7: CURRENT RESEARCH IN MATHEMATICS EDUCATION

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Current research in mathematics education can be characterized as large in quantity, poor but improving in quality, and diverse. During the past five years over a thousand studies of mathematics instruction have been reported. The poor quality of most studies can be attributed to too much interest in mathematical components and too little concern for experimental design, measurement, or analysis. The diversity of this immense number of studies was a major problem in preparing this review.

The school level categorization of past *Review* issues was not followed. The intent of the present organization is to highlight certain problem areas that cut across school levels. (For school level reviews see Glennon and Callahan, 1968, and Willoughby, 1969.) For this issue research reports have been categorized into the following eight areas: 1) mathematical learning from an association learning framework; 2) mathematical learning from an activity learning framework; 3) mathematical problem solving and creative behavior; 4) mathematics teaching; 5) the effectiveness of instructional programs; 6) the association of learner characteristics with mathematical achievement; 7) attitudes toward mathematics; and 8) the evaluation and measurement of mathematics achievement.

These eight areas reflect the stimulation of mathematics education by the new courses developed in the early 1960's and the growing interest and involvement of mathematics educators and psychologists in systematic studies of the learning and teaching of mathematics. At the beginning of this decade, the United States was swept with enthusiasm for improving scientific and mathematical training programs. Now as the decade draws to a close, salesmanship has given way to questioning and, in some cases, to careful inquiry. This does not imply that the objectives of the reform movement were failures; there is no question that the content of mathematics courses needed to be updated. However, the educational process is complicated. Changes in content alone were not sufficient to produce drastic changes in mathematical learning. There is considerable agreement about what should be taught, but not about how it should be taught

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(Begle, 1968). The implications of the content changes for children learning in various settings was not and is not clearly understood (see Heath, 1964).

The challenge of the early curriculum reforms to the orthodox views of curricula and learning was significant. The most astute educational scholars quickly saw that the advocated changes were more than an exchange of new concepts for old. "The greatest novelty in the new curricula," as Cronbach (1965, p. 121) pointed out, "is not the content, not the instructional methods, not the grade placement of topics [but] . . . the objective from which all else stems." The essential curricular aim was to have mathematics taught as a discipline—a system of thought as the specialist knows it, which includes the systematic contents of the discipline and its procedures.

The potentially most important avenues of research led to attempts to examine theoretically the learning of mathematics as a discipline. This research, however, did not follow a simple pattern. Researchers followed such leads as the structure of mathematics so convincingly portrayed by Bruner (1960), the hierarchical nature of mathematics championed by Glaser (1968), or the discovery process of learning central to many new curricula and recently criticized carefully by a number of scholars (see Shulman and Keislar, 1966). The categorization scheme chosen for this *Review* is an attempt to organize the recent activities into reasonable areas for summarization. The two categories on mathematical learning were suggested by Bourne (1966). He characterized contemporary trends in psychological learning theories either as associational—those picturing students as recipients of information—or as hypothesis testing—those picturing students as active participants in the process of learning. Hiemer and Kieren summarized the important research in these areas in their chapters.

Kilpatrick summarized research on mathematical problem solving and creativity in his chapter. To many nonmathematicians, mathematics is simply a set of concepts and skills to be acquired or mastered. However, to mathematicians, the ideas of the discipline have always been a framework from which they could create new ideas or new solutions to problems. The last chapter by James Fey is a summary of research on the teaching of mathematics. Due to space limitations these chapters include only the most important studies of the past five years. No chapter can be considered a comprehensive review of an area.

In this chapter trends in research on the learning and teaching of mathematics are discussed following a brief summary of the research in areas 5-8 listed at the beginning of the chapter. Treatment of these four areas is brief because the quality of recent research was not high enough to warrant separate discussion, the area overlapped substantially with other areas, or recent excellent reviews were published elsewhere.

Summary of Studies Related to the Effectiveness of Instructional Programs

The products of the curriculum reform movement made the most obvious contribution to changing educational practices. New texts, materials, instructional procedures, and even organizations for instruction flooded the educational marketplace. However, new materials or procedures of high quality are not insured by their being developed by well-funded scholars. Development is an engineering process designed to produce products. The only way to establish quality is by systematic evaluation.

