Asymptotic Performance of Successive Interference Cancellation in the Context of Linear Precoded OFDM Systems.

M. Debbah[1], *P. Loubaton*[2], *M. de Courville*[3]

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[1] Mobile Communications Group Institut Eurecom 2229 Route des Cretes B.P. 193 06904 sophia antipolis cedex, France - Tel: +33(0)493002686 - Fax: +33(0)493002627-e-mail: debbah@eurecom.fr

[2]Laboratoire système de communication, Université de Marne la Vallée, Cité Descartes, 5 Boulevard Descartes Champs sur Marne 77454 Marne la Vallée, France - Tel: +33 (0)1 60 95 72 93 - Fax:+33 (0)1 60 95 72 14 - e-mail: loubaton@univ-mlv.fr

[3]Centre de Recherche Motorola Paris, Espace Technologique Saint-Aubin 91193 Gif-sur-Yvette France - Tel: +33 (0)1 69 35 25 21 - Fax: +33 (0)1 69.35.25.01 - e-mail: courvill@crm.mot.com

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Abstract

Recent results on the asymptotic empirical eigenvalue distribution of random matrices have enabled the study of the asymptotic limits of Linear Precoded OFDM systems with MMSE equalization. In this contribution, we extend these results to the MMSE Successive Interference Cancellation (MMSE SIC) detector and quantify the non-linearity gain for certain type of precoding matrices.

Keywords

Free Probability, MC-CDMA, Random Spreading, OFDM, MMSE, Spectral Efficiency, Successive Interference Cancellation

I. Introduction

Linear Precoded OFDM (LP-OFDM) systems have drawn a lot of research interest lately. This type of modulation was first proposed in 1993 in the multi-user context known as Multi-Carrier CDMA [16]. In [15], this scheme has been extended to the single user scenario called LP-OFDM, in which, the information is linearly precoded before transmission. This procedure is simply expressed by the fact that a K dimensional transmit information vector is pre-multiplied by a $N \times K$ matrix before transmission on a frequency selective fading channel. In the case of the maximum likelihood detector, previous work have already analyzed the optimum choice of the precoder when $K = N$ [1] or the impact of limited diversity when $K \leq N$ [13]. The analysis has been recently extended to the MMSE receiver by Debbah et al. [2] (in the context of LP-OFDM and downlink MC-CDMA) and Peacock et al. [5] (in the context of uplink MC-CDMA). Based on the fact that the number of carriers is high in OFDM wireless networks, the influence of the precoding matrix and the ratio $\frac{K}{N}$ on the performance of the MMSE receiver have been analyzed. In order to obtain tractable expressions of the SINR, an attractive approach already used in the context of multi-users CDMA systems was proposed which models the precoder as a certain type of random matrix [9], [7]. The study is conducted in the asymptotic regime (by asymptotic, it is understood that the number of symbols K to be sent tends to infinity, the number of carriers N tends to infinity while the ratio remains constant) and it is shown that the SINR converges almost surely to a deterministic term independent of the particular realization of the random precoding matrix. Although simple, the sub-optimality of the MMSE receiver has nevertheless motivated in recent years the search for efficient non-linear detection schemes to increase the spectral efficiency [8] with only polynomial complexity [3]. Unfortunately, the gain achieved by these non-linear detectors is not straightforward to analyze and depends on many factor such as the ratio $\frac{K}{N}$ or the nature of the precoder. In this contribution, due to its simplicity, we derive the performance of a well known non-linear detector, the MMSE SIC receiver (which is known to achieve capacity [10]), and quantify for a given system load the non-linearity gain for two type of random codes: random i.i.d and random isometric (see section II-B for details and justification).

