Control and monitoring of a froth flotation column using a personal computer

by R. Pal, PhD and J. Masliyah, PhD

A computer-automated froth flotation circuit was designed and developed. Control and monitoring of the flotation column was carried out using a personal computer equipped with an input/output interface device. Process control strategy and analysis of the system are presented. The system hardware and software that were designed and developed are discussed.

Keywords: Flotation, automation, control, monitoring

List of symbols

a_0	Magnitude of a step change in the set point
A	Cross-sectional area of the column
A/D	Analogue-to-Digital module
C_{n-1}	Controller output at the $(n-1)$th sampling instant
C_n	Controller output at the nth sampling instant
$C(z)$	z-transformation of PI algorithm
DP	Differential pressure
D/A	Digital-to-analogue module
$D(z)$	Discrete transfer function of PI control algorithm
G_A	Transfer function of amplifier
G_f	Transfer function of pump speed controller
G_m	Transfer function of the measuring element
G_p	Transfer function of the process
h	Interface level above the middle DP cell
h_0	Height of the column from the location of the middle DP cell to the froth outlet
h'	Interface deviation variable
$h'(s)$	Laplace transformation of $h'(t)$
$h'(z)$	z-transform of discrete time function $h'(nT)$
h'_{sp}	Set point deviation variable
$h'_{sp}(z)$	z-transform of discrete time function $h'_{sp}(nT)$
$H(s)$	Transfer function of hold element
I_m	Current output from measuring element, middle DP cell
I'_m	Current output deviation variable
$I'_m(s)$	Laplace transformation of $I'_m(t)$
$I'_m(z)$	z-transform of discrete time function $I'_m(nT)$
K_0	Calibration constant of middle DP cell
K_1	Constant defined in Eqn (18)
K_A	Gain of amplifier
K_c	Proportional gain of the controller
K_f	Gain of the speed controller
K_m	Gain of the middle DP cell
L/C	Level controller
MH	Measured position of the interface level
M_{ex}	Mass flow rate of non-gaseous material leaving from the top of the column
M_F	Feed mass flow rate
M_T	Tailings mass flow rate
M_{Ts}	Steady-state value of tailings mass flow rate
M_w	Wash water mass flow rate
n	Sampling instant (where $nT = t$)
PT	Pressure transmitter, same as DP cell
P_m	Pressure measured by the middle DP cell in cm of water column
Q_T	Tailings volumetric flow rate
Q_{Ts}	Steady-state value of Q_T
Q'_T	Deviation variable of tailings volumetric flow rate (Eqn (9))
$Q'_T(s)$	Laplace transformation of $Q'_T(t)$
s	Variable in Laplace transformation
SH	Set position of the interface level
T	Sampling time period
$Y(z)$	Discrete transfer function of height converter, Eqn (12)
z	Variable in z-transform

Greek Letters

α_B	Gas hold-up in bubbly zone
α_F	Gas hold-up in froth zone
ϵ_{n-1}	Error signal at the $(n-1)$th sampling instant
ϵ_n	Error signal at the n^{th} sampling instant
$\epsilon(z)$	z-transform of the discrete time function $\epsilon(nT)$
ρ_B	Mean density in bubbly zone
ρ_F	Mean density in froth zone
ρ_T	Density of the tailings stream
ρ_w	Density of water
τ_I	Controller integral time
τ_R	Ratio of sampling period to integral time

Department of Chemical Engineering, University of Alberta, Edmonton, Alberta T6G 2G6, Canada

1. Introduction

Froth flotation, a unit operation for separating selected solids from aqueous suspensions by the use of air bubbles, is used widely in the mineral processing industry (Kitchener, 1984). Fig 1 shows a schematic of a flotation cell. The feed to the cell is a mineral-solids/water slurry which also contains a small amount of a frother. Air is introduced into the cell through the agitator shaft to generate small bubbles (typically 1 mm in diameter). As the air bubbles rise through the slurry, they pick-up hydrophobic mineral particles and carry

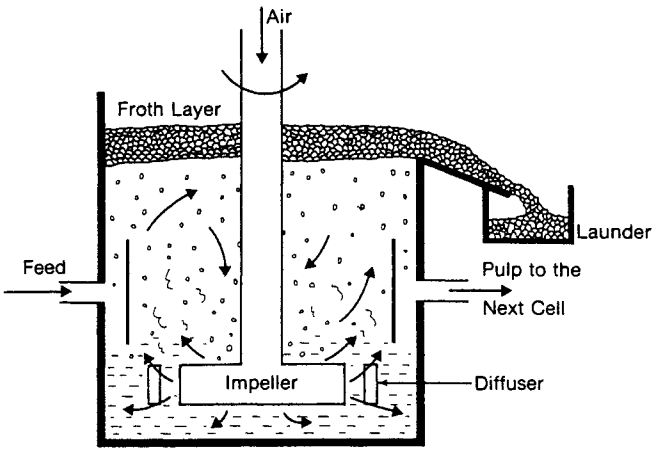

Fig 1 Schematic of a conventional flotation machine

them to the top of the cell. The mineral/air froth formed at the top of the cell finally flows away to a launder. Typically, the flotation cells contain 15–30% by volume of dispersed air and 10–25% by volume of solids.

Flotation cells, although used extensively in the mineral processing industry, have some inherent drawbacks. The product recovery from these machines is usually low. The grade of the recovered solids is poor as well. The main causes for low recovery and poor grade are a poor froth stability and a significant recovery of undesirable hydrophilic particles, referred to as gangue. The other disadvantages of the flotation cells are: poor process controllability; and high energy and maintenance costs due to mechanical agitation.

A recent advancement in froth flotation technology is 'froth flotation columns' which are essentially counter-current bubble columns (Dobby and Finch, 1986). Fig 2 shows a schematic of a froth flotation column. Other than the feed and gas inputs, there is one more input stream to the column: the wash water. The output streams from the column are the tailings and the froth. Flow conditions within the column are such that two distinct zones are formed: the bubbly zone and the froth zone.