Unfortunately, the science of curriculum evaluation is in its infancy, and little of the vast resources available in the last decade has been used to nurture it (although some evaluation strategy was followed by most developers). Student performance on some test at the end of instruction was usually employed to assess effectiveness. Neither ongoing effectiveness nor the utility of what was learned was examined systematically; evaluative judgments were often based on a single terminal measure. A stereotype of most evaluations would include a description of the data gathered and a comparison of that data with data from a nonequivalent group.

Descriptive Studies

In the scientifically weakest but perhaps most practical evaluations, innovators such as Davis (1905) and Johntz (1967) admitted the difficulties of systematic determination of effectiveness. Davis (1965) simply described the diversity of classroom activity, the amount of active student participation, and the creative classroom experiences as evidence to support the effectiveness of the third and fourth years of the Madison Project. Johntz (1967) examined the effect of a trained mathematician teaching disadvantaged elementary school children. He expected the scholar to communicate advanced notions of mathematics to the children and to arouse their interest in schoolwork. Observers' comments were used to describe the effectiveness of Project SEED (Special Elementary Education for the Disadvantaged). Thus, evidence other than student achievement can and should be used to examine curricular effectiveness. It is unfortunate that systematic procedures for gathering and reporting this kind of evidence were not used.

Reports of some studies such as Shah's (1969) only contained test results. No attempt was made to relate the results obtained to other data. Shah taught children of ages 7 to 11 a carefully developed set of geometry lessons. He administered a content test and reported generally high scores; the assumption was that without the lessons, low scores could be expected. Another common evaluation procedure is to report only gain scores. Pollack (1967) reported the results of a study on the use of the School Mathematics Study Group (SMSG) set of films and accompanying text to retrain elementary school teachers. Although a variety of instructional procedures was used, generally uniform and large gain scores were reported for scores derived from nominally parallel tests.

If descriptive procedures alone are to be useful, they must be comprehensive. Cronbach (1963) challenged developers to gather descriptive data during development by systematic procedures, subsequently called formative evaluations of the product. Exploratory studies by Coulson (1964) and Romberg and Roweton (1969) reported the use of formative procedures in developing instructional programs. Coulson began with an instructional program on geometric inequalities and used information gathered in tutorial sessions to modify the materials. To make a number of revisions in materials and procedures, Romberg and Roweton periodically gathered data on an instructional program being developed for kindergarten and first grade students.

Comparative Studies

Comparative evaluations are more common than strictly descriptive ones. Typically comparisons are made on data from experimental and control groups. However, true control groups are rare; they are usually alternative treatment groups. Studies by Biddle (1967) and Berger and Howitz (1967) were typical of group comparison studies. Two groups were given the same opportunity to learn a course of study, one group learned in a traditional way and the other in an experimental way. In Biddle's study the course was programed geometry, and in Berger and Howitz's study the course was a discovery program for general mathematics students. Although results of such studies differ on some details, the usual finding is "no significant differences." Occasionally differences between groups are found; Biddle found that, although there were no significant differences in achievement, the "regular" class had better retention.

Some studies in this category compare effects of "modern" and "conventional" programs over longer periods of time. Hungerman (1967) and Grafft and Ruddel (1968) examined students at the end of Grade 6 who had been in either the SMSG curriculum or a conventional program during Grades 4, 5, and 6. Hungerman found that the SMSG students were superior on a contemporary test and the non-SMSG students were superior on a conventional test. Grafft and Ruddel reported large differences favoring the SMSG students on three measures: understanding, thought processes, and a transfer test. No differences were reported on a computation test.

In other studies, such as those by Suppes and Binford (1964) and Herriot (1967), the relative effectiveness of a program in quite different populations of subjects was evaluated. Suppes and Binford concluded that bright fifth graders could learn and use symbolic logic as well as college students. The treatment time was approximately the same for both groups but considerably distributed for the fifth graders. In an exploratory study, Herriot examined the differences between regular seventh and ninth grade students taking a one-year SMSG course and students classified as "slow learners" who were enrolled in two-year courses covering basically the same material. Despite variation in the way in which schools classify students as "slow learners," such students clearly did benefit from the slower pace of instruction, although they failed to achieve as well as the regular students.

