BLIND CHANNEL ESTIMATION FOR OFDM SYSTEMS BASED ON NON-REDUNDANT LINEAR PRECODING

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ABSTRACT

In this paper we consider a novel approach that can be applied on a single transmit/receive antenna OFDM system, to increase multipath diversity, and at the same time, enable blind channel estimation at the receiver. A non-redundant linear precoder is applied on each pair of blocks before they enter the OFDM system. It is composed of unitary sub-matrices, which spread each symbol onto multiple sub-carriers, thus enabling robustness to subcarrier nulls. Due to the introduced structure, the channel can be estimated at the receiver based on cross-correlation operations, without increasing transmission power.

1. INTRODUCTION

During a wireless transmission, signal pulses are broadened in time as they travel through the multipath channel, thus limiting the speed at which data can be sent without suffering intersymbol interference (ISI). One technique to combat ISI is Orthogonal frequency division multiplexing (OFDM). The information bearing symbols are divided into different streams and each stream is sent over a different sub-carrier. On each sub-carrier, the symbol rate is low in comparison to the channel delay spread, thus ISI can be prevented. OFDM is viewed as the most promising approach for future generation wireless systems, which will support transmission of multimedia information (e.g., video).

A weakness of OFDM systems is lack of multipath diversity. As each symbol is transmitted via a single flat channel, in case that channel is in deep fade, the symbol can be corrupted. Several approaches have been proposed to compensate for the problem, such as coded OFDM [14], and linear precoded OFDM [13], both of which introduce redundancy to the data at the expense of bandwidth efficiency. A linear constellation precoding scheme was developed in [7] to guarantee symbol detectability when channel nulls occur assuming that channel state information was available at the receiver. Multiple-antenna OFDM systems have also been proposed in [6] for increasing multipath diversity.

In OFDM system, each sub-carrier experiences an independent attenuation and phase shift, which needs to be compensated for before signal recovery. An estimation of these parameters can be obtained with pilot/training symbols at the expense of bandwidth [5]. Blind channel estimation methods avoid the use of pilot symbols and thus is well motivated for applications requiring high spectral-efficiency [10, 3, 8, 11]

In this paper we consider a novel approach that can be applied on a single transmit/receive antenna OFDM system, to increase multipath diversity, and at the same time, enable blind channel estimation at the receiver. A non-redundant linear precoder is applied on each pair of blocks before they enter the OFDM system. It is composed of unitary sub-matrices, which spread each symbol onto multiple sub-carriers, thus enabling robustness to subcarrier nulls. At the same time, it introduces some structure to the transmitted blocks, which enables blind channel estimation at the received based on simple cross-correlation operations without increasing transmission power. The approach presented here constitutes an improvement over that of [9], where the concept of linear block-pair precoding was introduced. Unlike the method of [9], it spreads power among symbols evenly. Also, the precoding matrix used here is better conditioned than that of [9], resulting in less amplification of noise during equalization.

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2. THE PROPOSED BLOCK PRECODING

Let $d_{n,i},\ n = 0, ..., N-1$ denote the information bearing symbols of the i-th block. Assume that $d_{n,i}$ are zero-mean i.i.d., temporally white with unit variance, and spatially uncorrelated. And N is the number of subcarriers for the single user OFDM system. The precoder, applied before the OFDM system, maps every two consecutive blocks to a pair of coded blocks by a linear transformation,i.e.,

$$\begin{bmatrix} \mathbf{s}_{2i} \\ \mathbf{s}_{2i+1} \end{bmatrix} \triangleq \mathbf{W} \begin{bmatrix} \mathbf{d}_{2i} \\ \mathbf{d}_{2i+1} \end{bmatrix}; \tag{1}$$

where

$$\mathbf{W} = \frac{1}{\sqrt{\alpha^2+1}} \begin{bmatrix} \mathbf{W}_1 & \alpha\mathbf{W}_1 \\ \alpha\mathbf{W}_2 & j\mathbf{W}_2 \end{bmatrix} \tag{2}$$