The feed slurry enters the column at the top of the bubbly zone and flows downward. The air bubbles generated by a porous sparger at the bottom of the column rise upwards against the slurry. The air bubbles pick up hydrophobic mineral particles from the slurry and carry them upwards to the froth zone. It should be mentioned that the rising air bubbles also pick up some of the hydrophilic solid particles from the slurry and carry them to the froth zone. These hydrophilic particles are usually entrained in the water being carried upwards in the bubble wakes.

The froth zone of a flotation column resembles a packed bubble bed. In this zone, water is sprinkled continuously from the top. This water which drains downwards through the rising packed froth bed has two important functions: first, to wash the froth of its hydrophilic particles; and second, to increase the froth stability. Thus few or no hydrophilic particles are recovered in froth flotation columns. Moreover, the product recovery is high owing to an increase in the froth stability.

Froth flotation columns have further advantages such

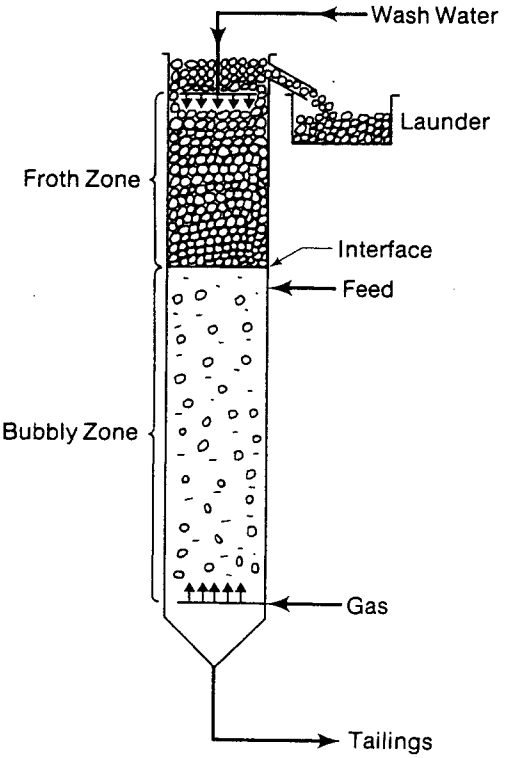

Fig 2 Schematic of a froth-flotation column

as: better process controllability; and significant energy and maintenance savings as there is no mechanical agitation involved in generating the air bubbles.

Although froth flotation columns appear very promising, they are far from being understood completely. Extensive experimental and theoretical research is required for their complete characterisation (Pal and Masliyah, 1989; 1990a; 1990b).

In order to be able to obtain reliable experimental data from laboratory columns, it is essential that the data are collected under steady-state operating conditions. Thus it is important to ensure that the column remains in steady state during the operation. It is further required that the steady state be reached as quickly as possible after the start-up. The key factor in achieving steady state in flotation columns is the interface level between the bubbly and the froth zones. The interface level must be brought to steady state at a pre-set value.

Presently, the steady-state operation of laboratory flotation columns is achieved manually. This is, indeed, a very tedious exercise and usually requires two operators to operate the column. Maintaining the interface level at a desired position is difficult, if not impossible. Furthermore, the interface position is not replicated in different experimental runs. This may lead to misinterpretation of the experimental data, as mineral recovery is a function of the froth depth.

The broad objective of the present study is to design and develop a computer-automated froth flotation column. Specific objectives include:

a. to design and develop a column flotation circuit adaptable for computer control and monitoring;
b. to develop a control strategy to bring the column quickly under steady-state conditions after start-up;

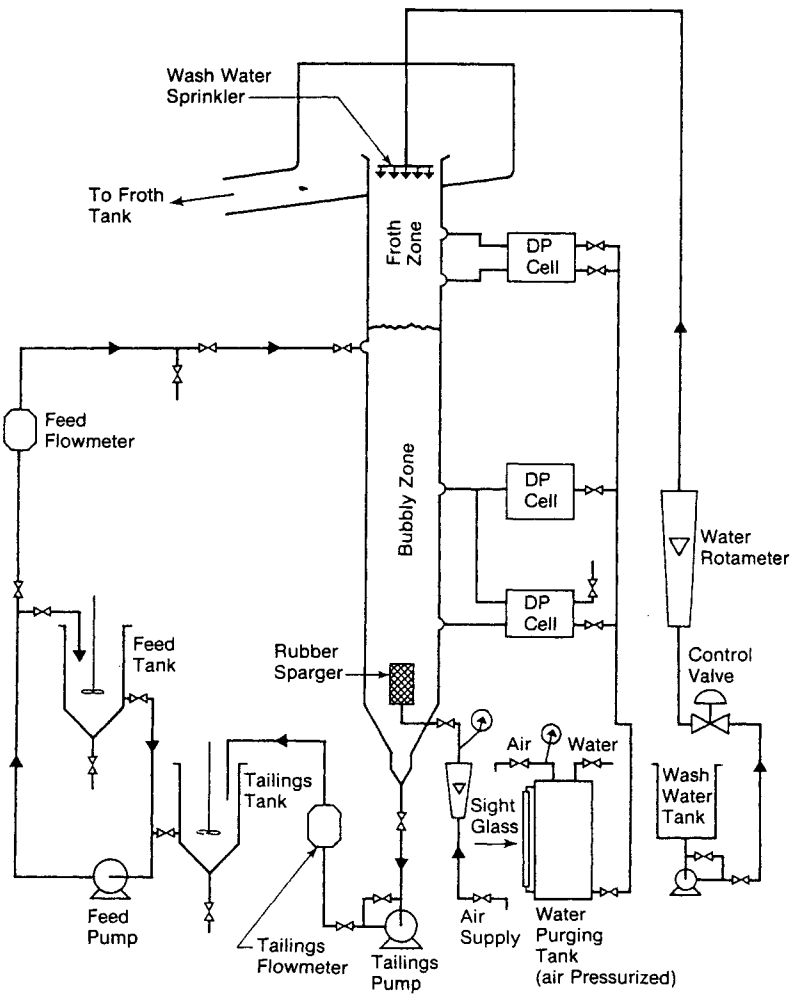

Fig 3 Schematic of the froth-flotation circuit

c. to design and develop hardware and software to achieve control and monitoring of the column;
d. to develop process dynamics of the system; and
e. to study the dynamic behaviour of the system.