II. LP-OFDM

A. Model

The baseband frequency domain block equivalent model of a LP-OFDM system is depicted in figure .2. The receiver front-end is formed by a symbol-matched filter followed by sampling at the symbol rate. The input symbol stream is serial to parallel converted, then the resulting K-dimensional symbol vector $\mathbf{s} = (s_1, \cdots, s_K)^T$ (a white vector process with

$E\left(\mathbf{s}\mathbf{s}^H\right) = \mathbf{I}_K)$ is multiplied by a $N \times K$ matrix $\mathbf{W}_{N,K}$ where $N \geq K$. This N-dimensional vector $\mathbf{x} = \mathbf{W}_{N,K}\mathbf{s}$ is parallel to serial converted and the corresponding generated data stream is sent across a non selective Rayleigh fading channel. After serial to parallel conversion, due to inter-carrier interference generated by the precoder, the N–dimensional received vector $\mathbf{y} = (y_1, \cdots, y_N)^T$ can be expressed as a function of the emitted symbol vector $\mathbf{s}$:

$$\mathbf{y} \;=\; \mathbf{H}_N \mathbf{W}_{N,K} \mathbf{s} \;+\; \mathbf{n} \tag{1}$$

where $\mathbf{n}$ is a additive white Gaussian noise such that $E\left(\mathbf{n}\mathbf{n}^H\right) = \sigma^2 \mathbf{I}_N$, and where $\mathbf{H}_N = \mathrm{diag}([\mathrm{h}_1, \ldots, \mathrm{h}_\mathrm{N}])$ is the $N \times N$ diagonal complex matrix bearing on its diagonal the channel gains.

In the remainder of this paper, channel knowledge and perfect synchronization at the receiver is assumed. Due to a lack of space, we will only consider the ergodic channel case. We assume fast fading environments in which time and frequency interleaving is performed on the components of $\mathbf{x} = \mathbf{W}_{N,K}\,\mathbf{s}$. The purpose of interleaving is to scramble the precoded data stream in order to provide the coefficients $(h_k)_{1,\ldots N}$ with ergodic properties (we denote by $p(t)$ the probability density of the random variables $(|h_i|^2)_{i=1,\ldots,N}$ and suppose that for each continuous bounded function f : $\mathbb{R} \to \mathbb{R}, \lim_{N\to+\infty} \frac{1}{N}\sum_{k=1}^{N} f(|h_k|^2) = \int f(t)p(t)\,dt$ almost surely). This strategy incurs a delay in order to retrieve all the components of $\mathbf{W}_{N,K}\,\mathbf{s}(n)$ before ≪ de-spreading ≫.

We set $E(|h_i|^2) = 1$ and $E(|s_i|^2) = 1$ so that σ^2 defined by $E(\mathbf{n}\mathbf{n}^H) = \sigma^2 \mathbf{I}_N$ represents the inverse of the SNR at the receiver input. The input SNR is defined as: $\lambda = \frac{1}{\sigma^2}$

B. Types of codes

Two types of codes will be analyzed:

- random i.i.d matrix: coefficients of the precoding matrix are modeled as i.i.d random variables. This choice is justified in order to get interpretable expressions of the SINR. The i.i.d. case study is based on mathematical results related to the "limiting distribution of eigenvalues" of some large random matrices with independent and identically distributed entries (see e.g. [7],[9]). As a particular example and for implementation simplicity sake, the coefficients can be chosen randomly from the set (1,-1).
- random Haar distributed isometric matrix: In general, Walsh-Hadamard codes are used in LP-OFDM schemes but the Haar distribution is introduced for calculus purpose. Moreover, as shown later, these matrices do not incur any performance loss. A $N \times N$ random unitary matrix is said to be Haar distributed if its probability distribution is invariant by right (or equivalently left) multiplication by deterministic unitary matrices. We restrict our study to isometric matrices (obtained by extracting $K < N$ columns from a Haar unitary matrix) since perfect synchronization is ensured between the codes at the transmitter. In this case, new tools, borrowed from the so-called **free probability theory** [11] are used to analyze the SINR formula. One drawback of the isometric codes is that it is impossible to take into account a scenario in which the redundancy factor $\alpha = \lim_{N\to\infty} \frac{K}{N}$ is greater than 1. In order to deal with this case, we consider co-isometric codes. Instead of using codes whose columns are orthogonal, one may use a matrix code such as the rows are orthogonal. In this context, we model the precoding matrix $\mathbf{W}_{N,K}$ as a random matrix obtained first by extracting N rows from a Haar distributed unitary $K \times K$ matrix, and second by multiplying the resulting matrix by the scaling factor $\alpha^{1/2}$. Therefore,