α is a predefined real number assumed to be known to the transmitter and receiver, and $j = \sqrt{-1}$. To keep the power constant, it should hold:

$$\mathbf{W}_1\mathbf{W}_1^H = \mathbf{I}_N, \quad \mathbf{W}_2\mathbf{W}_2^H = \mathbf{I}_N \tag{3}$$

The transformed symbols blocks $\mathbf{s}_i,\ i = 0, 1, \cdots$ are fed into conventional OFDM system. Let $h_k,\ k = 0, ..., L-1$ be the channel impulse response, with L being the channel length. The channel frequency response matrix is defined as: $\mathbf{H} = diag[H_0, H_1, \cdots, H_{N-1}]$ with $H_i = \sum_{k=0}^{L-1} h_k e^{-j\frac{2\pi}{N}ki}$. The received noisy block $\mathbf{y}_i$ can be expressed as:

$$\mathbf{y}_i = \mathbf{H}\mathbf{s}_i + \mathbf{v}_i \tag{4}$$

where $\mathbf{v}_i$ is the corresponding $N \times 1$ white Gaussian noise vector with zero mean and autocovariance: $E(\mathbf{v}_i\mathbf{v}_i^H) = \sigma_v^2\mathbf{I}_N$.

At the receiver, minimum mean square error (MMSE) equalizer followed by a minimum distance quantizer can be applied to recover the symbols, i.e.,

$$\hat{\mathbf{s}}_i = \mathbf{G}\mathbf{y}_i \tag{5}$$

and

$$\begin{bmatrix} \hat{\mathbf{d}}_{2i} \\ \hat{\mathbf{d}}_{2i+1} \end{bmatrix} = \frac{\sqrt{\alpha^2+1}}{\alpha^2-j} \begin{bmatrix} -j\mathbf{W}_1^H\hat{\mathbf{s}}_{2i} + \alpha\mathbf{W}_2^H\hat{\mathbf{s}}_{2i+1} \\ \alpha\mathbf{W}_1^H\hat{\mathbf{s}}_{2i} - \mathbf{W}_2^H\hat{\mathbf{s}}_{2i+1} \end{bmatrix} \tag{6}$$

where $\mathbf{G} = \hat{\mathbf{H}}^H(\hat{\mathbf{H}}\hat{\mathbf{H}}^H + \sigma_v^2\mathbf{I})^{-1}$ is equalizer matrix with $\hat{\mathbf{H}}$ as estimated channel gain matrix.

3. BLIND CHANNEL ESTIMATION

Under the assumption that the channel will remain the same over successive symbol blocks, we consider the statistics defined as:

$$\begin{aligned} \mathbf{R} &\triangleq E\{\mathbf{y}_{2i}\mathbf{y}_{2i+1}^H\} = \mathbf{H}E\{\mathbf{s}_{2i}\mathbf{s}_{2i+1}^H\}\mathbf{H}^H \\ &= \mathbf{H}\mathbf{W}_1E\{\bar{\mathbf{d}}_{2i}\bar{\mathbf{d}}_{2i+1}^H\}\mathbf{W}_2^H\mathbf{H}^H \\ &= \frac{\alpha(1-j)}{\alpha^2+1}\mathbf{H}\mathbf{W}_1\mathbf{W}_2^H\mathbf{H}^H = \frac{\alpha(1-j)}{\alpha^2+1}\mathbf{H}\mathbf{P}\mathbf{H}^H \end{aligned} \tag{7}$$