Scriven (1967), in replying to Cronbach's challenge to conduct formative evaluations, pointed out the need for summative evaluations. Such procedures call for multivariate comparisons among alternative programs, a sort of consumer report for curricula. The initial reports (School Mathematics Study Group, 1969) from the SMSG-directed National Longitudinal Study of Mathematical Ability (NLSMA) promise to be a prototype for demonstrating comparisons on a large number of measures between several groups defined by the text series used. Reports will be made for Grades 4 through 12. The data from NLSMA promises to be a major stimulus to research in mathematics education, as the SMSG texts have provided stimulation to mathematics instruction in the past decade.

Experimental Studies

In their designs for comparative studies on instructional programs, researchers attempt to control such factors as teachers or procedures. Thus, experimental studies are those in which researchers attempt to account or control for the variations in performance that might be attributable to other factors in the instructional situation. Eriksen and Ryan (1966) controlled for the teacher effect when comparing "modern" and "conventional" programs by having each teacher teach both programs. The most ambitious experimental study was done by Armstrong (1968). She experimentally controlled for teacher, instructional method, curriculum organization, classroom environment, media, and five learner variables. By determining weights associated with each variate in a series of canonical correlations, Armstrong demonstrated that the investigator who uses a global measure to assess experimental effects may obtain no significant differences because of cancelling interaction effects.

Other Studies

Curricula are not the only products of the reform movement in mathematics. Teaching procedures (particularly discovery teaching), manipulation aids, and even organizations for instruction have been produced. Studies of the effectiveness of teaching procedures and manipulative aids are reviewed by Fey and Kieren in this issue. The effectiveness of various organizational procedures for the teaching of mathematics at the elementary level was last reviewed by Weaver (1966), whose summary contained 92 references.

Although results of studies conducted since that time vary, organizational patterns in general appear to make little difference on mathematics achievement scores. However, in most studies both teachers and pupils preferred the alternative organizational patterns to the traditional age-grade self-contained class.

Although it has long been assumed that studying mathematics is useful (if not necessary) for many activities, very few studies have been done to validate this assumption. Not a single study on the utility of mathematics to learning tasks outside mathematics was reported. Grafft and Ruddel (1968) and Twelker (1967) examined the usefulness of mathematics to the later learning of mathematics. In a rather loosely designed comparative study. Grafft and Ruddel found that students who had studied SMSG in grades 4, 5, and 6 were able to learn a new modern topic in mathematics more rapidly and effectively than were students who had been in a traditional program. Twelker carefully designed an experiment on types of teacher-learner interaction in learning by discovery. Although he found no differences in learning a new task which could be attributed to type of prior learning, he demonstrated a methodological procedure that could be used with more contrasting treatments.

Additional References: Concannon (1966); Klinkerman and Bridges (1967); Lewis (1966); Price, Prescott, and Hopkins (1967).

Summary

Studies on the effectiveness of instructional programs, although numerous, were of poor quality. The major reason is that most efforts were expended in developing products. Researchers were primarily interested in the quality of the mathematics or the ingenuity of the technique and not in the changes in behavior produced. It may be that creative temperament of a developer is not compatible with the objectivity needed to evaluate a program or that program quality was affected by poor conceptualization, lack of adequate funds or other resources, inadequate evaluation instruments, or inappropriate use of experimental procedures and inferential statistics.

However, changes in the quality of evaluations seem to be imminent. There appears to be a growing awareness of the necessity for systematic evaluations and the unique problems they present. It is true that such educational spokesmen as Cronbach (1963), Scriven (1967), Stake (1967), and Wittrock (1966) have challenged developers to evaluate and offered them substantive procedural suggestions. However, since these scholars were not involved in developing mathematical materials, they were not heard. Only recently has the mathematics community been listening. Two documents in press, *Disciplined Inquiry for Education* and *Recommendations for Curriculum and Instruction Materials,* should help to improve evaluations. The first, edited by Cronbach and Suppes (See American Educational Research Association, 1969) was prepared by the Committee on Educational Research of the prestigious National Academy of Education. The authors of this report argued that decision-oriented studies (evaluations are one kind of decision-oriented studies) have been unsatisfactory, and they spelled out the unique functions and conditions of such studies. The second document, *Recommendations,* was written by Tyler and Klein (1967) for the Curriculum Evaluation Committee of AERA. This report was designed as a set of guidelines, recommendations, and technical standards for evaluating curricula. It was deliberately modeled after the influential *Standards for Educational and Psychological Tests and Manuals* (Joint Committee of the American Psychological Assoc., American Educational Research Assoc., and National Council on Measurement in Education, 1966).