2. Description of flotation circuit

Fig 3 shows a schematic of the froth flotation circuit. It consists of a glass flotation column, 10.4 cm in diameter and 2.1 m in height. The feed material is prepared in a feed tank equipped with a variable-speed mixer. The material from the feed tank is fed to the column by a variable-speed progressing-cavity pump. The feed flow rate is monitored by a magnetic flow meter. The air is introduced at the bottom of the column through a vertical rubber sparger. The wash water is introduced through a sprinkler installed at the top of the column, just below the froth outlet. The air and the wash-water flow rates are measured by means of rotameters. The tailings is withdrawn from the bottom of the column by a variable-speed progressing-cavity pump. The froth that leaves from the top of the column is collected in a froth tank, installed near the froth outlet.

The column is equipped with three differential pressure cells (DP cells). The top DP cell, which measures the static head over a certain height of the froth zone, enables mean density of the froth to be calculated. Similarly, the bottom DP cell enables mean density of the bubbly zone to be determined. The measurement from the middle DP cell is used to locate the position of the bubbly/froth zones interface.

3. Control strategy

Fig 4 illustrates the control strategy employed. The feed, air and wash water flow rates are set at the desired values. The wash water flow rate is set by means of a control valve (refer to Fig 3). The signal from the middle DP cell (PT-2) along with the signals from the top and bottom DP cells are used to calculate the interface position. The interface position is measured relative to the location of the middle DP cell, as shown in Fig 4. At any time, t, the interface position, $h(t)$ is given by:

$$h(t) = [p_m(t) - (\rho_F/\rho_w)h_0]/[(\rho_B/\rho_w) - (\rho_F/\rho_w)] \quad \ldots(1)$$

where $P_m(t)$ is the pressure measured by the middle DP cell in units of cm-water-column, ρ_F and ρ_B are the froth and bubbly zones' densities measured by the top DP cell and the bottom DP cell, respectively, ρ_w is the water density, and h_0 is the height of the column from the location of the middle DP cell to the froth outlet (the derivation of Eqn (1) is given in Appendix 1).

The above measured interface position is then compared with the set value. Depending upon the

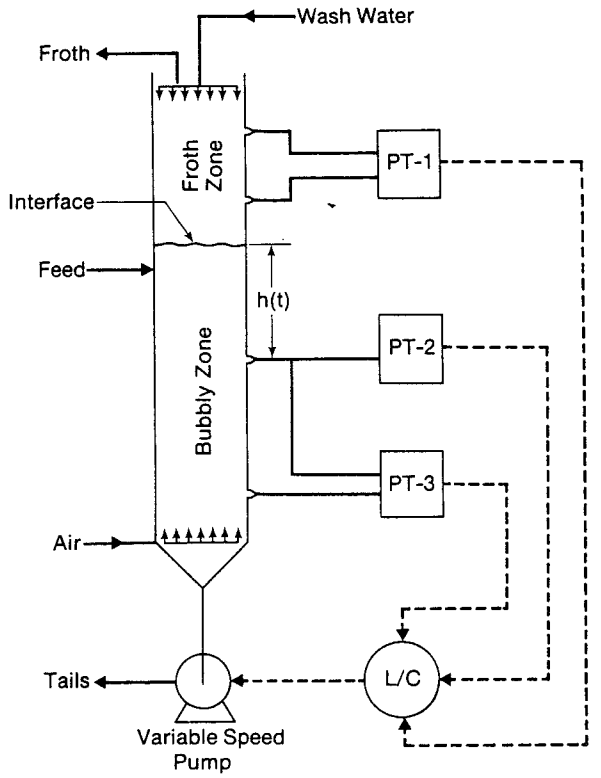

Fig 4 Illustration of the control strategy

difference between the measured and the set values, the tailings pump speed is varied by a level controller. The manipulation of the pump speed is continued until a steady state is reached.

4. System hardware and software

4.1 System hardware

Fig 5 shows the hardware components of the system. The main components include: a personal computer (Zenith 159); an Input/Output Interface device called Optomux (manufactured by Opto 22, California); two control elements – control valve and pump speed controller; three DP cells; and two flow rate sensors – feed and tailings, respectively.

The Interface device is equipped with five analogue-to-digital (A/D) modules and two digital-to-analogue (D/A) modules and the various supporting electronic devices such as the hold elements and the samplers required to perform the A/D and D/A conversions. The A/D modules are all AD3 type which perform the analogue-to-digital conversion of current inputs (4–20 mA, DC). The D/A modules are DA3 and DA4 types, respectively. The DA3 module gives a current output of 4–20 mA, DC and the DA4 module gives a voltage output of 0–5 V (DC). The former is used to operate the control valve and the latter is used to control the tailings pump speed.

There is one feedback control loop present in the system. The computer reads the signal from the middle DP cell at regular intervals of time. From this reading and the readings of the top and bottom DP cells, the computer calculates the position of the interface level (Eqn (1)). The computer then sends a signal to the tailings pump speed controller according to the deviation of the measured interface position from the set position.