Since $\mathbf{H}$ is diagonal matrix, it holds:

$$\mathbf{R} = \gamma \begin{bmatrix} P_{00}|H_0|^2 & \cdots & P_{0,N-1}H_0H_{N-1}^* \\ \vdots & \ddots & \vdots \\ P_{N-1,0}H_{N-1}H_0^* & \cdots & P_{N-1,N-1}|H_{N-1}|^2 \end{bmatrix} \tag{8}$$

where P_{kj} denotes the (k,j)-th element of matrix $\mathbf{P}$ and $\gamma = \frac{\alpha(1-j)}{\alpha^2+1}$ is a constant. Thus, the channel recovery would depend on the structure of $\mathbf{P}$. For example, if all of $\mathbf{P}$'s j-th column elements are non-zero, an estimate of the channel within the scalar H_j^* can be obtained through entry by entry division of $\mathbf{R}$ and $\mathbf{P}$, i.e.,:

$$\hat{H}_k = \frac{R_{kj}}{\gamma P_{kj}}, \quad k = 0, 1, \cdots, N-1 \tag{9}$$

So, as long as the matrix $\mathbf{P} = \mathbf{W}_1\mathbf{W}_2^H$ has sufficient number of linearly independent non-zero elements, the channel can be estimated up to a complex scalar ambiguity, which could be removed by introducing a pilot subcarrier either before precoding or after precoding procedure. One of the simplest case is to require that $\mathbf{P}$ have at least one non-zero column, in which case the channel can be obtained via (9).

In our scheme, we choose $\mathbf{W}_1$ to be a Hadamard matrix and $\mathbf{W}_2$ to be an identity matrix so that the channel can be estimated by (9) even if the channel has spectal nulls. Since all the entries in the resulting matrix $\mathbf{P}$ are non-zero, every column could be used for estimation. Also, the precoder $\mathbf{W}_1$ would spread each symbols onto multiple subcarries and provide multipath diversity.

4. PERFORMANCE ANALYSIS

Let us consider the equalized output as below:

$$\begin{aligned} \hat{\mathbf{d}}_{2i} &= \frac{1}{\alpha^2-j}(\alpha^2\mathbf{W}_2^H\mathbf{G}\mathbf{H}\mathbf{W}_2 - j\mathbf{W}_1^H\mathbf{G}\mathbf{H}\mathbf{W}_1)\mathbf{d}_{2i} \\ &\quad + \frac{j\alpha}{\alpha^2-j}(\mathbf{W}_2^H\mathbf{G}\mathbf{H}\mathbf{W}_2 - \mathbf{W}_1^H\mathbf{G}\mathbf{H}\mathbf{W}_1)\mathbf{d}_{2i+1} \\ &\quad + \frac{\sqrt{\alpha^2+1}}{\alpha^2-j}(\alpha\mathbf{W}_2^H\mathbf{G}\mathbf{v}_{2i+1} - j\mathbf{W}_1^H\mathbf{G}\mathbf{v}_{2i}) \\ &= \frac{1}{\alpha^2-j}(\alpha^2\mathbf{Q}_2 - j\mathbf{Q}_1)\mathbf{d}_{2i} + \frac{j\alpha}{\alpha^2-j}(\mathbf{Q}_2 - \mathbf{Q}_1)\mathbf{d}_{2i+1} \\ &\quad + \frac{\sqrt{\alpha^2+1}}{\alpha^2-j}(\alpha\mathbf{W}_2^H\mathbf{G}\mathbf{v}_{2i+1} - j\mathbf{W}_1^H\mathbf{G}\mathbf{v}_{2i}) \end{aligned} \tag{10}$$