$\mathbf{W}_{N,K}$ satisfies

$$\mathbf{W}_{N,K}\mathbf{W}_{N,K}^{H} = \alpha\mathbf{I}_N \tag{2}$$

The scaling factor α in (2) guarantees that the average power allocated to each component of $\mathbf{s}$ is 1 as is the case when $\alpha < 1$. The case $\alpha > 1$ is only interesting for a comparison basis with the i.i.d case.

III. Successive Interference Cancellation Detector

Results on the performance of the MMSE receiver have been previously derived in [2]. However, the sub-optimality of the MMSE receiver has raised considerable interest [8] for non-linear multiuser detectors. The price for those improvements is of course receiver complexity. Basically, these non-linear methods borrow equalization schemes from the CDMA context [6], [4].

A. MMSE SIC algorithm

The algorithm relies on a sequential detection of the received block [14]. At the first step of the method, a MMSE equalization of matrix $\mathbf{M}_{N,K} = \mathbf{H}_N\mathbf{W}_{N,K}$ is performed. The output of the MMSE detector $\hat{\mathbf{s}} = [\hat{s}_1,\ldots,\hat{s}_K]^T$ is given by $\hat{\mathbf{s}} = \mathbf{W}_{N,K}^H\mathbf{H}_N^H\left(\mathbf{H}_N\mathbf{W}_{N,K}\mathbf{W}_{N,K}^H\mathbf{H}_N^H + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{y}$. Each component $\hat{s}_k$ of $\hat{\mathbf{s}}$ is corrupted by the effect of both the thermal noise and by the "multi-user interference" due to the contributions of the other symbols $\{s_l\}_{l\neq k}$. Denote $\mathbf{w}_k$ the column of $\mathbf{W}_{N,K}$ associated to element s_k, and $\mathbf{U}_{N,K}$ the $N\times(K-1)$ matrix which remains after extracting $\mathbf{w}_k$ from $\mathbf{W}_{N,K}$. Define $\mathbf{A}_{N,K} = \mathbf{H}_N\mathbf{W}_{N,K}\mathbf{W}_{N,K}^H\mathbf{H}_N^H + \sigma^2\mathbf{I}_N$. The component $\hat{s}_k$ after MMSE equalization has the following form: $\hat{s}_k = \eta_{\mathbf{w}_k}s_k + \tau_k$ where $\eta_{\mathbf{w}_k} = \mathbf{w}_k^H\mathbf{H}_N^H\left(\mathbf{A}_{N,K}\right)^{-1}\mathbf{H}_N\mathbf{w}_k$. and $\tau_k = \mathbf{w}_k^H\mathbf{H}_N^H\left(\mathbf{A}_{N,K}\right)^{-1}\mathbf{H}_N\mathbf{W}_{N,K}[s_1,\ldots,s_{k-1},0,s_{k+1},\ldots,s_K]^T + \mathbf{w}_k^H\mathbf{H}_N^H\left(\mathbf{A}_{N,K}\right)^{-1}\mathbf{n}$. It has been shown [17] that the additive noise τ_k can be considered as Gaussian when K and N are large enough. τ_k is therefore an asymptotically zero mean Gaussian noise of variance $V = E(\mid\tau_k\mid^2/\mathbf{H}_N,\mathbf{W}_{N,K}) = \eta_{\mathbf{w}_k}(1-\eta_{\mathbf{w}_k})$.[1]