One can safely predict that when the *Review* again looks at mathematics education a large number of well-conceived systematic evaluations will have been carried out.

Studies Relating Learner Aptitudes and Abilities to Mathematics Learning

The ugly iron curtain described by Stolurow (1965), which divides the psychological area of individual differences from the area of learning, still exists. Stolurow's scholarly review of this problem as it relates to mathematics is the most cogent analysis of the potential contribution the areas can make to each other and to the teaching of mathematics (also see Walbesser, 1965). Unfortunately, this valid analysis appears in an obscure conference report which undoubtedly has not been read by most mathematics educators. The studies in this area are reviewed here in two categories: the identification of mathematical abilities, and the development of mathematical abilities.

Mathematical Abilities

In contrast to the large volume of curricular studies, only a few studies have been done recently on identifying mathematical abilities. The classic studies by the Swedish psychologist Werdelin on mathematical ability (1958) and geometrical ability (1961) are still little known in this country. The promise in NLSMA's title (National Longitudinal Study of Mathematical Abilities) appears to be unfulfilled since all the initial reports (School Mathematics Study Group, 1969) are curricular comparisons.

Correlational techniques have been used to examine some factors affecting ability. Hedley (1968) studied the relationship of personality factors and mathematical ability. She found two canonical factors, one which contrasted computation and clerical skills with avoidance tendencies, femininity, and social service. The second contrasted high general intelligence and low computation with high sensitivity and self-doubt. Westbrook et al. (1965) and Leton and Kim (1966) correlated various intellectual abilities and mathematics achievement. Factor analyses (at grades 4, 5, and 6 in the Westbrook et al. study and at grade 9 in the Leton and Kim study) revealed numerical reasoning and ability to discern verbal meaning to be highly correlated with achievement.

Williams (1965) and Leiderman, Chinn, and Dunkley (1966) described the mathematical abilities of primary-school children. Williams worked with children entering kindergarten and Leiderman et al. studied culturally disadvantaged and advantaged kindergarten and first grade children, but in both investigations the variability of the populations on a variety of tasks related to mathematics was striking. Leiderman et al. found significant differences favoring advantaged over disadvantaged children on four of eleven measures at kindergarten and seven of eleven at grade 1.

A number of investigators continue to study the relationship between mathematics and reading ability. Since mathematics has its own symbolism and syntactics, it requires its own reading skills. Several investigators (e.g., Smith and Heddons, 1964) employed readability formulas to analyze mathematics texts. Others (e.g., Kane and Hater, 1968) tried to adapt standard reading techniques to the readability of mathematical English. Call and Wiggin (1966) demonstrated that a ten-day unit on the reading of mathematics helped students to solve work problems. Surprisingly, Gilmary (1967) found that remedial reading instruction had a positive effect on arithmetical computation achievement.

The Development of Mathematical Abilities

One of the phenomena of the past decade has been the intellectual stimulation and volume of research studies generated by the observations and theories of Jean Piaget. Piaget's vast conceptualization of human cognitive development is of particular import to mathematics educators since most of his observations have been on mathematical tasks (quantification, geometry, spatial relations, logic, etc.).

A review of all the Piagetian studies that have been done is not possible in this chapter. Fortunately, two Canadian scholars recently prepared excellent critical reviews from quite different points of view. Sullivan (1967) attempted to draw the implications of Piagetian theories for educational practice. He concluded that the contributions are more apparent than real because of uncritical extrapolation of Piaget's observations; clearly this is a fault not of Piaget but of his followers. Harrison (1969) carefully reviewed 80 studies and attempted an extrapolation of the findings to mathematics learning and instruction. He expressed optimism about the relevance of much of Piaget's work to instruction.

Additional References: Carroll (1963); Cronbach (1957); Klausmeier et al. (1969); Mehler and Bever (1967, 1968); Nuffield Mathematics Project (1967, 1968); Piaget (1968); Ripple and Rockcastle (1964).