4.2 System software

The software is developed in BASIC. Initially the software was run by BASIC INTERPRETER. As this

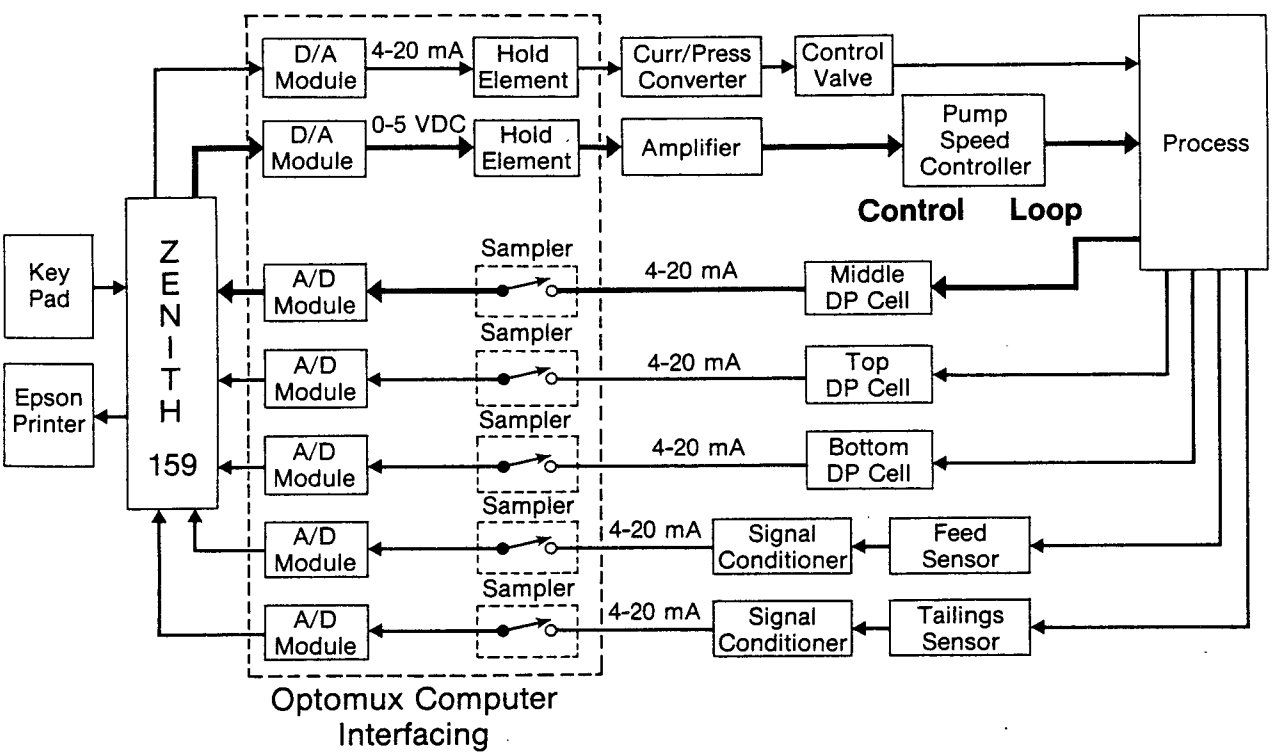

Fig 5 Hardware components of the system

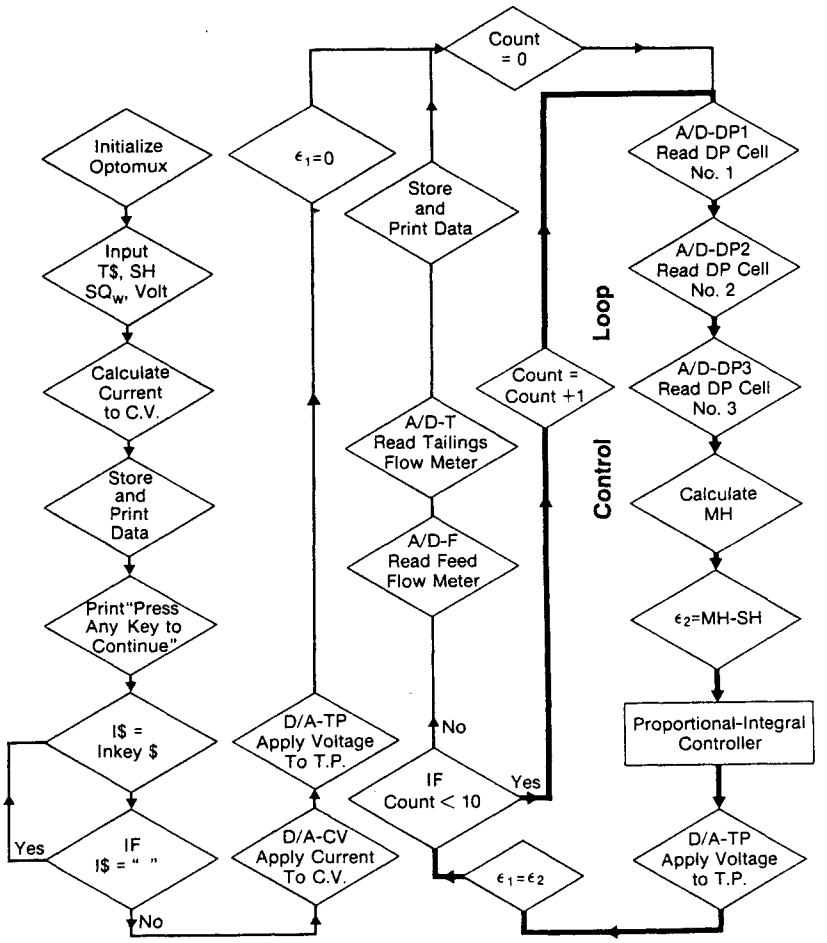

Fig 6 Column-control and monitoring algorithm

was found to be too slow, the software was compiled by a BASIC COMPILER; the compiled version was much faster.

Fig 6 shows the column control and monitoring algorithm. It performs two important functions: the interface level between the bubbly and the froth zones is brought to a set position (hence steady state) by controlling the tailings pump speed; and simultaneously the column data such as the feed flow rate, tailings flow rate, and the DP cells readings, are recorded as function of time.