The SINR $\beta_{\mathbf{w}_k}$ at the output k of the MMSE detector can thus be expressed as:

$$\begin{aligned}\beta_{\mathbf{w}_k} &= \frac{E[\mid\eta_{\mathbf{w}_k}s_k\mid^2/\mathbf{H}_N,\mathbf{W}_{N,K}]}{E[\mid\tau_k\mid^2/\mathbf{H}_N,\mathbf{W}_{N,K}]}\\ &= \frac{\eta_{\mathbf{w}_k}}{1-\eta_{\mathbf{w}_k}}\end{aligned}$$

In the case of i.i.d or isometric spreading, under certain conditions, it has been shown in [2] that when N grows towards infinity and $K/N\to\alpha$, the SINR $\beta_{\mathbf{w}_k}^K$ at the output k of a MMSE equalizer converges almost surely to a value $\beta(\alpha)$. Therefore, all the symbols enjoy asymptotically the same[2] SINR $\beta(\alpha)$. Since there is no optimal choice for detecting the first symbol, suppose that the algorithm starts by decoding symbol s_K. Assuming a perfect decision (this is possible if the information s_K has been encoded at a rate $\log_2(1+\beta_{\mathbf{w}_K}^K)$, the resulting estimated symbol $\hat{s}_K$ is subtracted from the vector of received samples in the following manner: $\mathbf{r}_2 = \mathbf{r}_1 - \hat{s}_K\mathbf{m}_K$ ($\mathbf{m}_i$ represents the i^{th} column of $\mathbf{M}_{N,K}$ and vector $\mathbf{r}_1 = \mathbf{y}$). This introduces one degree of freedom for the next canceling vector choice which enables to reduce the noise plus interference influence and yields an increase in the decision process reliability.

[1]this proof can be found in [12]

[2]In the case of finite systems, the SINR of each symbol is different. Therefore, only the symbol with the best SINR is detected first. There is an optimum ordering in the detection process (depending on the channel attenuations) which makes the analysis extremely difficult for finite dimensions.

The second step can be virtually represented by a completely new system of $K-1$ symbols $(s_1,...,s_{K-1})$ transmitted by an $N\times(K-1)$ isometric precoding matrix $\mathbf{W}_{N,K-1}$ on the same flat frequency fading channel. Equalizing with matrix $\mathbf{G}_2 = \mathbf{M}_{N,K-1}^H(\mathbf{M}_{N,K-1}\mathbf{M}_{N,K-1}^H + \sigma^2\mathbf{I})^{-1}$ where $\mathbf{W}_{N,K-1}$ denotes the matrix obtained by suppressing columns K of $\mathbf{W}_{N,K}$ and $\mathbf{M}_{N,K-1} = \mathbf{H}_N\mathbf{W}_{N,K-1}$, all the estimated symbols enjoy asymptotically an SINR of $\beta^{K-1}_{\mathbf{w}_{K-1}}$. Assuming that the information in s_{K-1} has been encoded at a rate $\log_2(1+\beta^{K-1}_{\mathbf{w}_{K-1}})$, one can reiterate the same process described at the beginning. The advantage of such a scheme is that $\beta^{K}_{\mathbf{w}_K} \leq \beta^{K-1}_{\mathbf{w}_{K-1}}$: one is able therefore to convey much more information on the second symbol (since the SINR increases) than with MMSE equalization.

B. Performance issues

In this section, we will derive the asymptotic SINR $\beta(x)$ for each symbol $k=[xN+1]$ ([] is the integer part operator) during the detection process. We assume hereafter that the detected symbols at each iteration are perfectly retrieved.
For each symbol $s_j, 1\leq j\leq K-l$, the SINR $\beta^{K-l}_{\mathbf{w}_j}$ is easily shown to be $\beta^{K-l}_{\mathbf{w}_j} = \frac{\eta^{K-l}_{\mathbf{w}_j}}{1-\eta^{K-l}_{\mathbf{w}_j}}$ where $\mathbf{w}_j$ is the j^{th} column of matrix $\mathbf{W}_{K-l}$ ($\eta^{K-l}_{\mathbf{w}_j} = \mathbf{w}_j^H\mathbf{H}_N^H\left(\mathbf{H}_N\mathbf{W}_{N,K-l}\mathbf{W}_{N,K-l}^H\mathbf{H}_N^H+\sigma^2\mathbf{I}_N\right)^{-1}\mathbf{H}_N\mathbf{w}_j$) Based on [2], the following result holds:

Proposition 1: Let k be the number of symbols to be detected and N the numbers of carriers. When $N\to\infty$ and $k/N\to x\leq 1$, the SINR $\beta^k_{\mathbf{w_j}}$ $(1\leq j\leq k)$ at the output of the MMSE SIC equalizer with i.i.d and random isometric Haar distributed matrices converges almost surely to a value $\beta(x)$ that is the unique solution of the equation:

- **Isometric case***:*

$$\int_0^\infty \frac{t}{xt+\sigma^2(1-x)\beta(x)+\sigma^2}\,p(t)\,dt = \frac{\beta(x)}{\beta(x)+1}\,. \tag{3}$$

- **i.i.d case***:*

$$\int_0^\infty \frac{t}{xt+\sigma^2\beta(x)+\sigma^2}\,p(t)\,dt = \frac{\beta(x)}{\beta(x)+1}\,. \tag{4}$$

Since $\beta(x)$ is a decreasing function of x, perfect retrieval of the symbols at each step of the algorithm reduces the interference and yields an increase in terms of performance. This is one of the asymptotic justification of the MMSE SIC algorithm. The following theorem will be used in order to derive the asymptotic performance of the MMSE SIC algorithm in terms of spectral efficiency and bit error probability :

Theorem 1: let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function. If $K\to\infty$, $N\to\infty$ and $K/N\to\alpha>0$, then $\frac{1}{K}\sum_{i=1}^K f(\beta^i_{\mathbf{w_i}}) \overset{a.s}{\to} \frac{1}{\alpha}\int_{x=0}^{\alpha} f(\beta(x))dx$ where $\beta^i_{\mathbf{w}_i} = \frac{\eta^i_{\mathbf{w}_i}}{1-\eta^i_{\mathbf{w}_i}}$ with $\eta^i_{\mathbf{w}_i} = \mathbf{w}_i^H\mathbf{H}_N^H\left(\mathbf{H}_N\mathbf{W}_{N,i}\mathbf{W}_{N,i}^H\mathbf{H}_N^H+\sigma^2\mathbf{I}_N\right)^{-1}\mathbf{H}_N\mathbf{w}_i$ and $\beta(x)$ solution of equation (3) or (4).
The proof can be found in the appendix.

BER Considerations: At each step of the SIC algorithm, perfect detection of the symbols is assumed. Even though such an assumption is not justified[3], the performance evaluation gives us an upper bound on the achievable performance. For a fixed K and N, the mean BER(K,N) is: $\frac{1}{N}\sum_{i=1}^K \mathrm{Q}\left(\sqrt{\beta_{\mathbf{w_i}}{}^{\mathrm{i}}}\right)$ (we consider here a QPSK constellation). Theorem 1 can be therefore applied to the continuous function $f:y\to Q(\sqrt{y})$ and yields $\lim_{N\to\infty}\mathrm{BER(K,N)} = \frac{1}{\alpha}\int_0^\alpha \mathrm{Q}\left(\sqrt{\beta(\mathrm{x})}\right)\,\mathrm{dx}$.