and

$$\begin{aligned} \hat{\mathbf{d}}_{2i+1} &= \frac{1}{\alpha^2-j}(\alpha^2\mathbf{W}_1^H\mathbf{G}\mathbf{H}\mathbf{W}_1 - j\mathbf{W}_2^H\mathbf{G}\mathbf{H}\mathbf{W}_2)\mathbf{d}_{2i+1} \\ &\quad + \frac{\alpha}{\alpha^2-j}(\mathbf{W}_1^H\mathbf{G}\mathbf{H}\mathbf{W}_1 - \mathbf{W}_2^H\mathbf{G}\mathbf{H}\mathbf{W}_2)\mathbf{d}_{2i} \\ &\quad + \frac{\sqrt{\alpha^2+1}}{\alpha^2-j}(\alpha\mathbf{W}_1^H\mathbf{G}\mathbf{v}_{2i} - \mathbf{W}_2^H\mathbf{G}\mathbf{v}_{2i+1}) \\ &= \frac{1}{\alpha^2-j}(\alpha^2\mathbf{Q}_1 - j\mathbf{Q}_2)\mathbf{d}_{2i+1} + \frac{\alpha}{\alpha^2-j}(\mathbf{Q}_1 - \mathbf{Q}_2)\mathbf{d}_{2i} \\ &\quad + \frac{\sqrt{\alpha^2+1}}{\alpha^2-j}(\alpha\mathbf{W}_1^H\mathbf{G}\mathbf{v}_{2i} - \mathbf{W}_2^H\mathbf{G}\mathbf{v}_{2i+1}) \end{aligned} \tag{11}$$

Define $\mathbf{e}_k$ as $N \times 1$ vector with 1 at the k-th entry and 0 at others, we can express SINR at k-th subcarrier for $2i$-th block as following:

$$SINR_k^{2i} = \frac{E_s^{2i}}{E_n^{2i} + E_i^{2i}} \tag{12}$$

where,

$$\begin{aligned} E_s^{2i} &= E_d|\mathbf{e}_k^H(\alpha^2\mathbf{Q}_2 - j\mathbf{Q}_1)\mathbf{e}_k|^2; \\ E_n^{2i} &= \frac{(\alpha^2+1)^2}{N}\sigma_v^2 tr(\mathbf{G}\mathbf{G}^H); \\ E_i^{2i} &= E_d\sum_{m\neq k}|\mathbf{e}_m^H(\alpha^2\mathbf{Q}_2 - j\mathbf{Q}_1)\mathbf{e}_m|^2 \\ &\quad + \alpha^2E_d\sum|\mathbf{e}_m^H(\mathbf{Q}_2 - \mathbf{Q}_1)\mathbf{e}_m|^2 \end{aligned} \tag{13}$$

The SINR at k-th subcarrier for $(2i+1)$-th block is:

$$SINR_k^{2i+1} = \frac{E_s^{2i+1}}{E_n^{2i+1} + E_i^{2i+1}} \tag{14}$$

where,

$$\begin{aligned} E_s^{2i+1} &= E_d|\mathbf{e}_k^H(\alpha^2\mathbf{Q}_1 - j\mathbf{Q}_2)\mathbf{e}_k|^2; \\ E_n^{2i+1} &= \frac{(\alpha^2+1)^2}{N}\sigma_v^2 tr(\mathbf{G}\mathbf{G}^H); \\ E_i^{2i+1} &= E_d\sum_{m\neq k}|\mathbf{e}_m^H(\alpha^2\mathbf{Q}_1 - j\mathbf{Q}_2)\mathbf{e}_m|^2 \\ &\quad + \alpha^2E_d\sum|\mathbf{e}_m^H(\mathbf{Q}_1 - \mathbf{Q}_2)\mathbf{e}_m|^2 \end{aligned} \tag{15}$$

When perfect channel information is available at the receiver, we have $\hat{\mathbf{H}} = \mathbf{H}$, $\mathbf{Q}_1 = \mathbf{Q}_2 = \mathbf{I}_N$, so

$$SINR_k^{2i} = SINR_k^{2i+1} = \frac{E_d(\alpha^4+1)}{\frac{(\alpha^2+1)^2}{N}\sigma_v^2 tr(\mathbf{G}\mathbf{G}^H)} \tag{16}$$

$SINR_k$ reaches maximum value when $\alpha = 0$ or $\alpha \to \infty$. However, the value of entries in estimation matrix $\mathbf{P}$ would be minimum according to (7), which will increase the channel estimation error. Imperfect channel estimation then result in larger inter-carrier interference due to the fact that $\mathbf{Q}_1$ is non-diagonal, and thus deteriorate the SINR. The choice of α should be a compromise between channel estimation error and SINR.