Summary

It seems plausible that many instructional procedures in mathematics could be guided by appropriate utilization of information on cognitive individual differences, but this is not yet the case. As individualization of instruction and computer management become a reality, aptitude and ability data should become extremely useful. Perhaps in the next decade, cracks in the iron curtain will appear.

Studies on Attitudes Toward Mathematics

Although conceptual and methodological problems plague many areas of inquiry, they are particularly acute in the study of attitudes toward mathematics. One of the anticipated outcomes of the modern mathematical programs was that students' attitudes toward mathematics would greatly improve. Although many investigators studied attitudes toward mathematics, evidence to support the claim of improved attitudes is still meager a decade after the new programs were introduced.

Attitude studies have not been fruitful for many reasons. First, attitudes have been operationally defined from scores on paper-and-pencil tests which are beset with insurmountable validity, internal consistency, and score stability problems. A theoretical formulation is needed which conceives of attitudes as a set of moderator variables that affect the subject's response to mathematical situations in observable and predictable ways. Second, most investigators use a single, global measure of attitudes toward mathematics. This is certainly not realistic, since there is probably a set of predispositions or feelings that vary from computation to problem solving, etc. Third, even if researchers knew what attitudes were and could measure them, they would need to identify the procedures that might be used to modify existing attitudes. There is no reason to believe that the new mathematics programs were appropriately designed to modify the underlying feelings of students or teachers.

Despite these problems, considerable work was done in the last decade on attitudes toward mathematics. Aiken (1969) prepared a careful, critical review of 99 of these studies.

Studies Related to Evaluation and Measurement of Mathematics Achievement

It is safe to generalize that, in most mathematics studies conducted during the 1960's, researchers used inappropriate or inadequate measuring devices to assess mathematics achievements. Since there were changes in content, objectives, and even in the purpose of gathering data, new tests were badly needed. However, most investigators continued to use tests developed earlier—tests designed to measure other objectives and constructed to maximize individual differences for counseling purposes rather than to measure the new objectives and to evaluate the effectiveness of instructional programs.

The time lag between changes in instructional programs and the development of appropriate tests is understandable, if unfortunate. Program developers were primarily interested in making the stimulus or content characteristics of mathematics valid, but not in describing the behavioral changes such content would produce. Most mathematicians were satisfied with describing their aims or goals as general declarations of intent; however, it is essential to convert their inspirational phrases into detailed descriptions of behavior that can be accepted as evidence of learning. Considerable work was done recently on specifying behavioral objectives. The belief that global, individual difference tests are inadequate sources of data for making program decisions about groups has led to ideas of mastery learning and criterion-referenced tests rather than comprehensive achievement and norm-referenced tests.

Objectives

The translation of goals into behavioral objectives has long been advocated by evaluators; the curriculum revolution in mathematics made that translation a necessity. Concurrent with the revolution the *Taxonomy of Educational Objectives: Handbook 1, Cognitive Domain,* (Bloom, 1956) was published. This useful classification of objectives proved to be widely followed in mathematics. Wood (1967), in summarizing the recent efforts of many mathematical groups to specify objectives, found that each owed considerable debt to the *Taxonomy*. Such influential mathematical groups as the College Entrance Examination Board (1960), SMSG-NLSMA (Romberg and Wilson, 1968), and the group conducting the International Study of Mathematics Achievement (Husén, 1967) adopted classifications of objectives similar to that outlined in the *Taxonomy*. These classifications have been used to construct more appropriate comprehensive mathematics tests (Epstein, 1968), to critique existing or newly developed tests (Romberg, 1967, 1968), to be one axis of a content by process matrix for the development of tests for specific behaviors (Romberg and Wilson, 1969), and to be a classification device for the development of an item bank (Wood, 1967).

Avital (1968) and Romberg (1967) attempted to validate taxonomic classifications with reference to mathematics learning at Grade 9. Both got high agreement among judges on item categorization and demonstrated a quasi-simplex ordering of the taxonomic categories; however, both investigators had some problem in interpreting achievement data by taxonomic levels. Romberg found that subscales derived by factor analytic techniques failed to have any taxonomic or even content characteristics; instead, they tended to be grouped on a familiar-unfamiliar continuum. This finding was supported by Pruzek (1967); he compared mathematicians' categorizations of items (content areas) from the Scholastic Achievement Test with factorially derived categories and found no similarity between categorizations. Perhaps achievement is most accurately described by the opportunity to learn and not by objectives. Although these studies may cast some doubt on the validity underlying taxonomic categorization, the utility of the taxonomic approach is unquestionable.