Figure (1) shows the BER in the uncoded case respectively for independent Rayleigh channel attenuations . Notice that only the random isometric Haar distributed case is illustrated. The theoretical curves closely match to the simulation

[3]With this assumption, the final step of the detection process achieves the performance of the single user bound while it is well established that due to the propagation of errors, such a result is not reached

results at high SNR, using a realistic number of sub-channels (N=256). At low SNR, a little gap appears due to the fact that uncoded transmissions have been simulated. In this case, the symbols are not retrieved perfectly and yield propagation errors. Indeed, for performing a good interference cancellation, due to the underlying feedback mechanism involved in the successive detection, the MMSE SIC algorithm should decode first the reliable carriers enjoying a greater SINR and then the most corrupted ones. Unfortunately, all the carriers share the same SINR resulting in practice to marginal performance gain when applying successive interference cancellation approaches. The theoretical curve in figure (1) shows also that even when ideal interference cancellation is performed (the best achievable BER), the performance gain in the uncoded case is poor in comparison to the great complexity of the algorithm. Indeed, the performance is mostly affected by the errors incurred at the first stages of the algorithm.

Spectral efficiency Considerations: The spectral efficiency is defined as the highest rate at which information can be sent with arbitrary low probability error with MMSE SIC detection. Our analysis (in which we consider perfect retrieval of the detected symbols) is justified if coding on each carrier k ($k = [xN+1]$) of figure (2) is carried at exactly the spectral efficiency rate $\log_2(1+\beta(x))$. For a fixed K and N, the mean spectral efficiency of re-encoded successive cancellation is: $\gamma(K,N) = \frac{1}{N}\sum_{i=1}^{K}\log_2(1+\beta^i_{\mathbf{w_i}})$. Theorem 1 can be therefore applied to the continuous function $f : y \to \log_2(1+y)$ and $\lim_{N\to\infty}\gamma(K,N) = \int_0^\alpha \log_2(1+\beta(x))\, dx$.

It is well known (see [10]) that for each K and N, $\gamma(K,N) = \frac{1}{N}\sum_{i=1}^{K}\log_2(1+\beta^i_{\mathbf{w_i}})$ actually coincides with the capacity $\frac{1}{N}\log_2(det(I+\frac{\mathbf{H}_N\mathbf{W}_{N,K}\mathbf{W}_{N,K}^H\mathbf{H}_N^H}{\sigma^2}))$. This equality is thus of course verified in the asymptotic regime. Therefore, the following result holds:

Theorem 2: let γ_{SIC} denote the spectral efficiency of the MMSE SIC algorithm and γ_{opt} the spectral efficiency when optimal detection is performed.

$$\gamma_{SIC} = \int_0^\alpha log_2(1+\beta(x))dx$$

and

$$\gamma_{opt} = \lim_{N\to\infty}\frac{1}{N}\log_2\left(det\left(\mathbf{I}_N + \frac{\mathbf{H}_N\mathbf{W}_{N,K}\mathbf{W}_{N,K}^H\mathbf{H}_N^H}{\sigma^2}\right)\right)$$

The following equality holds: $\gamma_{opt} = \gamma_{SIC}$

A remarkable result of theorem 2 in the ergodic case is that the MMSE SIC is able to achieve the single user bound without the knowledge of the channel realization at the transmitter (in order to encode information on each substream) but **only the channel statistics** (since $\beta(x)$ is deterministic). We believe that this is truly an important result as it does not require a channel feedback mechanism but only knowledge of the channel statistics. **Therefore, with time and frequency interleaving, the MMSE SIC is able to achieve optimal performance in limited diversity channels**.

In figure 3, we study the impact of the orthogonality of the precoder's columns with respect to the i.i.d case at 10 dB with i.i.d Rayleigh fading. Important conclusions can be drawn:

- with orthogonal codes, a great improvement with respect to the MMSE receiver is achieved when α is close to 1 (nearly 1.5 bit/s/Hz for $\alpha = 1$). When $0 \leq \alpha \leq 0.5$, there is no need into using such a detector. In this case, the high redundancy of the precoder is able to compensate the sub-optimality of the MMSE detector.
- with i.i.d codes, a surprising result shows that there is no loss in terms of spectral efficiency as long as $\alpha \to \infty$. Therefore, one can use a matrix with i.i.d entries and still achieve the best performance possible. However, since the

complexity of the SIC detector is related the number of inversions K, using i.i.d codes can be prohibitively to complex to use in this case. For $0 \leq \alpha \leq 1$, the SIC detector with i.i.d codes performs similarly as the MMSE detector with orthogonal codes. In this region, orthogonality pays off.

