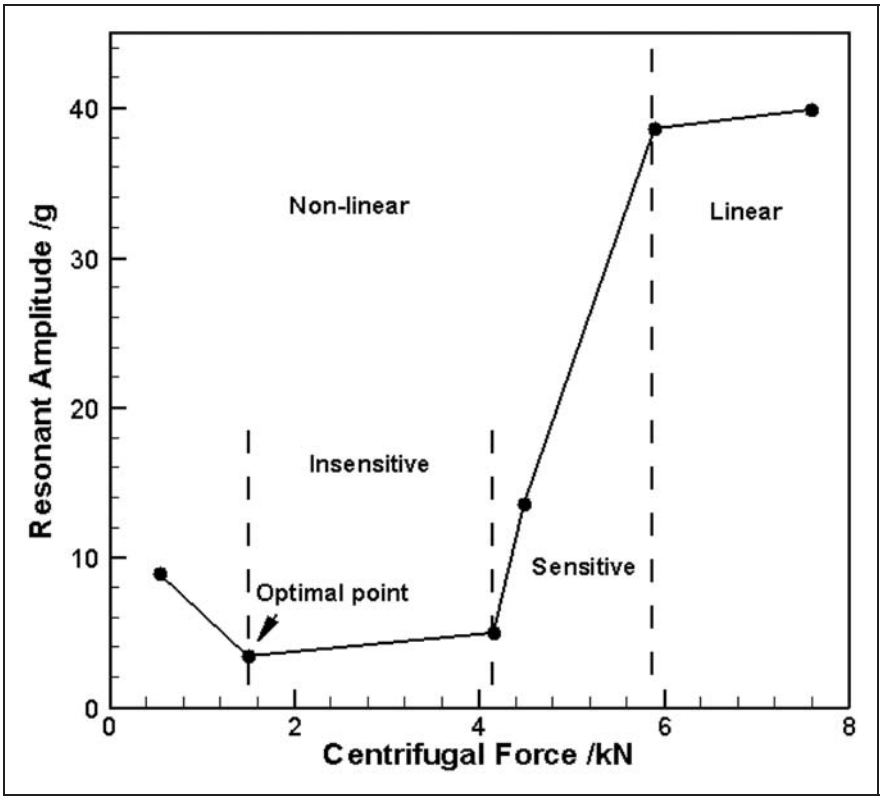

Figure 13. Resonant amplitude versus centrifugal force.

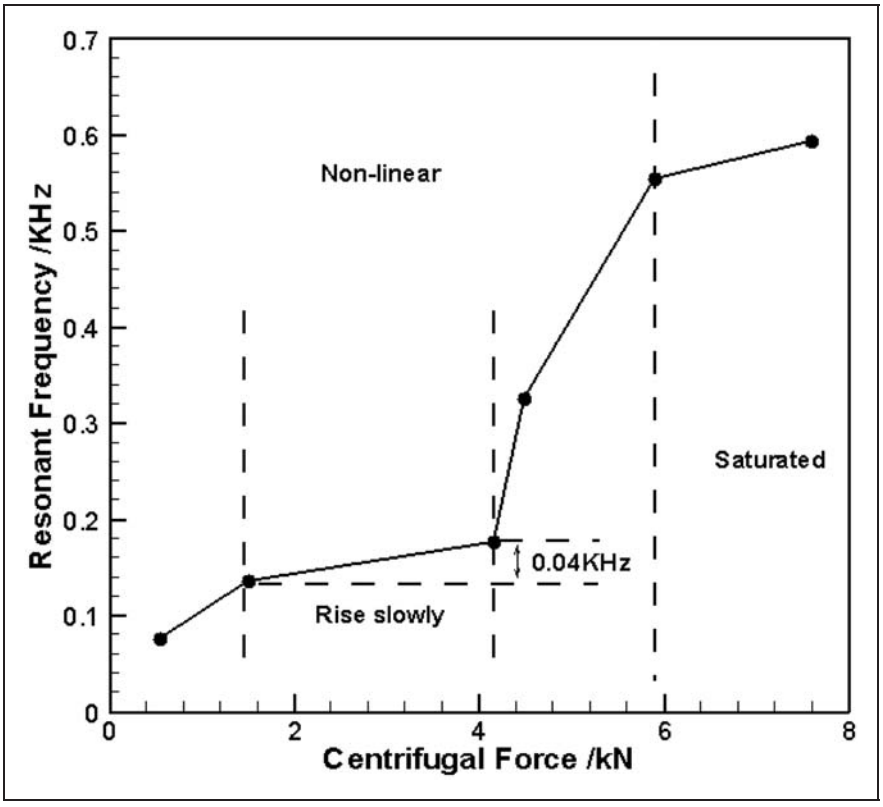

Figure 14. Resonant frequency versus centrifugal force.