A more specific procedure for specifying behavioral objectives, called *task analysis*, has long been advocated by several men (Glaser and Klaus, 1962; Gagné, 1967; Walbesser, 1968). However, with the exception of a few specific units for experimental studies, no comprehensive task analysis has been completed in mathematics. The study by Gagné and Paradise (1961) is a good example of the use of task analysis. Although it was not derived by task analysis, the most comprehensive set of objectives for a particular instrumental program is that associated with the Individually Prescribed Instruction (IPI) mathematics program for the elementary school prepared by the University of Pittsburgh Learning Research and Development Center (1967).

Mastery Learning and Criterion-Referenced Tests

Comprehensive tests given at the end of a course to compare experimental and traditional programs are almost useless for making practical decisions, so critical to development, about whether students have acquired the specific concept or skill taught in a lesson. Rather, precise evidence is needed at the time it is taught; specific information about expectations and not just comparing groups is required. Such learning has been called *mastery learning* (Bloom, 1968), and such tests *criterion-referenced tests* (Glaser and Klaus, 1962). Several studies associated with the IPI program (Cox and Graham, 1967; Cox and Vargas, 1966; Wang and Lindvall, 1969) were investigations of the utility of CRT's for mastery learning as well as of some of the methodological problems (like item selection) associated with the procedure. One of the problems with CRT's, namely appropriate sampling of test items from a well-specified domain, was examined by Hively, Patterson, and Page (1969). The area of mastery learning and criterion-referenced tests is just now becoming of interest to many measurement specialists. In the next few years substantial work will undoubtedly be done on this area. Additionally, prospects look bright for the development of achievement tests reflecting changes in mathematics education as those changes are made.

Additional References: Cahen, Romberg, and Zwirner (1968); Gorth (1969); Gray (1966); Heimer (1966); Walbesser and Carter (1969).

Other Areas of Research

Like all categorization schemes, the eight categories listed at the beginning of the chapter will not encompass all studies. In particular, comparative education studies and programatic research, now receiving greater emphasis, do not fit the scheme. The efforts of some organizations—the School Mathematics Study Group, the Institute for Mathematical Studies in the Social Sciences, and the U. S. Office of Education Research and Development Centers at Wisconsin or Pittsburgh—are not adequately described by a simple listing of some of the reports in various categories.

Comparative Education

One of the most widely discussed cross-cultural studies conducted in the last few years was the International Study of Achievement in Mathematics (Husén, 1967). The study is a true comparative educational study, with mathematics achievement as the dependent variable. School children in Australia, Belgium, England, Finland, France, Israel, Japan, the Netherlands, Scotland, Sweden, the United States, and West Germany were included in the study. Data were collected in 1964 by means of a series of multiple-choice tests especially devised for international use. The tests were administered in each country to samples of thirteen-year-olds and pupils finishing the secondary school. Considering that more than 130,000 students, 13,000 teachers, and 5,000 schools from 12 countries were included, this project ranks as a major technical achievement and clearly demonstrates the feasibility of a large-scale international study of educational problems; however, the educational value of the findings is not as clear. The goal of the project was to identify social and educational practices that influence students' achievement in mathematics, information of value for educational planning and curriculum development. Extreme care was taken to include a representative sample of schools from each country. Although the authors recognized that the study was correlational and did not allow causal relationships to be identified, the discussions and conclusions frequently betray an uncritical acceptance of certain causal interpretations. The most interesting finding was that students' opportunity to learn the test materials (as judged by their teachers) correlated very highly (.95) with the between-country differences in mean achievement, although the magnitude of the observed differences in performance among students in different countries was considerable. If the differences were primarily due to the opportunity to learn the ideas included on the test, then the differences among countries are only a function of the differences in the curriculum and not in other types of social or educational practices. Although the results are somewhat clouded by the extreme redundancy in reporting results and the possibly inadequate procedures for analyzing the data, the two-volume report describes a major international accomplishment.