The first step of the algorithm is initialisation of the Optomux computer interfacing. The purpose of initialisation is to select the desired operating characteristics of Optomux from the several available options. The most important feature to be selected is the configuration of the module positions. Optomux board has 16 module positions (numbered 0 to 15) where one can install either an A/D module or a D/A module. The default state assumes that all the module positions are occupied by the A/D modules.

In the second step, the program requests the necessary information pertaining to the experiment such as the title of the experiment (T$), the set values of the interface position (SH) and the wash water flow rate (SQ_w), and the approximate value of voltage to the tailings pump. The program then calculates the value of the current to be supplied to the wash water control valve. This is calculated from the calibration curve – ie, a plot of wash water flow rate versus current to the control valve.

In the next step, the program waits for the operator to press any computer key to continue. This gives an opportunity to the operator to start the experiment which involves activating the feed pump and opening the air-line valve at a desired air flow rate.

On pressing any key, the computer starts interacting with the process. The wash water flow line is opened by supplying current to the control valve through a D/A–CV module (CV refers to control valve). The tailings pump is started by sending a voltage signal to the pump speed controller through a D/A–TP module (TP refers to tailings pump). The signals from the three DP cells are read, and from these signals the location of the bubbly/froth zone interface is determined (Eqn (1)). The measured location (MH) of the interface is compared with the set location (SH) and the error (ϵ) is calculated:

$$\epsilon = \mathrm{MH} - \mathrm{SH} \qquad (2)$$

Based on the error value, the control action (voltage to tailings pump) is determined using the 'velocity form' of the Proportional–Integral (PI) equation (Eqn (13)).

In the feedback control loop, there is a counter. Each time the program goes through the loop, the value of counter is incremented by one. After every ten counts, the program jumps to a different branch as shown in Fig 6. In this branch, the computer reads the feed and

tailings flowmeters and stores all the data including the feed flow rate, tailings flow rate, the DP cells readings and the time on the hard disk. The counter is then re-initialised and the program comes back to the feedback control loop. Thus the computer performs the control and monitoring of the froth flotation column.

5. System analysis

5.1 Froth flotation column

Let the various variables be defined as follows:

M_F = Feed mass flow rate
M_w = Wash water mass flow rate
M_{ex} = Mass flow rate of non-gaseous material leaving from the top of the column
$M_T(t)$ = Tailings mass flow rate at time t
$h(t)$ = Position of interface above the middle DP cell at time t.

By overall mass balance:

$$\left\{\begin{matrix}\text{rate of}\\ \text{mass}\\ \text{accumulation}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of}\\ \text{mass}\\ \text{input}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{mass}\\ \text{output}\end{matrix}\right\} \quad ...(3)$$

$$M_F + M_w - M_{ex} - M_T(t) = \frac{d}{dt}[Ah\rho_B + A(h_0 - h)\rho_F] \quad ...(4)$$

where A = cross-sectional area of the column (ρ_B, ρ_F and h_0 were defined earlier in Eqn (1)). Eqn (4) can be simplified to:

$$M_F + M_w - M_{ex} - M_T(t) = A(\rho_B - \rho_F)\frac{dh}{dt} \quad ...(5)$$

as ρ_B, ρ_F and h_0 are constant. At steady state:

$$M_F + M_w - M_{ex} - M_{Ts} = 0 \quad ...(6)$$

where M_{Ts} = steady-state value of tailings mass flow rate. Note that only M_T and h are functions of time, and M_F, M_w and M_{ex} are all constant. From Eqns (5) and (6), we obtain:

$$M_T(t) - M_{Ts} = -A(\rho_B - \rho_F)\frac{dh}{dt} \quad ...(7)$$

The above equation can further be written as:

$$Q_T(t) - Q_{Ts} = -A\left(\frac{\rho_B - \rho_F}{\rho_T}\right)\frac{dh}{dt} \quad ...(8)$$

where $Q_T(t)$ is the tailings volumetric flow rate at time t, Q_{Ts} is the tailings volumetric flow rate at steady state and ρ_T is density of the tailings stream.

From the above equation, the transfer function of the process is given by:

$$G_p(s) = \frac{h'(s)}{Q'_T(s)} = -\frac{1}{A}\left(\frac{\rho_T}{\rho_B - \rho_F}\right)\frac{1}{s} \quad ...(9)$$

where h' and Q'_T are the deviation variables.

5.2 Measuring element

In the control loop, only the middle DP cell is to be considered as the measuring element. The DP cell is of a variable-capacitance type, having a very small time constant of about 0.2 s. The relationship between the current output (I_m) and the pressure input (P_m) to the DP cell is given by:

$$I_m = K_m P_m + K_0 \quad (10)$$

where K_m and K_0 are the calibration constants.

Using Eqns (1) and (10), it can be shown easily that the transfer function of the measuring element is given by:

$$G_m(s) = \frac{I'_m(s)}{h'(s)} = K_m[(\rho_B/\rho_w) - (\rho_F/\rho_w)] \quad ...(11)$$

where $I'_m(s)$ is a deviation variable.

5.3 Height converter

This is a software converter which determines the location of the interface level by using the signal from the middle DP cell. Its discrete transfer function is:

$$Y(z) = \frac{h'(z)}{I'_m(z)} = \frac{1}{K_m[(\rho_B/\rho_w) - (\rho_F/\rho_w)]} \quad ...(12)$$

where $h'(z)$ and $I'_m(z)$ are z-transform of discrete time functions h' and I'_m, respectively. ρ_B and ρ_F in the above equation are determined from the bottom and top DP cells measurements.

5.4 Controller

The controller uses 'velocity form' of the Proportional–Integral (PI) algorithm (Stephanopoulas, 1984a):

$$C_n = K_c\left\{\left(1 + \frac{T}{\tau_I}\right)\epsilon_n - \epsilon_{n-1}\right\} + C_{n-1} \quad ...(13)$$

where K_c is the proportional gain of the controller, T is the sampling time, τ_I is the integral time, C_n and ϵ_n are the controller output and error input signals at the nth sampling instant, and C_{n-1} and ϵ_{n-1} are the controller output and error input signals at $(n-1)$th sampling instant.