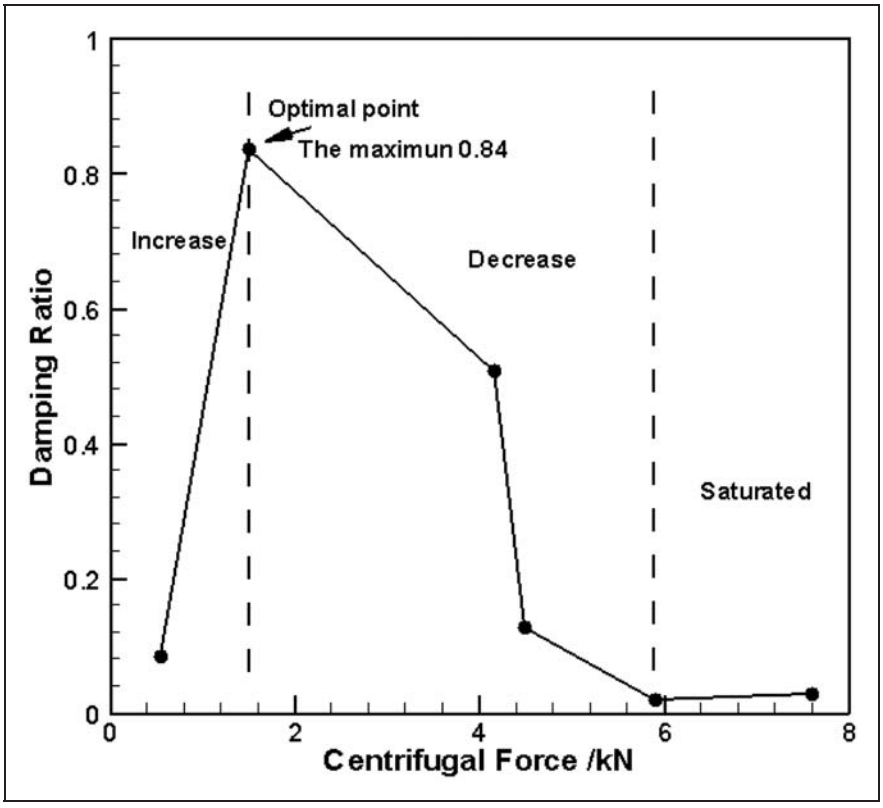

Figure 15. Damping ratio versus centrifugal force.

amplitude 3.42 g is only 22.4% of the biggest value 39.88 g in experimental frequency range. These data indicate that the damping effect of blade with dovetail attachment is obvious. The various values of centrifugal forces not only affect the resonance frequency in test systems, but also impact the peak amplitude of blade, and there is an optimal centrifugal force makes the minimum amplitude peak and the best damping effect.

The relationship between resonant amplitude and the applied centrifugal force is shown in Figure 13. When the centrifugal force is less than 1.5 kN, the curve exhibits downward status. The resonant amplitude increases when centrifugal force changes from 1.5 to 6 kN. When the centrifugal force exceeds 6 kN, the resonant amplitude maintains a constant value. According to this test data, when centrifugal force gets 1.5 kN, the resonant amplitude is the minimum and is about 3.42 g. That is to say 1.5 kN is the optimal centrifugal force for the compressor blade in experimental frequency range. The minimum value 3.42 g is about 8.5% of the biggest value 40 g. Another special centrifugal force in this test is 4 kN. The peak amplitude climbs very small just from 3.42 to 5.03 g, when the centrifugal force changes from 1.5 to 4 kN. While, in the range of 4 to 6 kN, the change rate of peak amplitude becomes very sensitive and it means that the damping effect is significant during this centrifugal force range. When the centrifugal force exceeds 6 kN, the contact surface between dovetail attachment and groove becomes too tight to produce friction force, the peak amplitude is saturated. As the friction damping does not work, the test blade system is similar to a linear system.The relationship between resonant frequency and applied centrifugal force is shown in Figure 14. The curve in this figure is relatively simple compared with others, in which the resonance frequency increases from 76 to 593 Hz with the increase of centrifugal force. Corresponding to Figure 13, in different centrifugal force range, the curve behaves in different change rate expressing different damping characteristics.The damping ratio versus applied centrifugal force curve is depicted in Figure 15. The damping ratio climbs quickly at the beginning until the point (centrifugal force 1.5 kN, damping ratio 0.84), which gives optimal damping characteristics, then, the curve decreases with the centrifugal force increasing. From Figure 13 when centrifugal force achieves 1.5 kN the resonant amplitude is the minimum value in experimental frequency range, at this time, the damping ratio reaches the biggest 0.84, which is 42 times bigger than

the minimum damping ratio 0.02. When the centrifugal force exceeds 6 kN, the two contact surface stick very tight, the friction damping faded away and the damping ratio reaches the minimum.

When the centrifugal force is a small value, the rubbing effect of the contact surfaces is very slight, the friction consumed energy is very small, accordingly, and the blade vibration attenuation is a little. The consumed energy will increase with centrifugal force because of the increased rubbing. When the centrifugal force exceeds the saturation value, friction damping disappears and the system becomes a linear cantilever beam system.

Conclusions

A test system is presented to investigate the dynamic responses of blade with dovetail attachment. The dynamic responses of a group of plane blades and a compressor blade are measured in consideration of variable centrifugal force. The experimental results show that the amplitude of resonant response decreases at the beginning and then increases when the centrifugal force increases. There is an optimal centrifugal force under which resonant amplitude is minimal and damping effect is the best. There is softening spring effect when friction forces worked on the blade system. When centrifugal force is changed from 0.3 to 2 kN, the dynamic response of plane blade is sensitive to centrifugal force. From the results of compressor blade, it is found that, when centrifugal force is 1.5 kN, the resonant amplitude reaches the least value 3.42 g, which is only 22.4% of the biggest value 39.88 g. It means that 1.5 kN is the optimal centrifugal force. The resonant response of compressor blade retains a low level when the centrifugal force in the range from 1.5 to 4 kN.

Funding

This work is financially supported by the National Basic Research Program of China (Grant No. 2011CB706505), Natural Science Foundation of China (Grant No. 50675169), and state 863 Programs of China (Grant No. 2007AA04Z1A1).

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Appendix

Notation

f	frequency of the harmonic excitation force (Hz)
g	gravity acceleration (m/s^2)
N	axial load/centrifugal force (kN)
N_s	saturated centrifugal force (kN)
P	exciting force (N)
P_a	amplitude of the excitation force (N)
R	response of the blade (g)
t	time variable (s)
θ	angle between two contact surfaces of the dovetail attachment (degree)
ζ	damping ratio