5. SIMULATION RESULTS

Simulations are carried out according to the IEEE 802.11a standard, which calls for blocks of length $N = 64$, and cyclic prefix of length $N_{cp} = 8$. The simulated channel was a 3-tap ($L = 3$) FIR filter, each tap being independently generated according to Jakes' model [4, Ch 1] using the same parameters but different initial phases. Since the Jakes' model is deterministic for the duration of one realization, we generated different channels by randomly selecting time intervals from the output of the simulator. It is anticipated that the third-generation European cellular standards will operate on trains moving as fast as $500km/h$ [12]. For that case, at 5.2 GHz carrier frequency, 54 Mbps data rate, 64 QAM modulation and 3/4 coding (802.11a standard [1]), the maximum Doppler frequency normalized with the Nyquist frequency would be to $f_d = 2 \times 10^{-4}$. We tested this fast-varying channel case, and also provide performance curves for slow varying channel with $f_d = 10^{-6}$, which corresponds to user speed roughly $2.5km/h$.

The results we obtained based on $M = 50$ Monte Carlo runs, where between runs, independent realizations of noise, input and channel were used. The NMSE is computed according to:

$$NMSE = \frac{1}{M} \sum_{i=1}^{M} \frac{\sum_{k=0}^{N-1} |H_k^i - \hat{H}_k^i|^2}{\sum_{k=0}^{N-1} |H_k^i|^2} \tag{17}$$

where $H_k^i, \hat{H}_k^i$ are the i-th realization of the channel and its estimation, respectively. The received blocks were divided into groups of $2J$ blocks, where $2J$ was selected so that the channel did not change significantly during each $2J$-block segment. For the proposed method, $\hat{\mathbf{H}}$ was obtained based on:

$$\mathbf{R}(k) = \lambda \mathbf{R}(k-1) + (1-\lambda)\mathbf{R}(k), \quad k \geq 2 \tag{18}$$

where

$$\mathbf{R}(k) = \frac{1}{J} \sum_{i=kJ}^{(k+1)J-1} \mathbf{y}_{2i} \mathbf{y}_{2i+1}^H \tag{19}$$

Although no channel length is needed, if available, channel length information could be used to improve the estimation. One method to achieve that, known as denoising [2], is to enforce the channel length L in the time domain, that is, to perform IDFT on $\hat{H}_k$, discard the last $N - L$ samples the IDFT output, and then perform (N-point) DFT on the truncated results.

Moreover, for high SNR, the NMSE can be further reduced for fixed J. This can be achieved by exploiting additional information available in the autocorrelation of the received signal. An additional channel magnitude estimate can be extracted as follows:

$$|\hat{H}_k|^2 = \frac{1}{\sigma_d^2} \mathbf{Z}_{k,k}^i, \; k = 0, ..., N-1 \tag{20}$$

where

$$\mathbf{Z}^i = \frac{1}{2J} \sum_{l=2iJ}^{2(i+1)J-1} \mathbf{y}_l \mathbf{y}_l^H \tag{21}$$

Although $\mathbf{Z}_{k,k}^i$ contains the effect of noise, at high SNR it produces a better magnitude estimate than (9). In out simulations we used the above described autocorrelation based approach to obtain channel magnitude information.

Figure 1(a) presents the NMSE of the channel gain estimation for fast varying channel. Once the channel was estimated, the obtained estimate was used to recover the transmitted symbols via an MMSE equalizer and a minimum distance quantizer. Fig. 1(b) shows the resulting Bit-Error-Rate (BER) as a function of SNR, defined here as E_d/σ_v^2, for 4-QAM, 16-QAM and 64-QAM. Correspondingly, the NMSE and BER for slow varying channel are shown in Fig. 2. Since the cross-correlation matrix is obtained based on finite pair of symbol blocks, i.e., J, for small J, the estimation error will always equal some non-zero value, and therefore the BER would level off at high SNR. The error floor can be lowered by increasing J.