Eqn (13) upon z-transformation yields:

$$C(z) = K_c\left\{\left(1 + \frac{T}{\tau_I}\right)\epsilon(z) - z^{-1}\epsilon(z)\right\} + z^{-1}C(z) \quad ...(14)$$

$$D(z) = \frac{C(z)}{\epsilon(z)} = K_c\left\{1 + \frac{(T/\tau_I)}{1 - z^{-1}}\right\} \quad ...(15)$$

$D(z)$ is the discrete transfer function of the velocity–PI control algorithm. $C(z)$ and $\epsilon(z)$ are z-transforms of the discrete time functions C and ϵ.

5.5 Hold element

The purpose of the hold element is to keep the value of the (discrete-time) controller output signal constant for the entire period until the new signal comes along. The transfer function of hold element is as follows:

$$H(s) = \frac{1 - \exp(-sT)}{s} \quad ...(16)$$

5.6 Amplifier

The purpose of the amplifier is to amplify the 0–5 V DC signal from the hold element to the 0–10 V DC signal. Its transfer function is:

$$G_A = K_A \qquad ...(17a)$$

where K_A is the gain of the amplifier. Note that the time constant of the amplifier is negligibly small.

5.7 Pump-speed controller

The pump-speed controller controlled the speed and hence the flow rate through the tailings pump. Its transfer function is:

$$G_f = K_f \qquad ...(17b)$$

where K_f is the proportionality constant between input signal to the speed controller (0–10 V DC) and flow rate through the pump. The time constant of the pump is assumed to be negligibly small.

6. Overall transfer function of the system

Fig 7 shows the block diagram of the control system and the relation between the various transfer functions. As there is no disturbance involved in our system, we are interested in dynamic behaviour only with set-point changes. In other words, the control problem is of 'servo' type.

The closed-loop transfer function relating the sampled process output $h'(z)$ to the discrete set point changes, $h'_{sp}(z)$ is given by (see Appendix 2):

$$h'(z) = h'_{sp}(z) - \frac{(1-z^{-1})^2 h'_{sp}(z)}{1+(K_1+K_1\tau_R-2)\,z^{-1}+(1-K_1)(z^{-1})^2} \qquad ...(18)$$

where $K_1 = K_A K_f K_c \rho_T T/[A(\rho_B - \rho_F)]$ and $\tau_R = T/\tau_1$.

The closed-loop characteristic equation of the system is:

$$1+(K_1+K_1\tau_R-2)\,z^{-1}+(1-K_1)(z^{-1})^2 = 0 \qquad (19)$$

For the control loop to be stable, the values of K_1 and τ_R should be such that the magnitude of all the roots of the above equation are smaller than unity (Stephanopoulos, 1984b).

7. Dynamic behaviour of the system

For a step change in the set point, $h'_{sp}(z)$ is given by:

$$h'_{sp}(z) = \frac{a_0}{1-z^{-1}} \qquad ...(20)$$

where a_0 is the magnitude of the step change. From Eqns (18) and (20), we obtain:

$$h'(z) = \frac{a_0}{1-z^{-1}} - \frac{a_0(1-z^{-1})}{1+(K_1+K_1\tau_R-2)\,z^{-1}+(1-K_1)(z^{-1})^2} \qquad ...(21)$$

From the above equation, the response curves of the system are obtained using a software package PC/MATLAB (The Mathworks, Inc, MA, USA).

Fig 8 shows the effect of K_1 on the response of the system. The response becomes more oscillatory with a decrease in K_1. The effect of τ_R on the response of the system is shown in Fig 9. The response becomes more oscillatory with an increase in τ_R. For all the response curves shown in Figs 8 and 9, the amplitude of oscillation decreases with time, indicating that the system is stable for the chosen values of τ_R and K_1 (the roots of the characteristic equation, Eqn (19), lie inside the unit circle in the complex plane).