IV. Conclusion

In this contribution, an asymptotic analysis has been derived for the SIC MMSE detector. We show in particular that for loads $0 \leq \alpha \leq 0.5$, the MMSE receiver already achieves optimal performance. Moreover, if time and frequency interleaving is performed, a truly important result shows that the coding rate on each carrier depends **only on the channel statistics**. We also showed that in the uncoded case, SIC approaches yield a very small performance gain in terms of BER.

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APPENDIX

Proof of theorem 1

As we assume in the ergodic case that the probability distribution of the channel fading is compactly supported, it is possible to show that the SINRs for each user at each iteration are uniformly bounded, for all N and K, and for all realization of the precoding matrix. In the non-ergodic case, the channel gains are also bounded for a given channel response. Consequently, is it sufficient to show the result for a function f that is continuous and bounded by a value M.

First, we note $\alpha_N := \frac{K(N)}{N}$, and we define the random piecewise constant function defined on $[0, max(\alpha_N, \alpha)]$ for an event ω:

$$g_N(x) \triangleq \begin{cases} f(SINR(\lfloor xN+1 \rfloor, N)) & \text{if } x \leq \alpha_N \\ 0 & \text{if } x > \alpha_N \end{cases}$$

We can also define for all x, $g(x) \triangleq f(\beta(x))$. Thus we have $\frac{1}{K}\sum_{i=1}^{K} f(SINR(i,N)) = \int_{x=0}^{\alpha_N} g_N(x)dx$

Because of the fact that $\alpha_N \to \alpha$ and f is bounded by a real value M, we have $\int_{\alpha_N}^{\alpha} g_N(x)dx \to 0$. In order to prove the result, it is sufficient to show that a.s., $\int_{x=0}^{\alpha} g_N(x)dx \to \int_{x=0}^{\alpha} g(x)dx$

For a fixed $x \in [0,\alpha]$, $\mathbb{P}[f(SINR(\lfloor xN+1 \rfloor, N)) \nrightarrow f(\beta(x))] = 0$ (consequence of the almost sure convergence of $SINR(\lfloor xN+1 \rfloor, N)$ shown in [2], and of the fact that f is continuous). This result can be written as:$\forall x \in [0,\alpha], \mathbb{E}\left(\mathbf{1}_{\{g_N(x) \nrightarrow g(x)\}}\right) = 0$. It is then possible to consider x as the realization a random variable X, independent from ω, with a uniform density over $[0,\alpha]$; thus we can calculate$\mathbb{E}_x\mathbb{E}\left(\mathbf{1}_{\{g_N(x) \nrightarrow g(x)\}}\right)dx = 0$ The function that is integrated is positive, so Fubini's theorem can apply: $\mathbb{E}\int_{x=0}^{\alpha} \mathbf{1}_{\{g_N(x) \nrightarrow g(x)\}}dx = 0$

Consequently, as the integrated term is positive, $\mathbb{P}_\omega$-a.s., $\mathbb{E}_X \mathbf{1}_{\{g_N(x,\omega) \nrightarrow g(x)\}}dx = 0$. Let us consider one such event ω. We then have $g_N(x,\omega) \to g(x)$ a.s, and as g and g_N are uniformly bounded by a positive value M, $\mathbb{E}_X[g_N(x)] \to \mathbb{E}_X[g(x)]$. Indeed, we have

$$\begin{aligned} |\mathbb{E}_X[g_N(x)] - \mathbb{E}_X[g(x)]| &\leq \mathbb{E}_X[|g_N(x) - g(x)|] \\ &\leq \underbrace{\frac{1}{\alpha}\int_0^{\alpha} \mathbf{1}_{\{g_N(x) \to g(x)\}}|g_N(x) - g(x)|dx}_{\to 0 (\text{dominated convergence})} \\ &\quad + \underbrace{\frac{1}{\alpha}\int_0^{\alpha} \mathbf{1}_{\{g_N(x) \nrightarrow g(x)\}}|g_N(x) - g(x)|dx}_{\leq 2M \times \mathbb{P}_X(g_N(x) \nrightarrow g(x)) = 0} \end{aligned}$$

i.e.