As a benchmark, we also tested the performance of the training-based method used in the 802.11a standard, in which the channel is estimated once only, based on two training OFDM blocks, i.e.,

$$\hat{H}_{tr}(k) = \frac{1}{2} \sum_{i=1}^{2} y_{i,k}/s_{i,k}, \tag{22}$$

The obtained channel estimate is then used to recover a total of 600 OFDM blocks.

It can be seen that the proposed blind approach exhibits better performance as compared to the training-based one for the case of fast varying channels, while it is equivalent to the training-based one for slow varying channels. A better way to see the effect of the diversity introduced by the precoding matrix, maximum likelihood decoder was used for 4-QAM, $N = 4$ and fast varying channel case. The BER result is shown in Fig. 3, where one can see that the proposed method achieves much better performance that the training based one. Unfortunately, ML is too computationally intensive to be used with higher N and constellation orders.

6. CONCLUSIONS

We presented a novel approach for blind channel estimation for OFDM system over frequency selective channel. The proposed method results in good performance, while is bandwidth efficient, maintains transmission power, and is computationally simple. The proposed approach introducing multipath diversity which improves symbol detectability when channel nulls exist on some subcarriers.

7. REFERENCES

[1] IEEE "Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications," *IEEE Standard* 802.11a, Part 11, 1999.

[2] J. van de Beek, O. Edfors, and M. Sandell, S. K. Wilson, and P. O. Bӧjesson, "On channel estimation in OFDM system," in *Proc. Veh. Technol. Conf.*, Chicago, IL, July 1995, vol.2, pp.815-819.

[3] X. Cai and A. N. Akansu, "A subspace method for blind channel identification in OFDM systems," in *Proc. ICC*, New Orleans, LA, Jul. 2000, pp. 929-933.

[4] W. C. Jakes (ed), *Microwave Mobile Communications*,, IEEE Press, New York, 1994.

[5] Y. Li, "Pilot-symbol-aided channel estimation for OFDM in wireless systems," *IEEE Trans. Veh., Technol.*, vol.48, pp.1207-1215, Jul. 2000.

[6] Y. Li, "Simplified channel estimation for OFDM systems with multiple transmit antennas," *IEEE Trans. Wireless Commun.*, vol.1, pp.67-75, Jan. 2002.

[7] Z. Liu, Y. Xin, G. B. Giannakis, "Linear Constellation Precoding for OFDM with Maximum Multipath Diversity and Coding Gains," *IEEE Trans. Commun.*, vol.51, no.3, pp.416-427, Mar. 2003.

[8] B. Lu and X. Wang, "Bayesian blind turbo receiver for coded OFDM systems with frequency offset and frequency-selective fading," *IEEE J. Select. Areas. Commun.*, vol.19, pp.2516 -2527, Dec. 2001.

[9] R. Lin, A.P. Petropulu,"Linear Block Precoding for Blind Channel Estimation in OFDM Systems," *Seventh International Symposium on Signal Processing and Its Applications (ISSPA 2003)*, Paris, France, July 1-4, 2003.

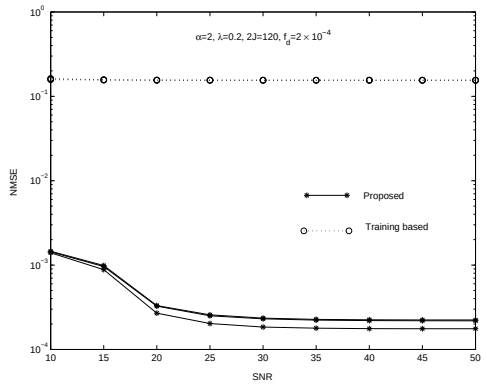
[10] B. Muquet, M. de Courville, P. Duhamel, "Subspace-based blind and semi-blind channel estimation for OFDM systems," in *IEEE Trans. on Signal Processing*, vol. 50 (7), pp. 1699-1712, July 2002.