A second but quite different interesting international project is the new series of translations of Russian works on research in mathematics education (Kilpatrick and Wirszup, 1969). The first two of the fifteen volumes which are in preparation are currently available. Volume I includes an introductory paper by N. A. Menchinskaya, "Fifty Years of Soviet Instructional Psychology," which is a fairly recent (1967) general survey of the literature. The books will contain one or more articles under general headings such as the Learning of Mathematical Concepts, the Structure of Mathematical Ability, and Problem Solving in Geometry. These translations should provide valuable information to researchers about significant work being done in a different culture.

Trends in Research in Mathematics Education

Speculations about what will happen in the future are always difficult. However, by projecting current work and considering the pronouncements and concerns of leaders in the field, certain trends become apparent. First, the largest percentage of recent studies are Ph.D. dissertations, and the number of these being done each year is increasing. In 1964, 75 doctoral dissertations in mathematics education were reported in *Dissertation Abstracts* (Summers, 1967); Mangrum and Morris (1969) found 97 theses in 1966; and Weaver (1969) reported 157 theses for 1968—an increase over 1964 of more than 100%. Two of the serious problems in the field are that few of the theses have been published and that few researchers have done work beyond their doctoral study. Mathematics education can be a mature field only if its research is widely disseminated and investigators continue to do research. Theses provide most students with their first chance to do research. The quality of such research is often not high, but doing a thesis is a learning experience. The most talented students should continue to do research. Unfortunately, the types of position available to people in mathematics education militate against continued research. Teacher preparation, curriculum development, and service to local schools are rewarded, but research is not. Fortunately, this situation is beginning to change. Leaders in the field are pointing to the growing awareness of the need for research. For example, three recent conferences dealt specifically with needed research. Fehr (1966) summarized a conference held in 1965 which dealt with unsolved problems and with the methods, resources, and means to investigate these problems. The National Council of Teachers of Mathematics and the University of Georgia (Hooton, 1967) sponsored a more ambitious conference dealing specifically with learning, teaching, and curricular problems. Long (1968) summarized a conference held by mathematicians at Cornell University who dealt primarily with the question of what should constitute a doctoral thesis in mathematics education for people majoring in mathematics. Outside of such conferences, a number of influential members of the mathematics education community have spoken out rather strongly on the need for research (E. G. Begle, 1968; Patrick Suppes, 1967; and Joseph Scandura, 1967).

There also were efforts to coordinate work in the field. The National Council of Teachers of Mathematics (NCTM) Research Advisory Committee sponsored a research monograph, cosponsored the Georgia Conference on Needed Research in Mathematics Education, established research sections at its annual meetings, and most recently sponsored a new journal designed specifically for researchers in the fields of mathematical education (see below). Within the AERA, a Special Interest Group in Mathematics Education was formed in 1968 and sponsored paper reading sessions and a symposium at the 1969 AERA Annual Meeting.

One outcome of the recent interest in research in mathematics education is the appearance of new publications for research reporting. In the past *The Mathematics Teacher* and *The Arithmetic Teacher*, prepared primarily for classroom teachers by the NCTM, published the largest percentage of articles dealing specifically with mathematics education. The first issue of NCTM's *Journal for Research in Mathematics Education* (David C. Johnson, editor), mentioned above, appeared in the Fall of 1969. *Investigations in Mathematics Education* from SMSG, edited by J. Fred Weaver, will provide abstracts of major published research on mathematics education in the United States. *Educational Studies in Mathe-*

matics, edited by Henry Freudenthal, is a new international journal published in Holland for educational leaders rather than teachers. The *Journal of Structural Learning*, first published in June 1969, is not solely for mathematics education, although its editor, Z. P. Dienes, and many of its editorial staff are members of the mathematics community. By giving more visibility to research results, these new publications should help to keep mathematicians in the research field.

Four trends in the progress of mathematics education research are apparent. First, there will be more programatic research. Concerted efforts will be made by researchers working in a setting which provides both time and resources for investigating problems from a long-range perspective. Second, there will be more and better basic research. Much programatic research will deal with basic problems about human acquisition of concepts and skills. Third, as stated earlier there will be better curriculum evaluations. Fourth, there will be better tests developed for use in mathematics education.

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