$$\int_{x=0}^{\alpha} g_N(x)dx \to \int_{x=0}^{\alpha} g(x)dx$$

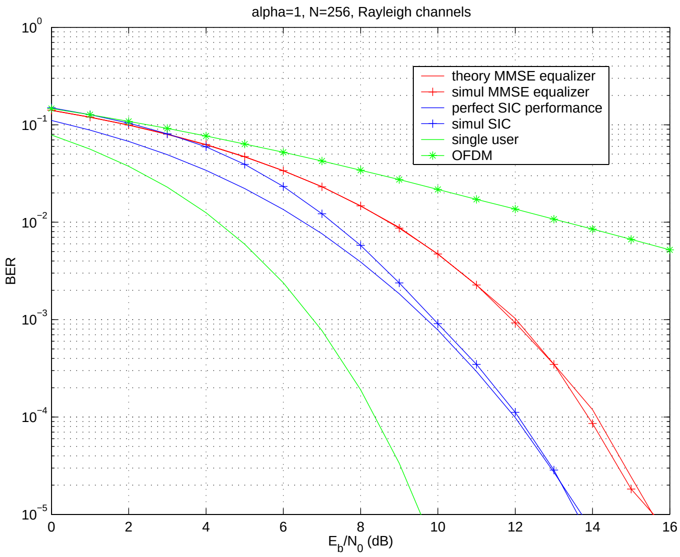

Fig. 1. Performance of SIC MMSE algorithm, Rayleigh channel (ergodic channel).

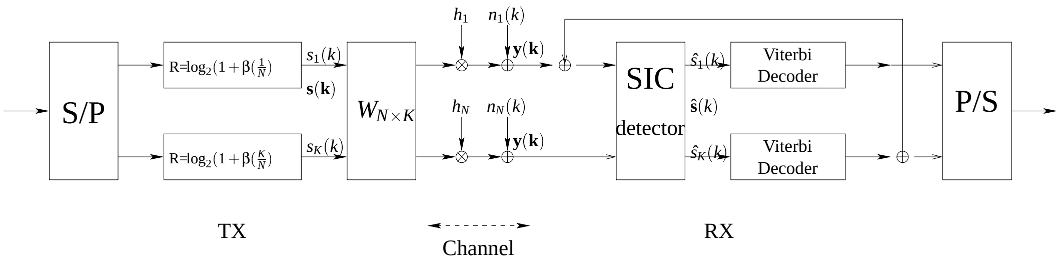

Fig. 2. coded system model.

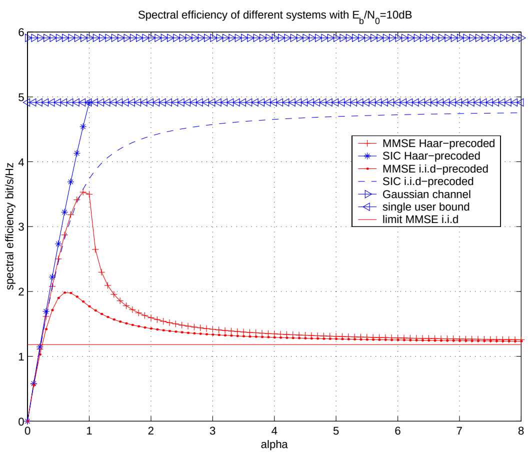

Fig. 3. Spectral efficiency versus α for various detectors at 10dB, Rayleigh channel (ergodic channel).