[11] A. P. Petropulu, R. Zhang and R. Lin, "Blind OFDM Channel Estimation through Simple Linear Precoding,"*IEEE Trans. Wireless Commun.*, to appear in 2003.

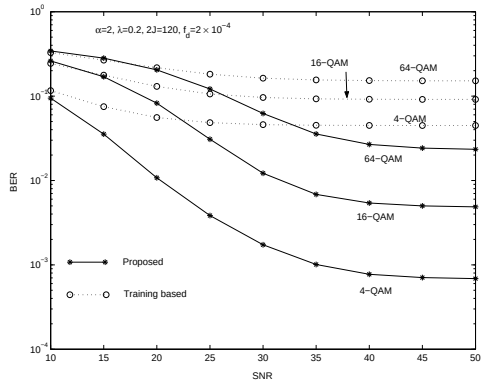
[12] "Frames muliple access proposal for UMTS radio interface-SMG2," *in Wkshp. UMTS Radio Interface Technol.*, Dec. 1996.

[13] Z. Wang and G. B. Giannakis, "Linear precoded or coded OFDM against wireless channel fades?," *proc. of SPAWC*, Taiwan, ROC, pp. 267-270, March 2001.

[14] 'W. Zou and Y. Wu, "COFDM: An overview," *IEEE Trans. on Broadcasting*, vol. 41(1), pp. 1-8, March 1995.

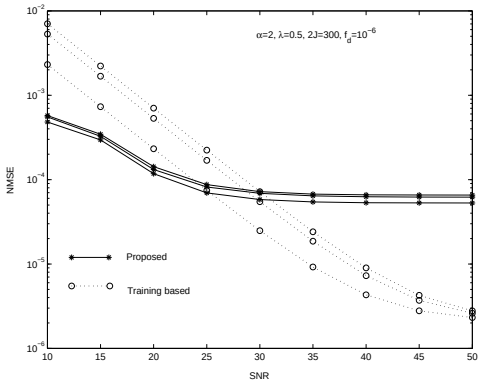


(a)

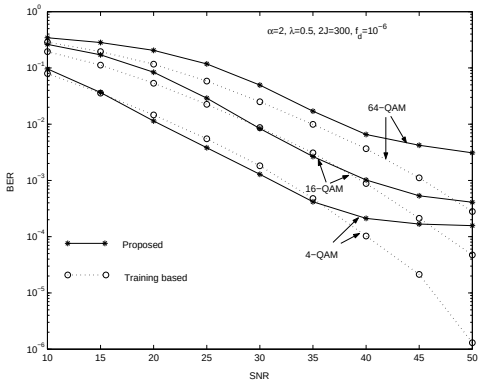


(b)

Fig. 1. Fast varying channel: Proposed vs Training based methods (a) NMSE versus SNR in dB. (b) BER performance.



(a)



(b)

Fig. 2. Slowly varying channel: Proposed vs Training based methods (a) NMSE versus SNR in dB. (b) BER performance.

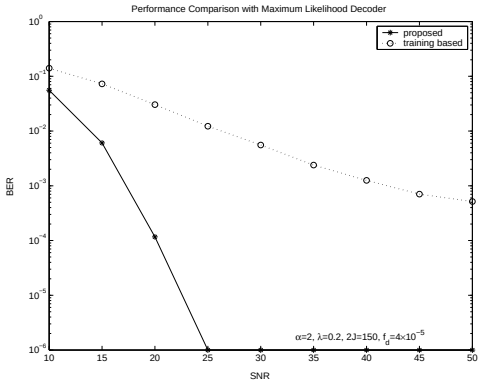


Fig. 3. BER performance of the proposed approach vs. training based methods for maximum likelihood decoder.