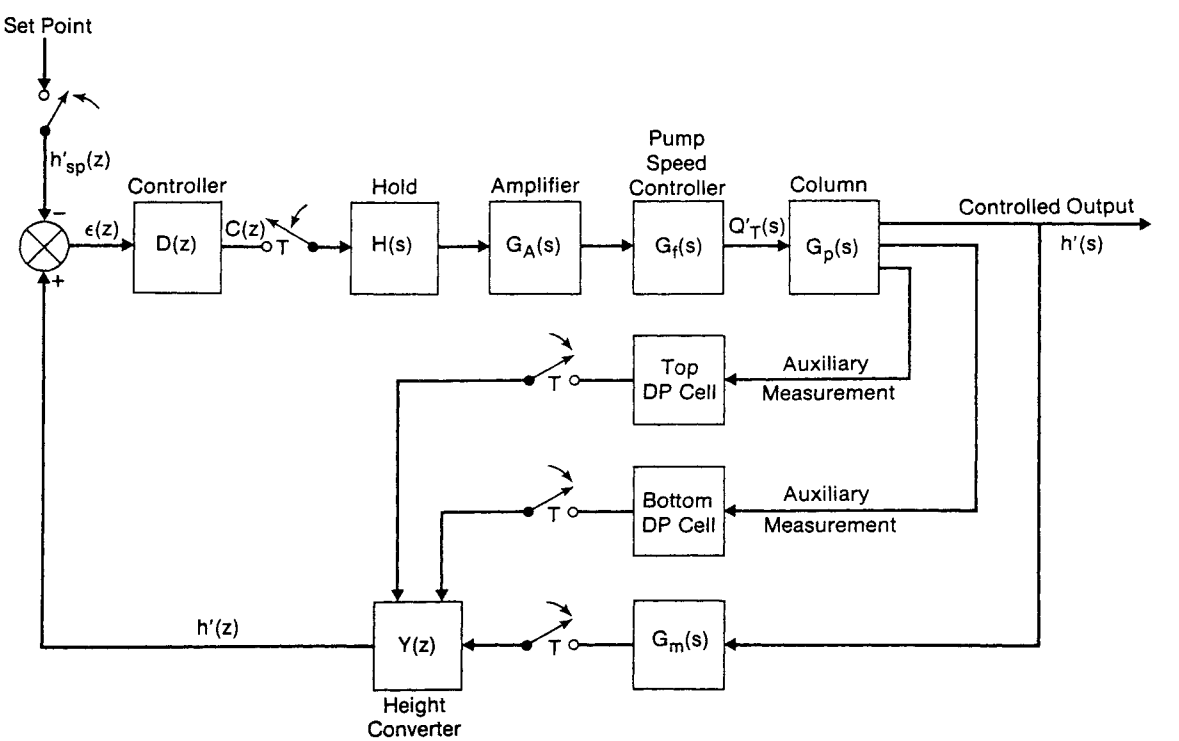

Fig 7 Block diagram of the control system

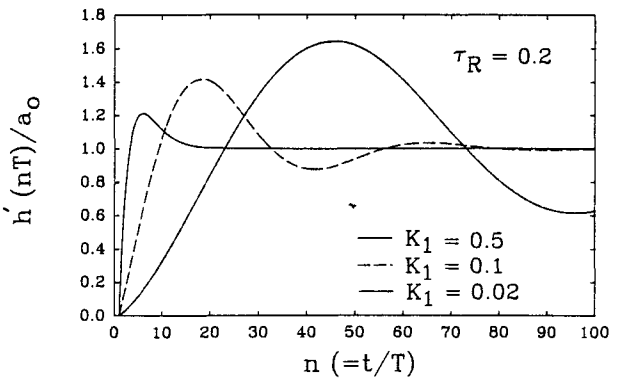

Fig 8 Response of the system to a step change in the set point – Effect of K_1

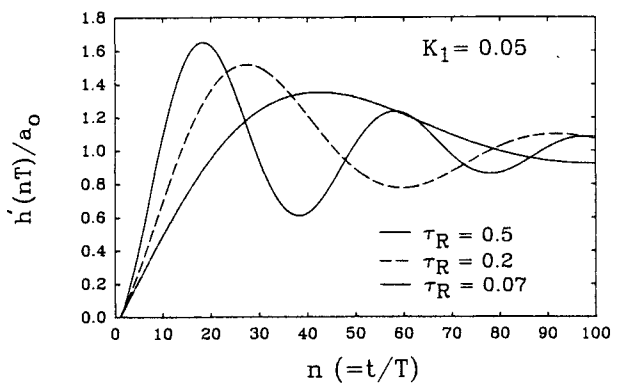

Fig 9 Response of the system to a step change in the set point – Effect of τ_R

8. Experimental work

To observe the experimental response of the system, experiments were carried out with an aqueous surfactant solution. The surfactant used was Dow froth 250-C (polypropylene glycol methyl ether) at a concentration of 30 mg/l. The flow rates of the feed, air and wash water were such that the following values of gas holdup in the bubbly (α_B) and froth (α_F) zones were obtained: $\alpha_B = 0.12$; and $\alpha_F = 0.765$. The corresponding values of bubbly and froth zones densities were: $\rho_B = 880\,kg/m^3$; and $\rho_F = 235\,kg/m^3$.

The controller settings were selected to be $K_c = 0.1\,V/cm$ and $\tau_I = 80\,s$. Note that the controller gain (K_c) and the integral time (τ_I) are linked to the factors K_I and τ_R as follows (refer to Eqn (18)):

$$K_c = \frac{K_I A}{K_A K_f T}\left(\frac{\rho_B - \rho_F}{\rho_T}\right) \qquad ...(22)$$

and

$$\tau_I = T/\tau_R \qquad (23)$$

where the values of the various calibration constants of the system are: $K_A = 2$, $K_f = 2.75\,cm^3/(s.V)$, $A = 85.54\,cm^2$, $(\rho_B - \rho_F)/\rho_T) = 0.645$, and $T = 16\,s$. The values of K_I and τ_R calculated from the above equations are: $K_I = 0.158$; and $\tau_R = 0.2$.

Fig 10 shows the system's response to a step change in the set-point (a step change of 10 cm was made in the set-point interface level while the system was at steady state). The response is quite good for the selected values of the controller constants. Steady state is reached at n of 40 (about 10 min). The agreement between the theoretical and experimental responses is excellent.

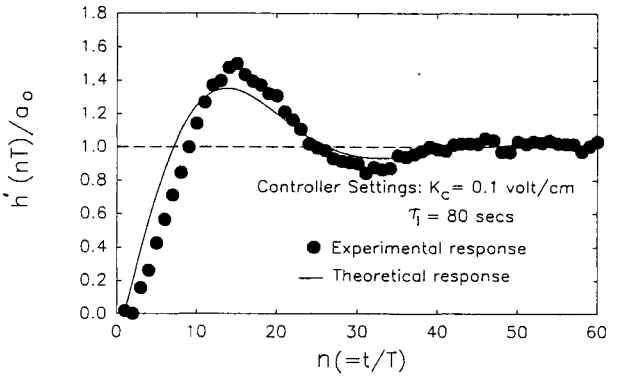

Fig 10 Experimental response of the system to a step change in the set point at $K_c = 0.1\,V/cm$ and $\tau_I = 80\,s$

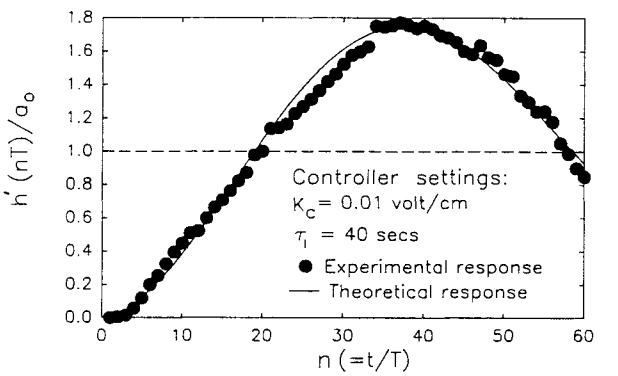

Fig 11 Experimental response of the system to a step change in the set point at $K_c = 0.01\,V/cm$ and $\tau_I = 40\,s$

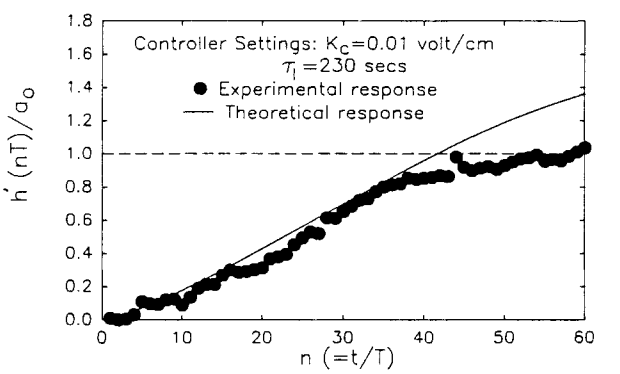

Fig 12 Experimental response of the system to a step change in the set point at $K_c = 0.01\,V/cm$ and $\tau_I = 230\,s$

Figs 11 and 12 show the experimental and theoretical responses of the system at other values of the controller settings (K_c and τ_I). These figures indicate clearly that the response of the system could be quite poor if the controller settings are not selected appropriately.

9. Conclusions

Froth flotation columns can be brought to steady-state quickly after a start-up by controlling the interface

level between the bubbly and froth zones. The interface level can be controlled at a pre-set value by manipulating the tailings flow rate. Hardware and software were developed for a newly designed computer-controlled flotation column. The system analysis enabled the controller settings to be determined quite readily.

10. Acknowledgements

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Appendix 1

Derivation of Eqn (1):

Pressure measured by the DP cell (refer to Fig 13) is given by:

$$P'_m = \rho_F g(h_0 - h) + \rho_B g h \quad \ldots (A1)$$

where ρ_F and ρ_B are froth zone and bubbly zone densities, and g is acceleration due to gravity. From Eqn (A1):

$$h(t) = (P'_m - \rho_F g h_0)/[(\rho_B - \rho_F)g] \quad (A2)$$

If P'_m is now expressed in terms of water-column, ie,

$$P'_m = \rho_w g P_m \quad (A3)$$

we obtain the following expression for $h(t)$:

$$h(t) = [P_m(t) - (\rho_F/\rho_w)h_0]/[(\rho_B/\rho_w) - (\rho_F/\rho_w)] \quad (1)$$

Appendix 2

The closed-loop transfer function relating the sampled process output $h'(z)$ to the discrete set point changes, $h'_{sp}(z)$ can be developed as follows. From the block diagram of the control system (Fig 7):

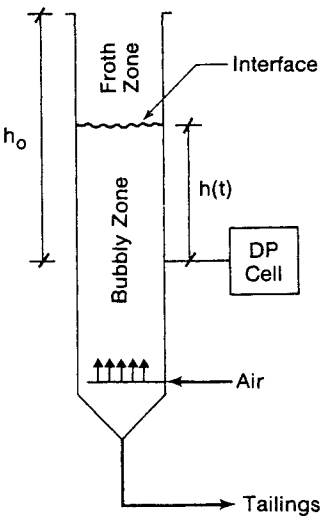

Fig 13 Schematic of the column

$$\frac{h'(z)}{C(z)} = HG_A G_f G_p(z) \quad \ldots (A4)$$

where $HG_A G_f G_p(z)$ is the Pulse Transfer Function (Stephanopoulas, 1984b) which is given by

$$HG_A G_f G_p(z) = Z[H(s) G_A(s) G_f(s) G_p(s)] \quad (A5)$$

Z here indicates z-transformation. From the comparator, one obtains

$$\epsilon(z) = h'(z) - h'_{sp}(z) \quad (A6)$$

and from the digital controller one gets

$$C(z) = D(z)\epsilon(z) = D(z)[h'(z) - h'_{sp}(z)] \quad (A7)$$

Substituting Eqn (A7) into Eqn (A4), we get:

$$h'(z) = \frac{-HG_A G_f G_p(z) D(z)}{1 - HG_A G_f G_p(z) D(z)} h'_{sp}(z) \quad \ldots (A8)$$

The pulse transfer function can be written as:

$$HG_A G_f G_p(z) = Z\left[\left(\frac{1-\exp(-sT)}{s}\right) K_A K_f\left(\frac{-\rho_T}{A(\rho_B - \rho_F)s}\right)\right] \quad \ldots (A9)$$

Upon performing z-transformation on the right side of the above equation, we obtain:

$$HG_A G_f G_p(z) = \frac{-K_A K_f \rho_T}{A(\rho_B - \rho_F)}\left[\frac{Tz^{-1}}{(1-z^{-1})^2} - \frac{T(z^{-1})^2}{(1-z^{-1})^2}\right] \quad \ldots (A10)$$

From the above equation and the expression for $D(z)$ (Eqn (15)), the following equation can be derived:

$$1 - HG_A G_f G_p(z) D(z) = 1 + K_1\left(1 + \frac{\tau_R}{1-z^{-1}}\right)\left(\frac{z^{-1}}{1-z^{-1}}\right) \quad \ldots (A11)$$

where $\tau_R = T/\tau_1$ and $K_1 = K_A K_f K_c \rho_T T/[A(\rho_B - \rho_F)]$.

Thus:

$$h'(z) = h'_{sp}(z) - \frac{(1-z^{-1})^2 h'_{sp}(z)}{1 + (K_1 + K_1\tau_R - 2)z^{-1} + (1-K_1)(z^{-1})^2} \quad \ldots (A12)$$